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Applied and Numerical Harmonic Analysis

Advances in Mathematical Finance

Michael C. Fu
Robert A. Jarrow
Ju-Yi J. Yen
Robert J. Elliott
Editors

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Michael C. Fu
Robert H. Smith School of Business
Van Munching Hall
University of Maryland
College Park, MD 20742
USA

Robert A. Jarrow
Johnson Graduate School of Management
451 Sage Hall
Cornell University
Ithaca, NY 14853
USA

Ju-Yi J. Yen
Department of Mathematics
1326 Stevenson Center
Vanderbilt University
Nashville, TN 37240
USA

Robert J. Elliott
Haskayne School of Business
Scurfield Hall
University of Calgary
Calgary, AB T2N 1N4
Canada

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In honor of Dilip B. Madan on the occasion of his 60th birthday

ANHA Series Preface

The *Applied and Numerical Harmonic Analysis (ANHA)* book series aims to provide the engineering, mathematical, and scientific communities with significant developments in harmonic analysis, ranging from abstract harmonic analysis to basic applications. The title of the series reflects the importance of applications and numerical implementation, but richness and relevance of applications and implementation depend fundamentally on the structure and depth of theoretical underpinnings. Thus, from our point of view, the interleaving of theory and applications and their creative symbiotic evolution is axiomatic.

Harmonic analysis is a wellspring of ideas and applicability that has flourished, developed, and deepened over time within many disciplines and by means of creative cross-fertilization with diverse areas. The intricate and fundamental relationship between harmonic analysis and fields such as signal processing, partial differential equations (PDEs), and image processing is reflected in our state-of-the-art *ANHA* series.

Our vision of modern harmonic analysis includes mathematical areas such as wavelet theory, Banach algebras, classical Fourier analysis, time-frequency analysis, and fractal geometry, as well as the diverse topics that impinge on them.

For example, wavelet theory can be considered an appropriate tool to deal with some basic problems in digital signal processing, speech and image processing, geophysics, pattern recognition, biomedical engineering, and turbulence. These areas implement the latest technology from sampling methods on surfaces to fast algorithms and computer vision methods. The underlying mathematics of wavelet theory depends not only on classical Fourier analysis, but also on ideas from abstract harmonic analysis, including von Neumann algebras and the affine group. This leads to a study of the Heisenberg group and its relationship to Gabor systems, and of the metaplectic group for a meaningful interaction of signal decomposition methods. The unifying influence of wavelet theory in the aforementioned topics illustrates the justification for providing a means for centralizing and disseminating information from the broader, but still focused, area of harmonic analysis. This will be a key role of *ANHA*. We intend to publish with the scope and interaction that such a host of issues demands.

Along with our commitment to publish mathematically significant works at the frontiers of harmonic analysis, we have a comparably strong commitment to publish major advances in the following applicable topics in which harmonic analysis plays a substantial role:

Antenna theory	*Prediction theory*
Biomedical signal processing	*Radar applications*
Digital signal processing	*Sampling theory*
Fast algorithms	*Spectral estimation*
Gabor theory and applications	*Speech processing*
Image processing	*Time-frequency and*
Numerical partial differential equations	*time-scale analysis*
	Wavelet theory

The above point of view for the *ANHA* book series is inspired by the history of Fourier analysis itself, whose tentacles reach into so many fields.

In the last two centuries Fourier analysis has had a major impact on the development of mathematics, on the understanding of many engineering and scientific phenomena, and on the solution of some of the most important problems in mathematics and the sciences. Historically, Fourier series were developed in the analysis of some of the classical PDEs of mathematical physics; these series were used to solve such equations. In order to understand Fourier series and the kinds of solutions they could represent, some of the most basic notions of analysis were defined, e.g., the concept of "function." Since the coefficients of Fourier series are integrals, it is no surprise that Riemann integrals were conceived to deal with uniqueness properties of trigonometric series. Cantor's set theory was also developed because of such uniqueness questions.

A basic problem in Fourier analysis is to show how complicated phenomena, such as sound waves, can be described in terms of elementary harmonics. There are two aspects of this problem: first, to find, or even define properly, the harmonics or spectrum of a given phenomenon, e.g., the spectroscopy problem in optics; second, to determine which phenomena can be constructed from given classes of harmonics, as done, for example, by the mechanical synthesizers in tidal analysis.

