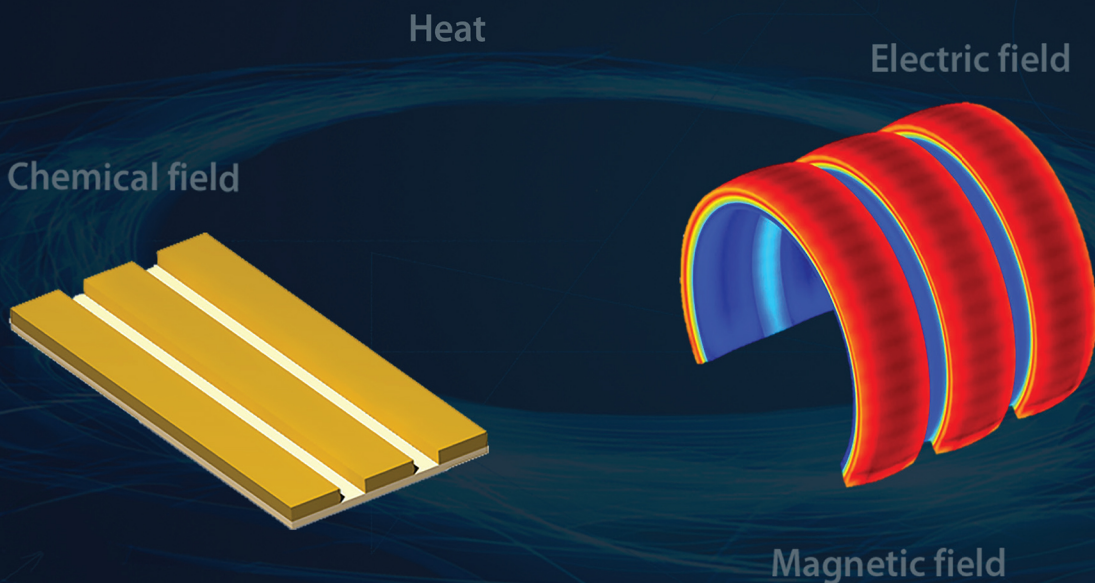


Rui Xiao

Multi-field Modeling of Soft Active Materials

Properties and Design



$$P = \frac{2}{J} \mathbf{F} \cdot \frac{\partial \Psi(\mathbf{C})}{\partial \mathbf{C}}$$

Multi-field Modeling of Soft Active Materials

Properties and Design

Rui Xiao

China

Author**Rui Xiao**

Zhejiang University
No. 38, Zheda Road
Xihu District
Hangzhou
China

Cover Design: Wiley

Cover Image: Courtesy of Rui Xiao

Typesetting: Lumina Datamatics

Library of Congress Card No.: applied for

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

Bibliographic information published by the Deutsche Nationalbibliothek

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available on the Internet at <http://dnb.d-nb.de>.

Print ISBN: 9783527354689

ePDF ISBN: 9783527850396

ePub ISBN: 9783527850389

oBook ISBN: 9783527850402

© 2026 WILEY-VCH GmbH,
Boschstraße 12, 69469 Weinheim, Germany

■ All rights reserved (including those of translation into other languages, text and data mining and training of artificial technologies or similar technologies). No part of this book may be reproduced in any form – by photoprinting, microfilm, or any other means – nor transmitted or translated into a machine language without written permission from the publishers. Registered names, trademarks, etc. used in this book, even when not specifically marked as such, are not to be considered unprotected by law.

The manufacturer's authorized representative according to the EU General Product Safety Regulation is WILEY-VCH GmbH, Boschstr. 12, 69469 Weinheim, Germany, e-mail: Product_Safety@wiley.com.

AI Disclaimer:

While the publisher and the authors have used their best efforts in preparing this work, including a review of the content of the work, neither the publisher nor the authors make any representations or warranties with respect to the accuracy or completeness of the contents of this work and specifically disclaim all warranties, including without limitation any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives, written sales materials or promotional statements for this work. The fact that an organization, website, or product is referred to in this work as a citation and/or potential source of further information does not mean that the publisher and authors endorse the information or services the organization, website, or product may provide or recommendations it may make. This work is sold with the understanding that the publisher is not engaged in rendering professional services. The advice and strategies contained herein may not be suitable for your situation. You should consult with a specialist where appropriate. Further, readers should be aware that websites listed in this work may have changed or disappeared between when this work was written and when it is read. Neither the publisher nor authors shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

