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Applied and Numerical Harmonic Analysis

Carlos Cabrelli
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Editors

Recent Developments in Real and Harmonic Analysis

In Honor of Carlos Segovia

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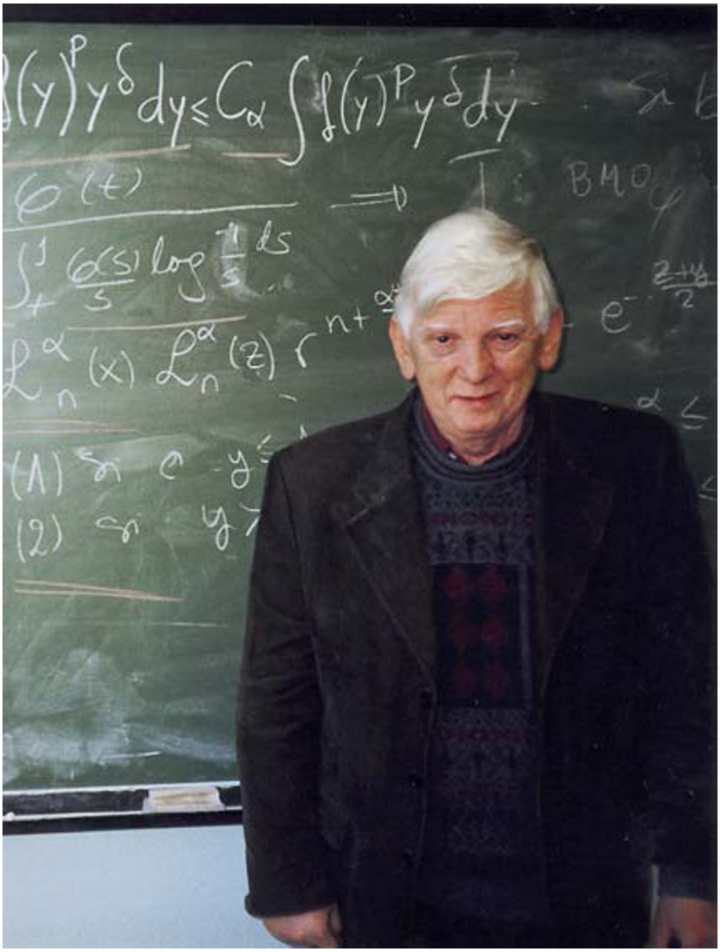
A nuestro querido Carlos Segovia



Segovia during a visit to the Math Department at the Universidad Autónoma de Madrid in December 2004.

ANHA Series Preface

The *Applied and Numerical Harmonic Analysis (ANHA)* book series aims to provide the engineering, mathematical, and scientific communities with significant developments in harmonic analysis, ranging from abstract harmonic analysis to basic applications. The title of the series reflects the importance of applications and numerical implementation, but richness and relevance of applications and implementation depend fundamentally on the structure and depth of theoretical underpinnings. Thus, from our point of view, the interleaving of theory and applications and their creative symbiotic evolution is axiomatic.

Harmonic analysis is a wellspring of ideas and applicability that has flourished, developed, and deepened over time within many disciplines and by means of creative cross-fertilization with diverse areas. The intricate and fundamental relationship between harmonic analysis and fields such as signal processing, partial differential equations (PDEs), and image processing is reflected in our state-of-the-art *ANHA* series.

Our vision of modern harmonic analysis includes mathematical areas such as wavelet theory, Banach algebras, classical Fourier analysis, time-frequency analysis, and fractal geometry, as well as the diverse topics that impinge on them.