Fourier analysis is also the natural setting for many other problems in engineering, mathematics, and the sciences. For example, Wiener's Tauberian theorem in Fourier analysis not only characterizes the behavior of the prime numbers, but also provides the proper notion of spectrum for phenomena such as white light; this latter process leads to the Fourier analysis associated with correlation functions in filtering and prediction problems, and these problems, in turn, deal naturally with Hardy spaces in the theory of complex variables.

Nowadays, some of the theory of PDEs has given way to the study of Fourier integral operators. Problems in antenna theory are studied in terms of unimodular trigonometric polynomials. Applications of Fourier analysis abound in signal processing, whether with the fast Fourier transform (FFT), or filter design, or the adaptive modeling inherent in time-frequency-scale methods such as wavelet theory. The coherent states of mathematical physics are translated and modulated Fourier transforms, and these are used, in conjunction with the uncertainty principle, for dealing with signal reconstruction in communications theory. We are back to the raison d'être of the *ANHA* series!

John J. Benedetto
Series Editor
University of Maryland
College Park

Preface

The "Mathematical Finance Conference in Honor of the 60th Birthday of Dilip B. Madan" was held at the Norbert Wiener Center of the University of Maryland, College Park, from September 29 – October 1, 2006, and this volume is a Festschrift in honor of Dilip that includes articles from most of the conference's speakers. Among his former students contributing to this volume are Ju-Yi Yen as one of the co-editors, along with Ali Hirsa and Xing Jin as co-authors of three of the articles.

Dilip Balkrishna Madan was born on December 12, 1946, in Washington, DC, but was raised in Bombay, India, and received his bachelor's degree in Commerce at the University of Bombay. He received two Ph.D.s at the University of Maryland, one in economics and the other in pure mathematics. What is all the more amazing is that prior to entering graduate school he had never had a formal university-level mathematics course! The first section of the book summarizes Dilip's career highlights, including distinguished awards and editorial appointments, followed by his list of publications.

The technical contributions in the book are divided into three parts. The first part deals with stochastic processes used in mathematical finance, primarily the Lévy processes most associated with Dilip, who has been a fervent advocate of this class of processes for addressing the well-known flaws of geometric Brownian motion for asset price modeling. The primary focus is on the Variance-Gamma (VG) process that Dilip and Eugene Seneta introduced to the finance community, and the lead article provides an historical review from the unique vantage point of Dilip's co-author, starting from the initiation of the collaboration at the University of Sydney. Techniques for simulating the Variance-Gamma process are surveyed in the article by Michael Fu, Dilip's longtime colleague at Maryland, moving from a review of basic Monte Carlo simulation for the VG process to more advanced topics in variation reduction and efficient estimation of the "Greeks" such as the option delta. The next two pieces by Marc Yor, a longtime close collaborator and the keynote speaker at the birthday conference, provide some mathematical properties and identities for gamma processes and beta and gamma random variables. The final article in the first part of the volume, written by frequent collaborator Robert Elliott and his co-author John van der Hoek, reviews the theory of fractional Brownian motion in the white noise framework and provides a new approach for deriving the associated Itô-type stochastic calculus formulas.

The second part of the volume treats various aspects of mathematical finance related to asset pricing and the valuation and hedging of derivatives. The article by Bob Jarrow, a longtime collaborator and colleague of Dilip in the mathematical finance community, provides a tutorial on zero volatility spreads and option adjusted spreads for fixed income securities – specifically bonds with embedded options – using the framework of the Heath-Jarrow-Morton model for the term structure of interest rates, and highlights the characteristics of zero volatility spreads capturing *both* embedded options and mispricings due to model or market errors, whereas option adjusted spreads measure only the mispricings. The phenomenon of market bubbles is addressed in the piece by Bob Jarrow, Phillip Protter, and Kazuhiro Shimbo, who provide new results on characterizing asset price bubbles in terms of their martingale properties under the standard no-arbitrage complete market framework. General equilibrium asset pricing models in incomplete markets that result from taxation and transaction costs are treated in the article by Xing Jin – who received his Ph.D. from Maryland's Business School co-supervised by Dilip – and Frank Milne – one of Dilip's early collaborators on the VG model. Recent work on applying Lévy processes to interest rate modeling, with a focus on real-world calibration issues, is reviewed in the article by Wolfgang Kluge and Ernst Eberlein, who nominated Dilip for the prestigious Humboldt Research Award in Mathematics. The next two articles, both co-authored by Ali Hirsa, who received his Ph.D. from the math department at Maryland co-supervised by Dilip, focus on derivatives pricing; the sole article in the volume on which Dilip is a co-author, with Massoud Heidari as the other co-author, prices swaptions using the fast Fourier transform under an affine term structure of interest rates incorporating stochastic volatility, whereas the article co-authored by Peter Carr – another of Dilip's most frequent collaborators – derives forward partial integro-differential equations for pricing knock-out call options when the underlying asset price follows a jump-diffusion model. The final article in the second part of the volume is by Hélyette Geman, Dilip's longtime collaborator from France who was responsible for introducing him to Marc Yor, and she treats energy commodity price modeling using real historical data, testing the hypothesis of mean reversion for oil and natural gas prices.