Contents

Foreword *vii*

Preface *ix*

Acknowledgments *xi*

1	Basics of Continuum Mechanics	1
1.1	Vectors and Tensors	1
1.1.1	Vector	1
1.1.2	Index Notation	2
1.1.3	Tensor	3
1.1.4	Gradient, Divergence, and Curl	4
1.2	Kinematics	5
1.2.1	Deformation Gradient	5
1.2.2	Strain Tensor	7
1.3	Stress	8
1.4	Balance Principles	9
1.4.1	Material Derivative and Spatial Derivative	10
1.4.2	Reynolds Transport Theorem	10
1.4.3	Conservation of Mass	11
1.4.4	Balance of Momentum	11
1.4.5	Balance of Angular Momentum	12
1.4.6	Balance of Mechanical Energy	13
1.4.7	Balance of Energy	14
1.4.8	Entropy Inequality	15
	References	17
2	Hyperelastic, Viscoelastic, and Damage Models	19
2.1	Introduction	19
2.2	Hyperelastic Models	21
2.2.1	Constitutive Equations of Hyperelastic Materials	21
2.2.2	Hyperelastic Models	24

2.2.3	Results	29
2.3	Viscoelastic Models	31
2.3.1	One-dimensional Small Strain Viscoelastic Models	31
2.3.2	Finite Deformation Viscoelastic Models	33
2.3.3	Results	38
2.3.4	Finite Element Simulation	39
2.4	Damage Models	41
2.4.1	Continuum Damage Model	41
2.4.2	Network Alteration Theory	42
2.4.3	Progressively Damage Model	42
2.4.4	Extended Network Alteration Theory	43
2.4.5	Extended Progressively Damage Model	45
2.4.6	Results	47
2.5	Conclusion	51
	References	53
3	A Thermomechanical Coupled Model for Amorphous Shape-memory Polymers	57
3.1	Introduction	57
3.2	Thermodynamics	59
3.2.1	Kinematics	59
3.2.2	Thermodynamic Framework	60
3.2.3	Constitutive Relationships	64
3.2.4	A Reduced Version of the Constitutive Model	67
3.3	Parameter Determination	68
3.4	Results	68
3.4.1	Performance of the Viscoelastic Model	68
3.4.2	Performance of the Effective Temperature Model	71
3.5	Discussion	73
3.6	Conclusion	74
	References	75
4	An Electromechanical Coupled Model for Dielectric Elastomers	79
4.1	Introduction	79
4.2	Theory	80
4.2.1	Thermodynamic Framework	81
4.2.2	An Electro-hyperelastic Model	84
4.2.3	An Electro-hyper-viscoelastic Model	86
4.2.4	Electromechanical Instability of Dielectric Elastomers	86
4.3	Finite Element Implementation	90
4.3.1	Constitutive Relation	90
4.3.2	Weak Forms	91
4.4	Conclusion	94
	References	95

5	A Magnetomechanical Coupled Model for Magnetoactive Soft Materials	99
5.1	Introduction	99
5.2	A Magnetomechanical Coupled Model for h-MREs	101
5.2.1	Kinematics	101
5.2.2	F-based Model	102
5.2.3	R-based Model	103
5.2.4	Comparing R-based and F-based Models	104
5.3	Magnetic Activated Shape-memory Polymers	105
5.3.1	Constitutive Theory	106
5.3.2	Parameter Determination	109
5.3.3	Simulation Results	111
5.4	Conclusion	112
	Appendix	113
	References	114
6	Multi-field Modeling of Liquid Crystal Elastomers	117
6.1	Introduction	117
6.2	General Theory for Liquid Crystal Elastomers	119
6.2.1	Liquid Crystal	119
6.2.2	Nematic Polymers	121
6.2.3	Neoclassical Model	122
6.3	Thermomechanical Coupled Model for Monodomain Liquid Crystal Elastomers	123
6.4	A Viscoelastic Micropolar Theory for Monodomain Liquid Crystal Elastomers	125
6.4.1	Kinematics	126
6.4.2	Balance of Linear and Angular Momentum	126
6.4.3	Balance of Energy and the Second Law of Thermodynamics	127
6.4.4	Constitutive Equations	128
6.4.5	Simplification for the Non-gradient Case	130
6.4.6	Free Energy Density	132
6.5	Theory for Polydomain Liquid Crystal Elastomers	134
6.6	Conclusion	139
	References	140
7	A Chemomechanical Model for Hydrogels	143
7.1	Introduction	143
7.2	Thermodynamics	145
7.3	Constitutive Relations	148
7.3.1	Neutral Gels	148
7.3.2	Fiber-reinforced Gels	149
7.3.3	Diffusion in Glassy Elastomers	150
7.3.4	Numerical Method	152

