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Aircraft Aerodynamic Design
Geometry and Optimization

Aerospace Series
Editors Peter Belobaba, Jonathan Cooper and Allan Seabridge

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GEOMETRY AND OPTIMIZATION

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Series Preface

The field of aerospace is multi-disciplinary and wide ranging, covering a large variety of products, disciplines and domains, not merely in engineering but in many related supporting activities. These combine to enable the aerospace industry to produce exciting and technologically advanced vehicles. The wealth of knowledge and experience that has been gained by expert practitioners in the various aerospace fields needs to be passed onto others working in the industry, including those just entering from university.

The *Aerospace Series* aims to be a practical, topical and relevant series of books aimed at people working in the aerospace industry, including engineering professionals and operators, allied professions such commercial and legal executives, and also engineers in academia. The range of topics is intended to be wide ranging, covering design and development, manufacture, operation and support of aircraft, as well as topics such as infrastructure operations and developments in research and technology.

Aerodynamics is the fundamental science that underpins the world-wide aerospace industry and enables much of the design and development of today's highly efficient aircraft. Much effort is devoted to the design of new aircraft in order to determine the wing and tail surface geometry that gives the optimum aerodynamic performance.

This book, *Aircraft Aerodynamic Design: Geometry and Optimization*, covers a range of different aspects of geometry parameterization that is relevant for aircraft lifting surface design. The emphasis is on the efficient construction of an aircraft geometry which can then be coupled to any flow solver and optimization package; however, most of the concepts can be applied to any engineering product. Starting with the underlying principles of geometric parameterization, the reader is taken through the fundamentals of 2D aerofoil optimization onto 3D wing synthesis and the computation of design sensitivities.The most important concepts are illustrated using a basic aerofoil analysis and a human powered wing design. All of the key ideas throughout the book are demonstrated using computer codes, making it easy for the readers to develop their own applications. The book provides a welcome addition to the Wiley Aerospace Series and complements other books on aerodynamic modelling and conceptual aircraft design.

Peter Belobaba, Jonathan Cooper and Allan Seabridge

Preface

In July 1978 the *Journal of Aircraft* published a paper titled ‘Wing design by numerical optimization’. The authors, Raymond Hicks of the NASA Ames Research Center and Preston Henne of the Douglas Aircraft Company, had identified a set of functions with ‘aerofoil-like’ shapes, which, when added to a baseline aerofoil in various linear combinations, generated other ‘sensible’ aerofoil shapes.

This, as a principle, was not new. After all, the National Advisory Committee for Aeronautics was already experimenting with *parametric aerofoils* in the 1930s. The formulation described by Hicks and Henne (1978) was a new aerofoil family generated in a novel way – building an aerofoil out of weighted shapes, much like one might build a musical sound from multiple harmonics. But this was not the real novelty; how they proceeded to use it was.

Combining the incipient technology of numerical flow simulation (they used a two-dimensional model) with a simple optimization heuristic and their new parametric geometry they performed an automated computational search for a *better aerofoil shape*.

Here is the idea that thus began to take shape and commence its ascent along the technology readiness level (TRL) ladder of the aerospace industry. A parametric geometry is placed at the heart of the aircraft design process. The *design variables* influencing its shape are adjusted in some systematic, iterative way, as dictated by an optimization algorithm. The latter is guided by a design performance metric, resulting from a physics-based simulation run on an instance of the parametric geometry.

The TRL rise was to be a slow one, for two reasons. First, because in a world largely reliant on drawing boards for years to come, this was a disruptive idea that would encounter much resistance in this notoriously risk-averse industry. Second, none of the links in the chain of tools required (numerical flow analysis, computational geometry and efficient optimization techniques) would be really ready for some fast optimization action until well into the 1990s.

There is a maxim known by most practitioners of the art, which states that an optimization algorithm will find the slightest flaws in the analysis code (usually comprising a mesher and a partial differential equation solver) and in the geometry model; that is, it will steer the design process precisely towards their weak areas.

This is not (only) due to Sod’s law – more fundamental effects are at play. Most computational analyses have a domain of ‘safe’ operation, outside of which they will either predict unphysically good or unphysically bad performance. Straying into the latter type of area will thus be a self-limiting deviation, but the former will lure the optimizer into ‘discovering’ amazingly good solutions that do not actually exist in ‘real’ physics. Sometimes these are obvious (what rookie optimization practitioner has not ‘discovered’ aerofoils that generate thrust instead of drag?), but more subtle pitfalls abound, and highlighting these remains a challenge in the path of the ubiquitous use of this technology.