The third part of the volume includes several contributions in one of the most rapidly growing fields in mathematical finance and financial engineering: credit risk. A new class of reduced-form credit risk models that associates default events directly with market information processes driving cash flows is introduced in the piece by Dorje Brody, Lane Hughston, and Andrea Macrina. A generic one-factor Lévy model for pricing collateralized debt obligations that unifies a number of recently proposed one-factor models is presented in the article by Hansjörg Albrecher, Sophie Ladoucette, and Wim Schoutens. An intensity-based default model that prices credit derivatives using utility functions rather than arbitrage-free measures is proposed in the article by Ronnie Sircar and Thaleia Zariphopoulou. Also using the utility-based pricing approach is the final article in the volume by Marek Musiela and Thaleia Zariphopoulou, and they address the integrated portfolio management optimal investment problem in incomplete markets stemming from stochastic factors in the underlying risky securities.

Besides being a distinguished researcher, Dilip is a dear friend, an esteemed colleague, and a caring mentor and teacher. During his professional career, Dilip was one of the early pioneers in mathematical finance, so it is only fitting that the title of this Festschrift documents his past and continuing love for the field that he helped develop.

Michael Fu
Bob Jarrow
Ju-Yi Yen
Robert Elliott
December 2006

Conference poster (designed by Jonathan Sears).

Photo Highlights (September 29, 2006)

Dilip delivering his lecture.

Dilip with many of his Ph.D. students.

Norbert Wiener Center director John Benedetto and Robert Elliott.

Left to right: CGMY (Carr, Geman, Madan, Yor).

VG inventors (Dilip and Eugene Seneta) with the Madan family.

Dilip's wife Vimla cutting the birthday cake.

Career Highlights and List of Publications

Dilip B. Madan

Robert H. Smith School of Business
Department of Finance
University of Maryland
College Park, MD 20742, USA
dmadan@rhsmith.umd.edu

Career Highlights

1971 Ph.D. Economics, University of Maryland
1975 Ph.D. Mathematics, University of Maryland

2006 recipient of Humboldt Research Award in Mathematics
President of Bachelier Finance Society 2002–2003
Managing Editor of *Mathematic Finance, Review of Derivatives Research*
Series Editor on Financial Mathematics for CRC, Chapman and Hall
Associate Editor for *Quantitative Finance, Journal of Credit Risk*

1971–1975: Assistant Professor of Economics, University of Maryland
1976–1979: Lecturer in Economic Statistics, University of Sydney
1980–1988: Senior Lecturer in Econometrics, University of Sydney
1981–1982: Acting Head, Department of Econometrics, Sydney
1989–1992: Assistant Professor of Finance, University of Maryland
1992–1997: Associate Professor of Finance, University of Maryland
1997–present: Professor of Finance, University of Maryland

Visiting Positions:
La Trobe University, Cambridge University (Isaac Newton Institute), Cornell University, University Paris,VI, University of Paris IX at Dauphine

Consulting:
Morgan Stanley, Bloomberg, Wachovia Securities, Caspian Capital, FDIC

Publications (as of December 2006 (60th birthday))