- 7.4 Results 153
 - 7.4.1 Neutral Gels and Fiber-reinforced Gels 153
 - 7.4.2 Diffusion in Glassy Polymers 155
- 7.5 Conclusion 157
 - References 157
- Index 161**

Foreword

One of the major advances in the field of mechanics comes from the emergence of *Mechanics of Soft Materials*. Its fast progress is facilitated by four factors: the finite deformation formulation becomes a necessity at its kinematic extreme; comparable effects are encountered due to mechanical, electrical, and magneto fields to leverage multi-physics coupling; the crossing between solid and fluid media makes its constitutive description dangling from non-Newtonian fluids to weakly shear-resistant solids; and the affinity to biological media leads to the interfacial matching between soft materials and human bodies and the intervention of biochemical processes. It is exactly built on this context; the book that addresses this need, *Multi-field Modeling of Soft Active Materials: Properties and Design*, is brought to the science and education forefront.

Prof. Rui XIAO is an energetic scholar with a destiny to engage in the mission of elucidating soft active materials. Rui got his PhD 10 years ago at Johns Hopkins University, and has been my young colleague at the Department of Engineering Mechanics, Zhejiang University, since 2019. He has been pursuing an academic career with a focus on the constitutive behavior of soft materials, encompassing shape-memory polymers, dielectric elastomers, magnetic-responsive elastomers, liquid crystal elastomers, and hydrogels. Though these contents are all hot topics in research, the current literature is mostly composed of the technical papers, seldom with heavyweight monographs to address comprehensively the mechanical essence of the subject. Rui's monograph has an ambitious goal: to build a knowledge framework on the mechanics characterization of soft active materials, from properties to designs, from mechanics foundation to multi-physics coupling, from shape memory to chemo-mechanical formulation. The methodology runs through continuum mechanics (Chapter 1), hyper-elastic and visco-elastic modeling (Chapter 2), and a spectrum of cases to exemplify multi-field modeling (Chapters 3–7). All formulations start from the thermodynamics framework, then derive constitutive relations, and finally to the finite element implementation. The content covers all major categories of soft materials: amorphous shape-memory polymers (Chapter 3), dielectric elastomers (Chapter 4), magnetoactive soft materials (Chapter 5), liquid crystal elastomers (Chapter 6), and hydrogels (Chapter 7).

With this book as the foundation, the readers may find a common approach to model soft active materials under multi-field situations. And even go beyond, to explore a pathway to construct and to model “intelligent soft materials” with self-healing, self-configuration, self-programmable, and self-growth capabilities.

A handwritten signature in black ink, reading 'Wei Yang' in a cursive style.

Wei Yang

Professor of Zhejiang University
Member of Chinese Academy of Science

Preface

Soft active materials, characterized by their capacity for large deformations in response to various fields, are pivotal in the design of smart structures for applications such as soft robotics and biomedical devices. This book presents a comprehensive exploration of the multi-field modeling of soft active materials, encompassing fundamental principles and recent advancements.

My engagement with this field commenced in 2009 during my doctoral studies, where my research centered on shape-memory polymers. At that time, the mechanics of soft matter was a nascent area, with investigations primarily focused on systems like hydrogels and dielectric elastomers. Concurrent with my PhD studies, I also conducted preliminary explorations in these domains. Subsequently, as I transitioned to independent research, the development of responsive soft materials, including magnetoactive elastomers, liquid crystal elastomers, and fiber-reinforced hydrogels, progressed rapidly. While theoretical models were developed to describe individual material systems, a consolidated resource detailing the commonalities in their multi-field coupling theories was absent. This gap motivated me to write this book.

This book addresses this need by providing a coherent theoretical framework applicable to these diverse responsive materials. It is my aspiration that this work will offer valuable insights to researchers and serve as a foundation for the development of models for increasingly complex systems in the future.