Along similar lines, parametric geometry modelling has its own pitfalls, deceptions and hurdles in the path of effective optimal design, and how to avoid (at least some of) them is the subject of this book.

Some of the principles discussed over the pages that follow can be applied to the geometry of any engineering product, but we focus on those aspects of geometry parameterization that are specific to external aircraft surfaces wetted by airflow. Some of the ideas are therefore linked to aerodynamics, and so we will touch upon the relevant aspects of aircraft aerodynamic design – from an engineering perspective. However, *this is not a book on aircraft aerodynamics*, and, for that matter, nor will it provide the reader with a recipe on how to design an aeroplane. Instead, it is an exposition of concepts necessary for the construction of aircraft geometry that can exploit the capabilities of an optimization algorithm.

The reader may wish to peruse the text simply to gain a theoretical appreciation of some of the issues of aircraft geometry parameterization, but there is plenty to get started with for the more practically minded too. All key concepts are illustrated with code, which can be run 'as is' or can form a building block in the reader's own code. After lengthy deliberations we selected two software platforms to use for this: Mathworks MATLAB® and Python. Some of the Python code calls methods from the OpenNURBS framework, which can be accessed through *Robert McNeel & Associates Rhino*, a powerful, yet easy to use, lightweight CAD package. Some of the code is reproduced in the text to help illustrate some of the formulations – in each case we selected one of the platforms mentioned above, but in most cases implementations in the others are available too on the website [www.wiley.com/go/sobester] accompanying the book.

Here is a brief sketch of the structure of this book.

After discussion of the general context of aircraft shape description and parameterization (*Prologue*), in the following chapter (*Geometry Parameterization: Philosophy and Practice*) we discuss the place of parametric geometries in aircraft design in general and we start the main threads that will be running through this book: the guiding principles of parametric geometry construction and their impact on the effectiveness of the optimization processes we might build upon them.

We next tackle the fundamental building blocks of all aircraft geometries, first in two dimensions (the chapter titled *Curves*), then in three (*Surfaces*). Two-dimensional sections through wings (and other lifting surfaces) are perhaps the most widely known and widely discussed aerodynamic geometry primitive, and we dedicate three chapters to them: a general introduction (*Aerofoil Engineering: Fundamentals*), a review of some of the key *Families of Legacy Aerofoils* and, arriving at the concept at the heart of this book, *Aerofoil Parameterization*.

Another classic two-dimensional view of aerodynamics is tackled in the chapter titled *Planform Parameterization*, thus completing the discussion of all the primitives needed to build a three-dimensional wing geometry – which we do in the chapter *Three-Dimensional Wing Synthesis*.

The ultimate point of geometry parameterization is, of course, the optimization of objective functions that measure the performance of the object represented by the geometry. Recent years have seen a strong push towards making this process as efficient as possible, and one of the enablers is the efficient computation of the sensitivities of the objective function with respect to the design variables controlling the shape. A number of ways of achieving this are discussed in the chapter titled *Design Sensitivities*.

The most important concepts are illustrated via examples throughout the book, but there are two more substantial such examples, which warrant chapters of their own: *Basic Aerofoil Analysis: A Worked Example* and *Human-Powered Aircraft Wing Design: A Case Study in Aerodynamic Shape Optimization*.

We then bring matters to a close by looking ahead and discussing the area where geometry parameterization is most acutely in need of further development – this is the chapter titled *Epilogue: Challenging Topological Prejudice*.

Parametric geometry is a vast subject, and a book dedicated even to one of its subsets – in this case, the parametric geometry of the external shape of fixed-wing aircraft – is unlikely to be comprehensive. We hope that, beyond a discussion of the formulations we felt to be the most important, this book succeeds in setting out the key principles that will enable the reader to 'discover', critically evaluate and deploy other formulations not discussed here. Moreover, it should assist in creating new models – essential building blocks of the design tools of the future.

Finally, we would like to acknowledge some of those who helped shape this text through discussions and reviews: Jennifer Forrester, Brenda Kulfan, Andy Keane, Christopher Paulson, James Scanlan, Nigel Taylor, David Toal and Sebastian Walter. We are also indebted to Tom Carter and Eric Willner at Wiley, whose patience and support made the long years of writing this book considerably easier.