1. The relevance of a probabilistic form of invertibility. *Biometrika*, 67(3):704–5, 1980 (with G. Babich).
2. Monotone and 1-1 sets. *Journal of the Australian Mathematical Society, Series A*, 33:62–75, 1982 (with R.W. Robinson).
3. Resurrecting the discounted cash equivalent flow. *Abacus*, 18-1:83–90, 1982.
4. Differentiating a determinant. *The American Statistician*, 36(3):178–179, 1982.
5. Measures of risk aversion with many commodities. *Economics Letters*, 11:93–100, 1983.
6. Inconsistent theories as scientific objectives. *Journal of the Philosophy of Science*, 50(3):453–470, 1983.
7. Testing for random pairing. *Journal of the American Statistical Association*, 78(382):332–336, 1983 (with Piet de Jong and Malcolm Greig).
8. Compound Poisson models for economic variable movements. *Sankhya Series B*, 46(2):174–187, 1984 (with E. Seneta).
9. The measurement of capital utilization rates. *Communications in Statistics: Theory and Methods*, A14(6):1301–1314, 1985.
10. Project evaluations and accounting income forecasts. *Abacus*, 21(2):197–202, 1985.
11. Utility correlations in probabilistic choice modeling. *Economics Letters*, 20:241–245, 1986.
12. Mode choice for urban travelers in Sydney. *Proceedings of the 13th ARRB and 5th REAAA Conference*, 13(8):52–62, 1986 (with R. Groenhout and M. Ranjbar).
13. Simulation of estimates using the empirical characteristic function. *International Statistical Review*, 55(2):153–161, 1987 (with E. Seneta).
14. Chebyshev polynomial approximations for characteristic function estimation. *Journal of the Royal Statistical Society, Series B*, 49(2):163–169, 1987 (with E. Seneta).
15. Modeling Sydney work trip travel mode choices. *Journal of Transportation Economics and Policy*, XXI(2):135–150, 1987 (with R. Groenhout).
16. Optimal duration and speed in the long run. *Review of Economic Studies*, 54a(4a):695–700, 1987.
17. Decision theory with complex uncertainties. *Synthese*, 75:25–44, 1988 (with J.C. Owings).
18. Risk measurement in semimartingale models with multiple consumption goods. *Journal of Economic Theory*, 44(2):398–412, 1988.
19. Stochastic stability in a rational expectations model of a small open economy. *Economica*, 56(221):97–108, 1989 (with E. Kiernan).
20. Dynamic factor demands with some immediately productive quasi fixed factors. *Journal of Econometrics*, 42:275–283, 1989 (with I. Prucha).

21. Characteristic function estimation using maximum likelihood on transformed variables. *Journal of the Royal Statistical Society, Series B*, 51(2):281–285, 1989 (with E. Seneta).
22. The multinomial option pricing model and its Brownian and Poisson limits. *Review of Financial Studies*, 2(2):251–265, 1989 (with F. Milne and H. Shefrin).
23. On the monotonicity of the labour-capital ratio in Sraffa's model. *Journal of Economics*, 51(1):101–107, 1989 (with E. Seneta).
24. The Variance-Gamma (V.G.) model for share market returns. *Journal of Business*, 63(4):511–52,1990 (with E. Seneta).
25. Design and marketing of financial products. *Review of Financial Studies*, 4(2):361–384, 1991 (with B. Soubra).
26. A characterization of complete security markets on a Brownian filtration. *Mathematical Finance*, 1(3):31–43, 1991 (with R.A. Jarrow).
27. Option pricing with VG Martingale components. *Mathematical Finance*, 1(4):39–56, 1991 (with F. Milne).
28. Informational content in interest rate term structures. *Review of Economics and Statistics*, 75(4):695–699, 1993 (with R.O. Edmister).
29. Diffusion coefficient estimation and asset pricing when risk premia and sensitivities are time varying. *Mathematical Finance*, 3(2):85–99, 1993 (with M. Chesney, R.J. Elliott, and H. Yang).
30. Contingent claims valued and hedged by pricing and investing in a basis. *Mathematical Finance*, 4(3):223–245, 1994 (with F. Milne).
31. Option pricing using the term structure of interest rates to hedge systematic discontinuities in asset returns. *Mathematical Finance*, 5(4):311–336, 1995 (with R.A. Jarrow).
32. Approaches to the solution of stochastic intertemporal consumption models. *Australian Economic Papers*, 34:86–103, 1995 (with R.J. Cooper and K. McLaren).
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34. Filtering derivative security valuations from market prices. *Mathematics of Derivative Securities*, eds. M.A.H. Dempster and S.R. Pliska, Cambridge University Press, 1997 (with R.J. Elliott and C. Lahaie)
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38. The discrete time equivalent martingale measure. *Mathematical Finance*, 8(2):127–152, 1998 (with R.J. Elliott).
39. The variance gamma process and option pricing. *European Finance Review*, 2:79–105, 1998 (with P. Carr and E. Chang).