For example, wavelet theory can be considered an appropriate tool to deal with some basic problems in digital signal processing, speech and image processing, geophysics, pattern recognition, biomedical engineering, and turbulence. These areas implement the latest technology from sampling methods on surfaces to fast algorithms and computer vision methods. The underlying mathematics of wavelet theory depends not only on classical Fourier analysis, but also on ideas from abstract harmonic analysis, including von Neumann algebras and the affine group. This leads to a study of the Heisenberg group and its relationship to Gabor systems, and of the metaplectic group for a meaningful interaction of signal decomposition methods. The unifying influence of wavelet theory in the aforementioned topics illustrates the justification for providing a means for centralizing and disseminating information from the broader, but still focused, area of harmonic analysis. This will be a key role of *ANHA*. We intend to publish with the scope and interaction that such a host of issues demands.

Along with our commitment to publish mathematically significant works at the frontiers of harmonic analysis, we have a comparably strong commitment to publish major advances in the following applicable topics in which harmonic analysis plays a substantial role:

Antenna theory	*Prediction theory*
Biomedical signal processing	*Radar applications*
Digital signal processing	*Sampling theory*
Fast algorithms	*Spectral estimation*
Gabor theory and applications	*Speech processing*
Image processing	*Time-frequency and*
Numerical partial differential equations	*time-scale analysis*
	Wavelet theory

The above point of view for the *ANHA* book series is inspired by the history of Fourier analysis itself, whose tentacles reach into so many fields.

In the last two centuries Fourier analysis has had a major impact on the development of mathematics, on the understanding of many engineering and scientific phenomena, and on the solution of some of the most important problems in mathematics and the sciences. Historically, Fourier series were developed in the analysis of some of the classical PDEs of mathematical physics; these series were used to solve such equations. In order to understand Fourier series and the kinds of solutions they could represent, some of the most basic notions of analysis were defined, e.g., the concept of "function." Since the coefficients of Fourier series are integrals, it is no surprise that Riemann integrals were conceived to deal with uniqueness properties of trigonometric series. Cantor's set theory was also developed because of such uniqueness questions.

A basic problem in Fourier analysis is to show how complicated phenomena, such as sound waves, can be described in terms of elementary harmonics. There are two aspects of this problem: first, to find, or even define properly, the harmonics or spectrum of a given phenomenon, e.g., the spectroscopy problem in optics; second, to determine which phenomena can be constructed from given classes of harmonics, as done, for example, by the mechanical synthesizers in tidal analysis.

Fourier analysis is also the natural setting for many other problems in engineering, mathematics, and the sciences. For example, Wiener's Tauberian theorem in Fourier analysis not only characterizes the behavior of the prime numbers, but also provides the proper notion of spectrum for phenomena such as white light; this latter process leads to the Fourier analysis associated with correlation functions in filtering and prediction problems, and these problems, in turn, deal naturally with Hardy spaces in the theory of complex variables.

Nowadays, some of the theory of PDEs has given way to the study of Fourier integral operators. Problems in antenna theory are studied in terms of unimodular trigonometric polynomials. Applications of Fourier analysis abound in signal processing, whether with the fast Fourier transform (FFT), or filter design, or the adaptive modeling inherent in time-frequency-scale methods such as wavelet theory. The coherent states of mathematical physics are translated and modulated Fourier transforms, and these are used, in conjunction with the uncertainty principle, for dealing with signal reconstruction in communications theory. We are back to the raison d'être of the *ANHA* series!

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Foreword

On April 3, 2007, Professor Carlos Segovia passed away in Buenos Aires, Argentina.

He graduated with a degree in Mathematics from the University of Buenos Aires in 1961. In 1967 he received his Ph.D. from the University of Chicago under the direction of Alberto P. Calderón. After two years in a postdoctoral position at Princeton University, he decided to return to Buenos Aires, Argentina. It is there that he spent his professional and political career (except for the period from 1975 to 1979, which he spent at the Universidade Estadual de Campinas, São Paulo, Brazil) at both the University of Buenos Aires and the Instituto Argentino de Matemática (IAM). He held important positions at both institutions: he was president of the University of Buenos Aires during 1982 and the director of the IAM from 1991 to 1998. In 1988 he was chosen to become a member of the Natural and Exact Sciences Academy of Argentina. In 1996 the Third World Academy of Sciences in Trieste (Italy) awarded him the Award in Mathematics.