The book begins with the basis of continuum mechanics in Chapter 1, introducing vectors, tensors, kinematics, stress, and balance principles. Chapter 2 details constitutive models for hyperelasticity viscoelasticity, and damage. While these topics are typically introduced in a standard *Continuum Mechanics* course, they serve as the foundation of multi-field models developed in the subsequent chapters.

Thermomechanical coupling in amorphous shape-memory polymers (SMPs) is discussed in Chapter 3, featuring the effective temperature model and its application to solvent-driven shape-memory behaviors. Chapter 4 explores electromechanical coupling in dielectric elastomers, including a thermodynamic framework and analysis of electromechanical instability. Chapter 5 presents magnetic-mechanical coupling in magnetoactive soft materials, comparing models for magnetorheological elastomers and incorporating glass transition mechanisms. Multi-field modeling of liquid crystal elastomers (LCEs)

is covered in Chapter 6, including theories for monodomain and polydomain LCEs. Finally, Chapter 7 discusses chemo-mechanical modeling of kinetic diffusion in hydrogels, addressing the coupling between solvent diffusion and large deformation.

Overall, this book aims to provide researchers, engineers, and graduate students in mechanical engineering, materials science, and biomedical engineering a robust understanding of soft active materials and their multi-field modeling.

Rui Xiao

Zhejiang University
Hangzhou, China

Acknowledgments

I owe my deepest gratitude to Professor Thao D. Nguyen, with whom I had the privilege of working during my PhD studies at Johns Hopkins University. Her guidance, unwavering support, and exacting standards for research have profoundly influenced not only my doctoral journey but also my subsequent career as an independent researcher.

I was honored to study and collaborate with Professor Paul Steinmann at Friedrich-Alexander University of Erlangen-Nürnberg as a Humboldt Research Fellow. His exceptional expertise and insightful perspectives have inspired me in countless ways.

I am deeply indebted to my esteemed colleagues, Professors Wei Yang, Weiqiu Chen, Shaoxing Qu, Jin Qian, Jizhou Song, Zhe Chen, and Guoyong Mao, for their invaluable collaborations, which have been instrumental in shaping this book. Their contributions were essential to the realization of this work.

Special thanks are due to my dedicated students, especially Liqian Wang, Junwei Xu, Peng Sun, Zengting Xu, Lingrui Zhu, Kunquan Wang, and Le Du, for their meticulous work in preparing figures, reviewing the manuscript, and, most importantly, for the foundational research that directly informs a large portion of this book.

1

Basics of Continuum Mechanics

1.1 Vectors and Tensors

Continuum mechanics relies on vectors and tensors. We first introduce vector/tensor operations. In this book, scalars are denoted by italic lowercase letters, while vectors and tensors are represented by upright boldface letters.

1.1.1 Vector

If a physical quantity contains only magnitude information, such as time, mass, temperature, density, and length, it is referred to as a scalar. A vector represents a directed line segment in space. The length of the line segment indicates the magnitude of the vector, while the direction of the line segment represents the direction of the vector. It is used to describe physical quantities that have both direction and magnitude, such as displacement, velocity, acceleration, and force.

The addition of vectors follows the same rules as scalar addition, satisfying the commutative law and the associative law of addition:

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}, \quad (1.1a)$$

$$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w}). \quad (1.1b)$$

The sum of two vectors is still a vector, and the magnitude and direction of the resultant vector are determined according to the parallelogram law.

The product of a vector $\mathbf{u}$ and a scalar α is a new vector that has the same direction as $\mathbf{u}$ if $\alpha > 0$, or the opposite direction if $\alpha < 0$. The multiplication of a scalar and a vector satisfies the commutative law, the associative law, and the distributive law as follows:

$$\alpha \mathbf{u} = \mathbf{u} \alpha, \quad (1.2a)$$

$$(\alpha \beta) \mathbf{u} = \alpha (\beta \mathbf{u}), \quad (1.2b)$$

$$(\alpha + \beta) \mathbf{u} = \alpha \mathbf{u} + \beta \mathbf{u}, \quad (1.2c)$$

$$\alpha (\mathbf{u} + \mathbf{v}) = \alpha \mathbf{u} + \alpha \mathbf{v}. \quad (1.2d)$$

The dot product of vectors $\mathbf{u}$ and $\mathbf{v}$, denoted by $\mathbf{u} \cdot \mathbf{v}$, results in a scalar:

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos \theta, \quad (1.3)$$

where θ is the angle between two vectors $\mathbf{u}$ and $\mathbf{v}$. The dot product of vectors has the following properties:

- $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$,
- $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$.