40. Towards a theory of volatility trading. *Volatility*, ed. R.A. Jarrow, Risk Books, 417–427, 1998 (with P. Carr).
41. Valuing and hedging contingent claims on semimartingales. *Finance and Stochastics*, 3:111–134, 1999 (with R.A. Jarrow).
42. The second fundamental theorem of asset pricing theory. *Mathematical Finance*, 9(3):255–273, 1999 (with R.A. Jarrow and X. Jin).
43. Pricing continuous time Asian options: A comparison of Monte Carlo and Laplace transform inversion methods. *Journal of Computational Finance*, 2:49–74, 1999 (with M.C. Fu and T. Wang).
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45. Option valuation using the fast Fourier transform. *Journal of Computational Finance*, 2:61–73, 1999.
46. Spanning and derivative security valuation. *Journal of Financial Economics*, 55:205–238, 2000 (with G. Bakshi).
47. A two factor hazard rate model for pricing risky debt and the term structure of credit spreads. *Journal of Financial and Quantitative Analysis*, 35:43–65, 2000 (with H. Unal).
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49. Investing in skews. *Journal of Risk Finance*, 2(1):10–18, 2000 (with G. McPhail).
50. Going with the flow. *Risk*, 85–89, August 2000 (with P. Carr and A. Lipton).
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53. Optimal positioning in derivatives. *Quantitative Finance*, 1(1):19–37, 2001 (with P. Carr).
54. Pricing and hedging in incomplete markets. *Journal of Financial Economics*, 62:131–167, 2001 (with P. Carr and H. Geman).
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69. The effect of model risk on the valuation of barrier options. *Journal of Risk Finance*, 4:47–55, 2003 (with G. Courtadon and A. Hirsa).
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77. Pricing options on realized variance. *Finance and Stochastics*, 9:453–475, 2005 (with P. Carr, H. Geman, and M. Yor).
78. A note on sufficient conditions for no arbitrage. *Finance Research Letters*, 2:125–130, 2005 (with P. Carr).
79. Investigating the role of systematic and firm-specific factors in default risk: Lessons from empirically evaluating credit risk models. *Journal of Business*, 79(4):1955–1988, July 2006 (with G. Bakshi and F. Zhang).
80. Credit default and basket default swaps. *Journal of Credit Risk*, 2:67–87, 2006 (with M. Konikov).
81. Itô's integrated formula for strict local martingales. *In Memoriam Paul-André Meyer – Séminaire de Probabilités XXXIX*, eds. M. Émery and M. Yor, Lecture Notes in Mathematics 1874, Springer, 2006 (with M. Yor).
82. Equilibrium asset pricing with non-Gaussian returns and exponential utility. *Quantitative Finance*, 6(6):455–463, 2006.
83. A theory of volatility spreads. *Management Science*, 52(12):1945–56, 2006 (with G. Bakshi).
84. Asset allocation for CARA utility with multivariate Lévy returns. forthcoming in *Handbook of Financial Engineering* (with J.-Y. Yen).
85. Self-decomposability and option pricing. *Mathematical Finance,* 17(1):31–57, 2007 (with P. Carr, H. Geman, and M. Yor).
86. Probing options markets for information. *Methodology and Computing in Applied Probability,* 9:115–131, 2007 (with H. Geman and M. Yor).
87. Correlation and the pricing of risks. forthcoming in *Annals of Finance* (with M. Atlan, H. Geman, and M. Yor).

Completed Papers

88. Asset pricing in an incomplete market with a locally risky discount factor, 1995 (with S. Acharya).
89. Estimation of statistical and risk-neutral densities by hermite polynomial approximation: With an application to Eurodollar Futures options, 1996 (with P. Abken and S. Ramamurtie).
90. Crash discovery in options markets, 1999 (with G. Bakshi).
91. Risk aversion, physical skew and kurtosis, and the dichotomy between risk-neutral and physical index volatility, 2001 (with G. Bakshi and I. Kirgiz).
92. Factor models for option pricing, 2001 (with P. Carr).
93. On the nature of options, 2001 (with P. Carr).
94. Recovery in default risk modeling: Theoretical foundations and empirical applications, 2001 (with G. Bakshi and F. Zhang).
95. Reduction method for valuing derivative securities, 2001 (with P. Carr and A. Lipton).
96. Option pricing and heat transfer, 2002 (with P. Carr and A. Lipton).
97. Multiple prior asset pricing models, 2003 (with R.J. Elliott).