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Southampton, UK, 2014

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Prologue

Geometry is the *lingua franca* of engineering. Any conversation around a nontrivial design problem usually has even the most articulate engineer overcome, within minutes, by the desire to draw, sketch or doodle. Over the centuries the sketching tools have changed. However, Leonardo da Vinci wielded his chalk and pen for the same reason why today's engineers slide their fingertips along tablet computer screens, deftly creating three-dimensional geometrical models and navigating around them: the functionality and performance of an engineering product depends, to a very large extent, on its shape and size; that is, on its *geometry*.

Different fields of engineering place different levels of emphasis on geometry, but perhaps none focuses on it more sharply than the aerodynamic design of aircraft. The goal of the aerodynamics engineer is to create an object that, when immersed in airflow, will change the patterns of the latter in a desirable fashion,[1] and this is most readily achieved through the shaping and sizing of the object.

Sadly for aerodynamic engineers, their freedom to play with the form of the aircraft's flow-wetted surfaces is often curtailed by other departments competing for influence over the same piece of real estate: structural engineering, cost modelling, propulsion, control systems, cabin and payload, and so on. Moreover, the aerodynamic performance of an aircraft is usually multifaceted too: different phases of the same mission tend to drive the external shape in different directions. This tension between competing objectives is usually resolved in one of three ways:

1. *One of the goals trumps all others.* The shape usually gives this away – it is immediately clear to the trained observer that one interest drove the design of the aircraft and the others had to operate within very strict constraints defined by it.

   Consider Figure 1.1 as an example. It shows three unmanned aircraft. One has a delicate-looking, sleek airframe with very long and narrow wings: a glider (sailplane), the design of which was driven by the single-minded desire to maximize endurance. The structural and

[1] Aerodynamics engineers are also unique amongst the general public in regarding an aircraft as a stationary object, with the atmosphere 'flying' past it.

Aircraft Aerodynamic Design: Geometry and Optimization, First Edition. András Sóbester and Alexander I J Forrester.

Figure 1.1 Three unmanned air vehicles (UAVs), three main design drivers: NASA Towed Glider Air-Launch Concept (left, NASA image) – long endurance at low speeds; AeroVironment RQ-11 Raven (top right, USAF image) – structural robustness to cope with battlefield conditions; Boeing X-51 (USAF image) – extremely high speed.

cost engineers would no doubt have liked to have seen shorter, stubbier wings, upon which the air loads generate lighter bending moments, but this was a case of shape design in the service of aerodynamic efficiency, with little more than a glance toward other objectives.

The top right image shows a soldier launching a low-altitude, low-endurance surveillance platform. The ‘boxy’ fuselage and the short, wide wings identify this as a design driven by a desire for ruggedness and enough spare structural strength to allow the aircraft to cope with the rough handling likely in a battlefield environment – at the expense of aerodynamic efficiency.

Finally, the third picture shows a hypersonic research aircraft. It is not able to carry any payload, it has no landing gear (it is a single-use vehicle) and its endurance is measured in seconds. But it does ‘ace’ one objective: speed. Every feature of its geometry says ‘designed for a hypersonic dash’ (more on this extraordinary vehicle in the next chapter).

2. *A compromise* results, which balances all the competing goals. How to analyse all the trade-offs involved and how to make design decisions based on them is the core question of modern engineering design and we will discuss some of the relevant techniques in Chapter 2.
3. *The ‘all things to all departments’ solution.* The aircraft, or aircraft subsystem, is actually several designs rolled into the same packaging, with each design optimized for a particular goal. An in-flight ‘morphing’ process mutates the geometry from one shape to another, depending on the phase of the mission. Perhaps the most common embodiment of this principle is the high-lift system that enables many aircraft to cruise efficiently at high speed, but also generates sufficient lift at the low speeds typical of take-off and landing (see Figure 1.2).

Figure 1.2 Three wing geometries packaged into one and able to morph from one to another: the trailing edge of the wing of a Boeing 787-8 airliner in (from left to right) cruising flight, final approach and touch-down (photographs by A. Sóbester).

> This is a complex problem for the designer, as the challenge is not only to design multiple geometries, each optimized for, say, different flow regimes, but also to choreograph the transition process – all this without exceeding weight, cost and complexity constraints.

Ultimately, the external geometry of an aircraft results from the systematic analysis of the physics behind all the relevant objectives and constraints and the application of engineering optimization techniques to the resulting data to generate a solution that satisfies the design brief. There is an almost endless diversity of design briefs, of different relative importances of the objectives, of physics-based performance simulation capabilities and of design techniques, the result being an immense range of shapes – see Figure 1.3 for a small selection. This diversity is a measure of how serious a mathematical challenge shape modelling is.