The cross product of vectors $\mathbf{u}$ and $\mathbf{v}$, denoted by $\mathbf{u} \times \mathbf{v}$, results in a vector. Its magnitude is given by:

$$|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}| \sin \theta. \quad (1.4)$$

The direction of $\mathbf{u} \times \mathbf{v}$ is perpendicular to the plane formed by $\mathbf{u}$ and $\mathbf{v}$, which follows the right-hand rule. The cross product of vectors has the following properties:

- $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$,
- $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$.

1.1.2 Index Notation

To introduce the coordinate (or component) expression of a vector in a right-handed orthogonal system, we first introduce a set of fixed basis vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$, called a Cartesian basis, with the following properties:

$$\mathbf{e}_1 \cdot \mathbf{e}_2 = \mathbf{e}_1 \cdot \mathbf{e}_3 = \mathbf{e}_2 \cdot \mathbf{e}_3 = 0, \quad \mathbf{e}_1 \cdot \mathbf{e}_1 = \mathbf{e}_2 \cdot \mathbf{e}_2 = \mathbf{e}_3 \cdot \mathbf{e}_3 = 1. \quad (1.5)$$

These three vectors are of unit length and mutually orthogonal. A vector in three-dimensional space can be expressed as

$$\mathbf{u} = u_1 \mathbf{e}_1 + u_2 \mathbf{e}_2 + u_3 \mathbf{e}_3, \quad (1.6)$$

which can be written in summation notation as

$$\mathbf{u} = \sum_{i=1}^3 u_i \mathbf{e}_i. \quad (1.7)$$

This expression can be further simplified by using a dummy index to omit the summation symbol, written as

$$\mathbf{u} = u_i \mathbf{e}_i. \quad (1.8)$$

An index that repeats exactly twice in an expression is called a dummy index, which signifies the summation convention. An index that is not summed over in an expression is referred to as a free index. For example:

$$f_i = u_i v_j w_j, \quad (1.9a)$$

$$f_1 = u_1(v_1 w_1 + v_2 w_2 + v_3 w_3), \quad (1.9b)$$

$$f_2 = u_2(v_1 w_1 + v_2 w_2 + v_3 w_3), \quad (1.9c)$$

$$f_3 = u_3(v_1 w_1 + v_2 w_2 + v_3 w_3), \quad (1.9d)$$

where i is the free index and j is the dummy index.

The calculus operations involving scalars and vectors often carry clear physical significance. We begin by introducing two commonly used symbols in continuum mechanics: the Kronecker delta δ_{ij} and the permutation symbol ε_{ijk} . The Kronecker delta δ_{ij} is defined as follows:

$$\mathbf{e}_i \cdot \mathbf{e}_j = \delta_{ij} \equiv \begin{cases} 1, & \text{if } i=j, \\ 0, & \text{if } i \neq j. \end{cases}$$

The following are some of its useful properties:

$$\delta_{ii} = 3, \quad \delta_{ij} u_i = u_j, \quad \delta_{ij} \delta_{jk} = \delta_{ik}. \quad (1.10)$$

Another commonly used symbol is the permutation symbol ε_{ijk} , which is defined as follows:

$$\varepsilon_{ijk} = \begin{cases} 1, & \text{for even permutations of } (i, j, k) \text{ (i.e. } 123, 231, 312). \\ -1, & \text{for odd permutations of } (i, j, k) \text{ (i.e. } 132, 213, 321). \\ 0, & \text{if there is a repeated index.} \end{cases} \quad (1.11)$$

The permutation symbol has the following properties:

$$\varepsilon_{ijk} = \varepsilon_{jki} = \varepsilon_{kij}, \quad \varepsilon_{ijk} = -\varepsilon_{ikj}, \quad \varepsilon_{ijk} = -\varepsilon_{jik}. \quad (1.12)$$

1.1.3 Tensor

A second-order tensor can be viewed as a linear operator acting on a vector $\mathbf{u}$, producing a new vector $\mathbf{v}$ as

$$\mathbf{v} = \mathbf{A}\mathbf{u}. \quad (1.13)$$

A second-order tensor can be written in the index notation as

$$\mathbf{A} = A_{ij} \mathbf{e}_i \otimes \mathbf{e}_j. \quad (1.14)$$

where $\otimes$ is the dyad of vectors.