98. Absence of arbitrage and local Lévy models, 2003 (with P. Carr, H. Geman and M. Yor).
99. Bell shaped returns, 2003 (with A. Khanna, H. Geman and M. Yor).
100. Pricing the risks of deposit insurance, 2004 (with H. Unal).
101. Representing the CGMY and Meixner processes as time changed Brownian motions, 2006 (with M. Yor).
102. Pricing equity default swaps under the CGMY Lévy model, 2005 (with S. Asmussen and M. Pistorious).
103. Coherent measurement of factor risks, 2005 (with A. Cherny).
104. Pricing and hedging in incomplete markets with coherent risk, 2005 (with A. Cherny).
105. CAPM, rewards and empirical asset pricing with coherent risk, 2005 (with A. Cherny).
106. The distribution of risk aversion, 2006 (with G. Bakshi).
107. Designing countercyclical and risk based aggregate deposit insurance premia, 2006 (with H. Unal).
108. Sato processes and the valuation of structured products, 2006 (with E. Eberlein).
109. Measuring the degree of market efficiency, 2006 (with A. Cherny).

Contents

Part I

Variance-Gamma and Related Stochastic Processes

The Early Years of the Variance-Gamma Process

Eugene Seneta

School of Mathematics and Statistics FO7
University of Sydney
Sydney, New South Wales 2006, Australia
eseneta@maths.usyd.edu.au

Summary. Dilip Madan and I worked on stochastic process models with stationary independent increments for the movement of log-prices at the University of Sydney in the period 1980–1990, and completed the 1990 paper [21] while respectively at the University of Maryland and the University of Virginia. The (symmetric) Variance-Gamma (VG) distribution for log-price increments and the VG stochastic process first appear in an Econometrics Discussion Paper in 1985 and two journal papers of 1987. The theme of the pre-1990 papers is estimation of parameters of log-price increment distributions that have real simple closed-form characteristic function, using this characteristic function directly on simulated data and Sydney Stock Exchange data. The present paper reviews the evolution of this theme, leading to the definitive theoretical study of the symmetric VG process in the 1990 paper.

Key words: Log-price increments; independent stationary increments; Brownian motion; characteristic function estimation; normal law; symmetric stable law; compound Poisson; Variance-Gamma; Praetz t.

1 Qualitative History of the Collaboration

Our collaboration began shortly after my arrival at the University of Sydney in June 1979, after a long spell (from 1965) at the Australian National University, Canberra. After arrival I was in the Department of Mathematical Statistics, and Dilip a young lecturer in the Department of Economic Statistics (later renamed the Department of Econometrics). These two sister departments, neither of which now exists as a distinct entity, were in different Faculties (Colleges) and in buildings separated by City Road, which divides the main campus.

Having heard that I was an applied probabilist with focus on stochastic processes, and wanting someone to talk with on such topics, he simply walked into my office one day, and our collaboration began. We used to meet about once a week in my office in the Carslaw Building at around midday, and our meetings were accompanied by lunch and a long walk around the campus. This routine at Sydney continued until he left the University in 1988 for the University of Maryland at College Park, his alma mater. I remember telling him on several of these walks that Sydney was too small a pond for his talents.

I was to be on study leave and teaching stochastic processes and time series analysis at the Mathematics Department of the University of Virginia, Charlottesville, during the 1988–1989 academic year. This came about through the efforts of Steve Evans, whom Dilip and I had taught in a joint course, partly on the Poisson process and its variants, at the University of Sydney. The collaboration on what became the foundation paper on the Variance-Gamma (VG) process (sometimes now called the Madan–Seneta process) and distribution [21] was thus able to continue, through several of my visits to his new home. The VG process had appeared in minor roles in two earlier joint papers [16] and [18].

There was a last-to-be published joint paper from the 1988 and 1989 years, [22], which, however, did not have the same econometric theme as all the others, being related to my interest in the theory of nonnegative matrices.