Professor Segovia's mathematical work falls into the framework of harmonic analysis of the University of Chicago School of the 1960s. He started following in the footsteps of A.P. Calderón and A. Zygmund and continuously extended the subject during the last century. In his Ph.D. thesis, he characterized Hardy spaces of harmonic functions in dimension n by means of square functions. This contribution to the theory of Hardy spaces was very deep and is reflected in the main references of the subject. But without a doubt, his main contribution to mathematics is the sequence of articles, written in collaboration with his colleague and friend Roberto Macías, about spaces of homogeneous type. A space of homogeneous type is a quasi-metric space (without any additional structure) on which one can define a measure which possesses a certain doubling property; precisely, the measure of each ball of radius 2R is controlled by the measure of the ball of radius R (except for a constant that depends on the space). Macías and Segovia developed a satisfactory theory of Hardy spaces and Lipschitz functions in this context. Their ideas not only answered some of the relevant questions about Hardy spaces of that time but also inspired new branches and work in areas of harmonic analysis. “Macías–Segovia” is even today a must-know reference for papers on the subject.

Carlos Segovia was born in Valencia on December 7, 1937. His father was a physician in the army of the Spanish Republic, and his mother’s father came from Ferrol, Galicia. At the end of the Spanish civil war, the family had to leave Spain, and they established themselves in Argentina. Carlos Segovia spent his childhood and youth in Buenos Aires. His Spanish ancestry made the nickname “gallego” unavoidable in Argentina, but his strong “porteño” accent got him called “the Argentinian” in many places in Spain.

In 1988 he was appointed visiting professor by the Universidad Autónoma de Madrid, and from then on he very frequently visited the mathematical department of that university. On his many trips to Spain, he also visited several other universities, with Málaga and Zaragoza being his special favorites. He also was supported by the program for sabbatical leaves of the Education and Science Ministry of Spain and by research grants from the Spanish Ministry of External Affairs. In 1992 he received the title “Catedrático de Universidad” with a contract for five years.

Segovia’s health was relatively weak. In 1999 he suffered a stroke with complications that had him suffering for almost a year, spending several months in the hospital. At some moments he seemed close to death, but each time he survived with very difficult and painful recoveries. The consequences of that period (almost total paralysis of his right side and severe diabetes) did not interfere with his continuing to be the perfect collaborator and a scholarly person of good manners with an exquisite sense of humor. Moreover, he continued his research in mathematics with even stronger vigor; in fact, during his last years he maintained a twofold research agenda. With a group of students in Argentina, he developed a whole theory of “lateral Hardy spaces.” With collaborators from Argentina and Spain, traveling in spite of his physical difficulties at least once a year to Madrid from Buenos Aires, he worked on operators associated with generalized Laplacians. The effort involved in this recent work is, on the one hand, rewarded by the fact that many of his contributions in both lines of research have been published, or will be in the near future, in journals of high level. On the other hand, this effort increases our sadness that he is no longer with us.

Buenos Aires and Madrid
June 2009

Carlos Cabrelli
José Luis Torrea

Preface

From December 12 to 15, 2005, a number of harmonic analysts from all over the world gathered in Buenos Aires, Argentina, for a conference organized to honor Carlos Segovia Fernández on the occasion of his sixty-eighth birthday for "his mathematical contributions and services to the development of mathematics in Argentina."

The conference took place at the Instituto Argentino de Matemática (IAM). The members of the advisory committee were Luis Caffarelli, Cristian Gutiérrez, Carlos Kenig, Roberto Macías, José Luis Torrea, and Richard Wheeden. The local organizing committee members were Gustavo Corach (Chair), Carlos Cabrelli, Eleonor Harboure, Alejandra Maestripieri, and Beatriz Viviani.