A unit tensor $\mathbf{I}$ can be represented using the Kronecker delta symbol as

$$\mathbf{I} = \delta_{ij} \mathbf{e}_i \otimes \mathbf{e}_j = \mathbf{e}_i \otimes \mathbf{e}_i. \quad (1.15)$$

The dot product of two second-order tensors $\mathbf{A}$ and $\mathbf{B}$ is typically denoted as $\mathbf{AB}$, which is still a second-order tensor. The components of the dot product $\mathbf{AB}$ can be represented as

$$(\mathbf{AB})_{ij} = A_{ik} B_{kj}. \quad (1.16)$$

The double dot product of two second-order tensors results in a scalar, as given by the following expression:

$$\mathbf{A} : \mathbf{B} = A_{ij} \mathbf{e}_i \otimes \mathbf{e}_j : B_{kl} \mathbf{e}_k \otimes \mathbf{e}_l = A_{ij} B_{kl} (\mathbf{e}_i \cdot \mathbf{e}_k) (\mathbf{e}_j \cdot \mathbf{e}_l) = A_{ij} B_{ij}. \quad (1.17)$$

The transpose of a second-order tensor $\mathbf{A}$ is denoted as $\mathbf{A}^T$, with the following property:

$$(A^T)_{ij} = A_{ji}. \quad (1.18)$$

The trace of a second-order tensor $\mathbf{A}$ equals to the sum of the diagonal terms represented as

$$\text{tr}(\mathbf{A}) = A_{ii}. \quad (1.19)$$

The determinant of a tensor $\mathbf{A}$ is the determinant of the matrix $\mathbf{A}$, represented as

$$\det(\mathbf{A}) = \varepsilon_{ijk} A_{1i} A_{2j} A_{3k}. \quad (1.20)$$

For a nonsingular tensor, namely, $\det(\mathbf{A}) \neq 0$, there exists a unique inverse of the tensor, denoted as $\mathbf{A}^{-1}$, which satisfies as

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I}. \quad (1.21)$$

A tensor can be decomposed into a symmetric part and a skew part as

$$\text{sym}(\mathbf{A}) = \frac{\mathbf{A} + \mathbf{A}^T}{2}, \quad \text{skew}(\mathbf{A}) = \frac{\mathbf{A} - \mathbf{A}^T}{2}. \quad (1.22)$$

A second-order tensor has different component forms in different coordinate systems:

$$\mathbf{A} = A_{ij} \mathbf{e}_i \otimes \mathbf{e}_j \quad \text{or} \quad \mathbf{A} = \tilde{A}_{ij} \tilde{\mathbf{e}}_i \otimes \tilde{\mathbf{e}}_j. \quad (1.23)$$

These two component forms can be related through the transformation of the basis vectors corresponding to the two coordinate systems, as

$$\mathbf{e}_i = Q_{ij} \tilde{\mathbf{e}}_j \quad Q_{ij} = \mathbf{e}_i \cdot \tilde{\mathbf{e}}_j. \quad (1.24)$$

where Q_{ij} represents the direction cosines between the basis vectors of the two coordinate systems. $\mathbf{Q}$ is an orthogonal second-order matrix with $\mathbf{Q}^T = \mathbf{Q}^{-1}$.

Furthermore, we have:

$$\tilde{A}_{ij} = Q_{ki} A_{kl} Q_{lj}, \quad [\tilde{\mathbf{A}}] = [\mathbf{Q}]^T [\mathbf{A}] [\mathbf{Q}]. \quad (1.25)$$

A vector can be treated as a tensor with order 1. Thus, higher order tensors can also be defined in a similar way. For example, tensors with an order 3 or 4 can be defined as

$$\mathcal{A} = A_{ijk} \mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_k. \quad (1.26a)$$

$$\mathbb{A} = A_{ijkl} \mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_k \otimes \mathbf{e}_l. \quad (1.26b)$$

1.1.4 Gradient, Divergence, and Curl

We introduce the vector operator ∇ , commonly referred to as the Nabla operator, which is frequently used in computations:

$$\nabla(\bullet) = \frac{\partial(\bullet)}{\partial x_i} \mathbf{e}_i = \frac{\partial(\bullet)}{\partial x_1} \mathbf{e}_1 + \frac{\partial(\bullet)}{\partial x_2} \mathbf{e}_2 + \frac{\partial(\bullet)}{\partial x_3} \mathbf{e}_3. \quad (1.27)$$