After 1989 there was sporadic contact, mainly by e-mail, between Dilip and myself, some of which is mentioned in the sequel, until I became aware of work by another long-term friend and colleague, Chris Heyde, who divides his time between the Australian National University and Columbia University. His seminal paper [8] was about to appear when he presented his work at a seminar at Sydney University in March 1999. Among his various themes, he advocates the t distribution for returns (increments in log-price) for financial asset movements. This idea, as shown in the sequel, had been one of the motivations for the work on the VG by Dilip and myself, on account of a 1972 paper of Praetz [25].

I was to spend the fall semester of the 1999–2000 academic year again at the University of Virginia, where Wake (T.W.) Epps asked me to be present at the thesis defence of one of his students, and surprised me by saying to one of the other committee members that as one of the creators of the VG process, I "was there to defend my turf." Wake had learned about the VG process during my 1988–1989 sojourn, and a footnote in [21] acknowledges his help, and that of Steve Evans amongst others. Wake's book [2] was the first to give prominence to VG structure. More recently, the books of Schoutens [27] and Applebaum [1] give it exposure.

At about this time I had also had several e-mail inquiries from outside Australia about the fitting of the VG model from financial data, and there was demand for supervision of mathematical statistics students at Sydney University on financial topics. I was supervising one student by early 2002, so I asked Dilip for some offprints of his VG work since our collaboration, and then turned to him by e-mail about the problem of statistical estimation of parameters of the VG. I still have his response of May 20, 2002, generously telling me something of what he had been doing on this.

My personal VG story restarts with the paper [28], produced for a Festschrift to celebrate Chris Heyde's 65th birthday. In this I tried to synthesize the various ideas of Dilip and his colleagues on the VG process with those of Chris and his students on the t process. The themes are: subordinated Brownian motion, skewness of the distribution of returns, returns over unit time forming a strictly stationary time series, statistical estimation, long-range dependence and self-similarity, and duality between the VG and t processes. Some of the joint work with M.Sc. dissertation graduate students [30; 5; 6] stemming from that paper has been, or is about to be, published.

I now pass on to the history of the technical development of my collaboration with Dilip.

2 The First Discussion Paper

The classical model (Bachelier) in continuous time $t \geq 0$ for movement of prices $\{P(t)\}$, modified to allow for drift, is

$$Z(t) = \log\{P(t)/P(0)\} = \mu t + \theta^{1/2} b(t), \tag{1}$$

where $\{b(t)\}$ is standard Brownian motion (the Wiener process); μ is a real number, the drift parameter; and θ is a positive scale constant, the diffusion parameter. Thus the process $\{Z(t)\}$ has stationary independent increments in continuous time, which over unit time are:

$$X(t) = \log\{P(t)/P(t-1)\} \sim \mathcal{N}(\mu, \theta), \tag{2}$$

where $\mathcal{N}(\mu, \theta)$ represents the normal (Gaussian) distribution with mean μ and variance θ. When we began our collaboration, it had been observed for some time that although the assumed common distribution of the $\{X(t)\}$ given by the left-hand side of (2) for historical data indeed seemed symmetric about some mean μ, the tails of the distribution were heavier than the normal. The assumption of independently and identically distributed (i.i.d.) increments was pervasive, making the process $\{Z(t), t = 1, 2, \ldots\}$ a random walk.

Having made these points, our first working paper [12], which has never been published, retained all the assumptions of the classical Bachelier model, with drift parameter given by $\mu = r - \theta/2$. This arose out of a model in continuous time, in which the instantaneous rate of return on a stock is assumed normally distributed, with mean r and variance θ. Consequently, $\{e^{-rt}P(t)\}$ is seen to be a martingale, consistent with option pricing measure for European options under the Merton–Black–Scholes assumptions, where r is the interest rate. The abstract to [12] reads:

> Formulae are developed for computing the expected profitability of market strategies that involve the purchase or sale of a stock on the same day, with the transaction to be reversed when the price reaches either of two prespecified limits or a fixed time has elapsed.

This paper already displayed Dilip's computational skills (not to mention his analytical skills) by including many tables with the headings: Probabilities of hitting the upper [resp., lower] barrier. The profit rate r is characteristically taken as 0.002.