Unfortunately, Segovia was not able to attend due to health problems. The conference atmosphere was full of emotion, and many fond memories of Carlos were recalled by the participants.

It was at this meeting that the idea crystallized of writing a mathematical tour of ideas arising around Segovia's work. Unfortunately, one year after the conference, Carlos passed away and could not see this book finished.

The book starts with a chronological description of his mathematical life, entitled "Carlos Segovia Fernández." This comprehensive presentation of his original ideas, and even their evolution, may be a source of inspiration for many mathematicians working in a huge area in the fields of harmonic analysis, functional analysis, and partial differential equations (PDEs). Apart from this contribution, the reader will find in the book two different types of chapters: a group of surveys dealing with Carlos' favorite topics and a group of PDE works written by authors close to him and whose careers were influenced in some way by him.

In the first group of chapters, we find the contribution by Hugo Aimar related to spaces of homogeneous type. Roberto Macías and Carlos Segovia showed that it is always possible to find an equivalent quasi-distance on a given space of homogeneous type whose balls are spaces of homogeneous type. Aimar uses this construction to show a stronger version of the uniform regularity of the balls. Two recurrent topics in the work of Carlos Segovia were commutators and vector-valued analysis, and this pair of topics is the subject of the chapter by Oscar Blasco. He presents part of the work by Segovia related to commutators, and he extends it to a general class of Calderón–Zygmund operators. The words "Hardy, Lipschitz, and BMO" spaces were again recurrent in the work of Segovia. An analysis of the behaviour of the product of a function in some Hardy space with a function in the dual (Lipschitz space) is made in the chapter by Aline Bonami and Justin Feuto. In the last fifteen years Segovia was very interested in applying some of his former ideas in Euclidean harmonic analysis to different Laplacians. He made some contributions to the subject, as can be observed in the publications list included in the present book. Along this line of thought is the chapter by Liliana Forzani, Eleonor Harboure, and Roberto Scotto. They review some aspects of this "harmonic analysis" related to the case of Hermite functions and polynomials. The last Ph.D. students of Segovia were introduced by him to the world of "one-sided" operators, with special attention to weighted inequalities. Francisco Martín-Reyes, Pedro Ortega and Alberto de la Torre survey this subject in their chapter. As the authors say, they try to produce a more or less complete account of the main results and applications of the theory of weights for one-sided operators.

In the second group of chapters, the reader will find the chapter by Luis Caffarelli and Aram Karakhanyan dealing with solutions to the porous media equation in one space dimension. Topics such as travelling fronts, separation of variables, and fundamental solutions are considered. The chapter by Sagun Chanillo and Juan Manfredi considers the problem of the global bound, in the space L^2, of the Hessian of the solution of a certain second-order differential operator in a strictly pseudo-convex pseudo-Hermitian manifold. In the classical case, this global bound can be seen as a "Cordes perturbation method" of the boundedness of the iteration of the Riesz transforms. Well-posedness theory of the initial-value problem for the Kadomtsev–Petviashvili equations is treated in the chapter by Carlos Kenig; a connection with the Korteweg–de Vries equation is also discussed. A survey of recent results on the solutions and applications of the Monge–Ampère equation is written by Cristian Gutiérrez.

We thank all the contributors of this volume for their willingness to collaborate in this tribute to Carlos Segovia and his work.

We are grateful to John Benedetto for inviting us to include our book in his prestigious series *Applied and Numerical Harmonic Analysis*, to Ursula Molter and Michael Shub for their proofreading and helpful comments, and to Tom Grasso and Regina Gorenshteyn from Birkhäuser for their editorial help.

