



András Sóbester

Alexander I J Forrester

Aircraft Aerodynamic Design

Geometry and Optimization

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AIRCRAFT AERODYNAMIC DESIGN

GEOMETRY AND OPTIMIZATION

András Sóbester and Alexander I J Forrester

*Faculty of Engineering and the Environment, University of
Southampton, UK*

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Chapter 9

Figure 9.1 Aircraft with high, swept wings, featuring pronounced anhedral. Clockwise from top left: Harrier (AV8-B), Airbus A400M, British Aerospace BAe-146, Ilyushin Il-76 (photographs by A. Sóbester).

Figure 9.2 Aircraft with polyhedral wings. Anticlockwise from top: Jodel D18, Vought F4U-1D Corsair and the Mahoney Sorceress, a staggered biplane designed for the Reno Air Races (photographs by A. Sóbester).

Figure 9.3 Multisegment flaps on the polyhedral wing of a Vought Corsair (photograph by A. Sóbester).

Figure 9.4 CAD rendering of a box wing unmanned aircraft - an example of $\Gamma(\varepsilon)$ varying linearly between the end of the zero dihedral lower half of the wing and an upper half that can be viewed, from a purely geometrical standpoint, as having 180° dihedral.

Figure 9.5 Local dihedral angle variation defined along a wing-attached, spanwise coordinate axis ε , to model deformed shapes for fluid-structures interactions studies (Boeing 787-8; photographs by A. Sóbester).

Figure 9.6 Two wings (transonic transport on the left, box wing on the right) illustrating a conventionally orientated Cartesian system with its origin at the root of the leading edge and an additional curvilinear dimension ε attached to the leading edge.

Figure 9.7 The first step of building the box wing: a simple straight, plane wing with a NACA 5310 section.

Figure 9.8 Step 2 of generating a box wing: the basic wing is folded back onto itself through a linear transition.

Figure 9.9 Folded wing with aerofoil section camber transition at the folding point - both elements of the wing have positive camber.

Figure 9.10 Completed box wing.

[Figure 9.11 Blended winglets on an Embraer ERJ 190-200LR \(photograph by A. Sóbester\).](#)

[Figure 9.12 A simple, two-variable parameterization of a blended winglet geometry. Sweeps of the variable controlling the tip tangent \(left\) and the variable controlling the transition point \(right\).](#)

[Figure 9.13 Scimitar winglet generated by combining two instances of the parametric blended winglet. The two component winglets differ on the starting point of their transition, the wingtip tangent, as well as on the overall scaling factor.](#)

[Figure 9.14 Commuter-class turboprop wing sketch.](#)

[Figure 9.15 Constraint diagram of the commuter airliner design example. \$C_D^{\text{para}}\$ denotes parasitic drag, \$C_D^{\text{ind}}\$ stands for induced drag.](#)

[Figure 9.16 BAe Jetstream 31, seen here in the livery of the UK's National Flying Laboratory \(courtesy of G-NFLA\).](#)

[Figure 9.17 Three design points of the BAe Jetstream 31 against the constraint boundaries of the commuter turboprop design example.](#)

[Chapter 10](#)

[Figure 10.1 One-sided difference error when approximating \$\partial L \partial x_0\$ using Equation 10.4.](#)

[Figure 10.2 AlgoPy's computational graph of the forward part of Listing 10.2.](#)

[Figure 10.3 The Bézier spline aerofoil for which the derivatives of \$c_1\$ with respect to its surface are to be obtained. The circled control points are those for which the z-coordinate will be varied via an inverse design process in Section 10.4.](#)

Figure 10.4 Derivatives of $p_{\text{panel}}(x, \alpha, Re)$ with respect to aerofoil surface coordinates.

Figure 10.5 Derivatives of the Bézier spline aerofoil surface with respect to the control points (compare with the Bernstein polynomials in Figure 3.7).

Figure 10.6 A Ferguson spline aerofoil defined by $A_{\text{upper, lower}} = 0.0, 0.0, B_{\text{upper, lower}} = 1.0, 0.0, T_{\text{lower } A} = 0.0, -0.025, T_{\text{upper } A} = 0.0, 0.03, T_{\text{lower } B} = 0.75, 0.0, T_{\text{upper } B} = 0.9, -0.03$.

Figure 10.7 Derivatives of the Ferguson spline aerofoil surface with respect to the leading and trailing edge points (A and B), and leading and trailing edge tangents (T_A and T_B) (compare with the basis functions in Figure 3.12).

Figure 10.8 Derivatives of the Ferguson spline aerofoil surface with respect to the definition in Figure 7.2, showing the intuitive nature of this parameterization (i.e. the variables have clear associations with the shape of the aerofoil). Note that although some parameters have the same effect on the shape, they will, naturally, have different effects on the flow.

Figure 10.9 Derivatives of a NACA 4412 aerofoil surface with respect to the four-digit definition; that is, $z_{\text{cam}}^{\text{max}}, x_{\text{mc}}$ and t_{max} .

Figure 10.10 Initial, target and optimized c_p profiles from the inverse design process in Listing 10.9.

Figure 10.11 Initial and optimized aerofoils from the inverse design process in Listing 10.9.

Chapter 11

[Figure 11.1 Ferguson spline aerofoils with varying \$T_{upper_A}\$, produced by Listing 11.7.](#)

[Figure 11.2 XFOIL-calculated drag coefficients for Ferguson spline aerofoils with varying \$T_{upper_A}\$, produced by Listing 11.7. The curve fit is a 'moving least squares' - for example, see Forrester and Keane \(2009\).](#)

[Chapter 12](#)

[Figure 12.1 SUHPA in flight during the 2013 Icarus Cup at Sywell Aerodrome \(piloted by Bill Brooks; power-plant, Guy Martin; photograph by Fred To\).](#)

[Figure 12.2 SUHPA in flight. Note the high aspect ratio and low thickness/chord, highlighting the importance of the aero-structural trade-off \(piloted by Bill Brooks; power-plant, Guy Martin; photograph by Fred To\).](#)

[Figure 12.3 Planform of SUHPA after the three-variable NACA 44xx optimization \(Listing 12.1\).](#)

[Figure 12.4 Aerofoils and corresponding pressure profiles after the three-variable NACA 44xx optimization \(Listing 12.1\).](#)

[Figure 12.5 Aerofoils and corresponding pressure profiles after the five-variable NACA xxxx optimization.](#)

[Figure 12.6 Aerofoils and corresponding pressure profiles after the nine-variable Ferguson spline-based optimization.](#)

[Figure 12.7 Aerofoils and corresponding pressure profiles after the 27-variable Ferguson spline-based optimization.](#)