Dilip had had a number of items presented in the Economic Statistics Papers series before number 45 appeared in February 1981. These were of both individual and joint authorship. Several, from their titles, seem of a philosophical nature, for example:

- No. 34. D. Madan. Economics: Its Questions and Answers
- No. 37. D. Madan. A New View of Science or at Least Social Science
- No. 38. D. Madan. An Alternative to Econometrics in Economic Data Analyses

There were also technical papers foreshadowing things to come on his return to the University of Maryland, for example:

- No. 23. P. de Jong and D. Madan. The Fast Fourier Transform in Applied Spectral Inference

Dilip was clearly interested in, and very capable in, an extraordinarily wide range of topics. Our collaboration seems to have marked a narrowing of focus, and continued production on more specific themes. The incoming Head of the Department of Econometrics, Professor Alan Woodland, also encouraged him in this.

3 The Normal Compound Poisson (NCP) Process

Our first published paper [14], based on the the Economics Discussion Paper [13] dated February 1982, focuses on modelling the *second* differences in log-price: $\log\{P(t)/P(0)\}$ by the first difference of the continuous-time stochastic process $\{Z(t), t \geq 0\}$, where

$$Z(t) = \mu t + \sum_{i=1}^{N(t)} \xi_i + \theta^{1/2} b(t). \tag{3}$$

Here $\{N(t)\}$ is the ordinary Poisson process with arrival rate λ, and $\{b(t)\}$ is standard Brownian motion (the Wiener process), μ is a real constant, and $\theta > 0$ is a scale parameter. The ξ_i, $i = 1, 2, \ldots$, form a sequence of i.i.d. $\mathcal{N}(0, \sigma^2)$ rv's, probabilistically independent of the process $\{b(t)\}$. The process $\{Z(t),\ t \geq 0\}$, called in [14] the Normal Compound Poisson (NCP) process, is therefore a process with independent stationary increments, whose distribution (the NCP distribution) over unit time interval, is given by:

$$X|V \sim \mathcal{N}(\mu, \theta + \sigma^2 V), \text{where}\, V \sim \text{Poisson}(\lambda). \tag{4}$$

Hence the cumulative distribution function (c.d.f.) and the characteristic function (c.f.) of X are given by

$$F(x) = \sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} \Phi\left(\frac{x-\mu}{\theta+\sigma^2 n}\right), \tag{5}$$

$$\phi_X(u) = \exp\Big(i\mu u - u^2\theta/2 + \lambda(e^{-u^2\sigma^2/2} - 1)\Big). \tag{6}$$

Here $\Phi(\cdot)$ is the c.d.f. of a standard normal distribution.

The distribution of X is thus a normal with mixing on the variance, is symmetric about μ, and has the same form irrespective of the size of time increment t. It is long-tailed relative to the normal in the sense that its kurtosis value

$$3 + \frac{3\lambda\sigma^4}{(\theta+\sigma^2\lambda)^2}$$

exceeds that of the normal (whose kurtosis value is 3). When the NCP distribution is symmetrized about the origin by putting $\mu = 0$, it has a simple real characteristic function of closed form.

The NCP process from the structure (3) clearly has jump components (the ξ_is are regarded as "shocks" arriving at Poisson rate), and through the Brownian process add-on $\theta^{1/2}b(t)$ in (3), has obviously a Gaussian component. The NCP distribution and NCP process, and the above formulae, are due to Press [26]; in fact, our NCP structure is a simplification of his model (where $\xi_i \sim \mathcal{N}(\nu, \sigma^2)$) to symmetry by taking $\nu = 0$. Note that the nonsimplified model of Press is normal with mixing on both the mean and variance:

$$X|V \sim \mathcal{N}(\mu + \nu V, \theta + \sigma^2 V),$$

where as before $V \sim \text{Poisson}(\lambda)$. Normal variance–mean mixture models have been studied more recently; see [30] for some cases and earlier references.

The c.f.s of the symmetric stable laws,

$$e^{iu\mu - \gamma|u|^\beta},\ \gamma > 0,\ 0 < \beta < 2, \tag{7}$$

where γ is a scale parameter, were also fitted in this way, as was the c.f. of the normal (the case $\beta = 2$). The distributions fitted were thus symmetric about a central point μ, and the c.f.s, apart from allowing for the shift to μ, were consequently real-valued, and of simple closed form. Like the NCP, the process of independent stable increments has both continuous and jump components.

The continuous-time strictly stationary process of i.i.d. increments with stable law also has the advantage of having laws of the same form for an increment over a time interval of any length, and heavy tails, but such laws have infinite variance.

The data for the statistical analysis consisted of five series of share prices and six series on economic variables. The share prices taken were daily last prices on the Sydney Stock Exchange.