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Publications of Carlos Segovia

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It is an entirely vain endeavor to try to describe in a few pages the mathematical life of Carlos Segovia. The variety and richness of his deep results and proofs would need a whole book in order to put them into their proper context and to see what has been their ulterior influence. However, space limitation has the advantage that this chapter will probably be read by more people than if the exposition was as exhaustive as it deserves. This chapter has been written keeping in mind the idea of reaching more people than just the specialists. It can be thought of as a painting in which only a reduced number of strokes have been made, enough to give the viewer a rough idea of the completed work, but giving him the freedom to choose to finish those parts in which he is more interested.

The order of the exposition is supposed to be chronological when possible. It intends to give a quick presentation of the keystone results of Carlos Segovia; the proofs are omitted and we refer the interested reader to the original papers. The presentation starts with the fundamental period of his Ph.D. studies, which has a crucial influence on the remainder of his work, and it ends with a small exposition of the ideas which Carlos Segovia was dealing with in the very last period of his life. The chapter is divided into the following sections: Square functions, Spaces of homogeneous type, Weighted inequalities, One-sided operators, Vector-valued Fourier analysis, and Harmonic analysis associated with generalized Laplacians.

Carlos Segovia understood the work of a mathematician as a way to relish his lifetime surrounded by friends. In his career he had more than twenty co-authors; all of them had spent long evenings of hard work in which Carlos was struggling very hard with a stubborn result. He is standing in front of the blackboard with a cigarette in one hand and the chalk in the other. Clearly, this presentation is influenced by the author's personal experience and probably it would be different for any of the others co-authors of Professor Segovia. In spite of the space limitations, our intention is to review some of Segovia's cooperation with each of his co-authors.

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We mention that even before engaging himself in doctoral studies Segovia produced original mathematical results. His first co-authors were A. Benedek and R. Panzone, see [4] and [44].

1 Square functions

Let us consider the area function of Lusin given by

$$S_a(F)(x) = \left(\int_{\Gamma_a(x)} |F'(u+is)|^2\, du\, ds\right)^{1/2},$$

where $\Gamma_a(x) = \{(u,s) : |u-x| < as\}$ is the cone in $\mathcal{R}^2_+$ with vertex in the point $(x,0)$ and opening a. In 1965, A. P. Calderón, [6], proved the following characterization of the spaces H^p of analytic functions.

Theorem 1.1 *Let $F(t+is)$ be analytic in $s>0$ and belong to H^p, $0<p<\infty$, that is, $\|F\|_{H^p} = \sup_{s>0}(\int_{-\infty}^{\infty} |F(t+is)|^p\, dt)^{1/p} < \infty$. Then, there exist two positive constants c_1 and c_2, depending on a, and p only, such that*

$$c_1\|F_0\|_{L^p(\mathcal{R})} \le \|S_a(F)\|_{L^p(\mathcal{R})} \le c_2\|F_0\|_{L^p(\mathcal{R})},$$

where $F_0(t) = \lim_{s\to 0} F(t+is)$.

The function S_a is built in three steps: we begin by squaring a quantity, afterwards we integrate, and finally we take the square root. These three steps: square, integration, and square root are the essentials features of a huge family of functions and operators that are known by the generic name of "square functions." These square functions appeared in the 1920s and since then they have been the objects of great interest by people working in probability and harmonic analysis. In the work involving these functions one can find techniques of real and complex variables, functional analysis, probability, and Fourier series. It could be said that they produce a kind of fascination. As an example of their reputation among mathematicians, we quote two instances:

1. B. Bollobás speaking about a result related to some square function, see [5]: "The first result of this paper was proved in January 1975 in order to engage the interest of Professor J.E. Littlewood, who was in hospital at the time"

2. E. M. Stein [78] (monograph about the square functions in the work of A. Zygmund): "A deep concept in mathematics is usually not an idea in its pure form, but rather takes various shapes depending on the uses it is put to. The same is true of square functions. These appear in a variety of forms, and while in spirit are all the same, in actual practice they can be quite different. Thus the metamorphosis of square functions is all important."

Calderón was an expert in the theory of square functions. Theorem 1.1 is a clear example of its use; moreover, he also proved results intrinsic to the