In terms of an aeronautical engineer's relationship with geometry, much has changed since da Vinci began sketching flying vehicles. This technological transformation has been a nonlinear one too, with the migration from a two-dimensional drawing board into the three-dimensional space of virtual geometries stored in the memory of a computer constituting the greatest leap. However, the most important aspect of this revolution that began to gather momentum in the 1970s and 1980s was *not* the extension into the third dimension.

Rather, with the advent of the first computational geometry engines, engineers were suddenly able to make local, as well as large-scale, *changes* to the emerging geometry in a systematic and time-efficient manner. This step was particularly significant at the level of whole aircraft geometries – no longer did the change of a simple dimension on one component mean that tens or hundreds of other blueprints had to be redrawn. Instead, the computational geometry cascaded the changes as appropriate, and thus new configurations could be generated in a matter of seconds.

At the component level, generating hundreds or thousands of different candidate shapes became possible, which, in combination with the emerging field of numerical analysis codes capable of simulating the performance of these shapes, gave engineers a powerful design tool.

This new capability created a need for developments in the mathematics of shapes of relevance to aerodynamic design. A new breed of curve and surface models was needed that was equipped with simple handles: *design variables*, the values of which could be altered, leading to intuitive shape changes. This had previously been of little interest, as redrawing a

Figure 1.3 External (or *outer mould line*) geometry – even within the realm of fixed-wing aircraft, the variety and sophistication of surface shapes poses a serious geometry modelling and optimal design challenge (photographs by A. Sóbester).

blueprint or sanding the wind tunnel model of a geometry to a slightly adjusted shape had not required such formalisms.

The Hicks–Henne basis functions mentioned in the Preface (we will return to it later in this text) were one example of such developments. At the same time a much older idea gained a ‘second life’.

In the 1930s, long before the earliest electronic computers, the National Advisory Committee for Aeronautics (NACA) had developed a wing section model defined through two pairs of parametric polynomials. By altering the parameter values, a family of designs could be built up, comprising airfoils suitable for a wide range of applications. As we shall see in Chapter 6, members of this family are still ‘in service’ eight decades later on a number of aircraft. But, perhaps even more importantly, in the new era of computational geometry modelling and numerical flow simulation the NACA sections suddenly became the template for just the type of mathematical formulation demanded by the new technology. The three components of the modern computational design process architecture, as we know it today, crystallized: a parametric shape description, a physics-based simulation code capable of measuring the performance of a given instance of the parametric shape and a systematic means of varying the parameters in a way that enabled progress in an efficient manner towards the optimum solution.

This book is about the intellectual descendants of the NACA wing section model: parametric geometries that make the most of the combined capabilities of analysis codes and optimization heuristics. In addition to reviewing existing formulations and their place in the aerospace engineer’s tool set, we also set out the principles we recommend to be followed in designing new formulations. And, in an age when the computational cost of performance analysis is falling as predicted by Moore’s law and optimization heuristics represent a fully mature technology, the importance of the effective use of the third element, geometry parameterization, cannot be overstated.

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Geometry Parameterization: Philosophy and Practice

The subject of this book is the parametric geometry of bodies that are of interest in aerodynamic design. We use the term 'geometry' to cover two aspects of the mathematical representation of such bodies: their shape and scale (or size). Before we begin our exploration of the tremendous variety of aerodynamic shapes, we endeavour to clarify the place of both concepts in aerodynamics.

2.1 A Sense of Scale

2.1.1 Separating Shape and Scale

It is perhaps a little counter-intuitive to begin a chapter (and, indeed, a book) on geometry parameterization with a brief foray into fundamental aerodynamics, but the idea of separating the two fundamental building blocks of geometry is grounded in aerodynamics. We shall thus set out the reasons for this in what follows, in the context of the basic mechanics of viscous, incompressible flow.

At this point, then, the reader anxious to dive straight into the matters at the heart of this text – that is, how to build geometry models suitable for efficient optimization – may wish to skip the remainder of this section, simply taking note of the fact that it is often convenient, from a design point of view, to create some degree of isolation between the shape and the scale of a geometry. Those wishing to know *why*, should read on.

Consider the body immersed in viscous, incompressible flow, as shown in Figure 2.1. The velocity field within the fluid is described by the momentum equation

$$\frac{\partial \Omega}{\partial t} + \nabla \times (\Omega \times v) = \frac{\mu}{\rho} \nabla^2 \Omega, \tag{2.1}$$

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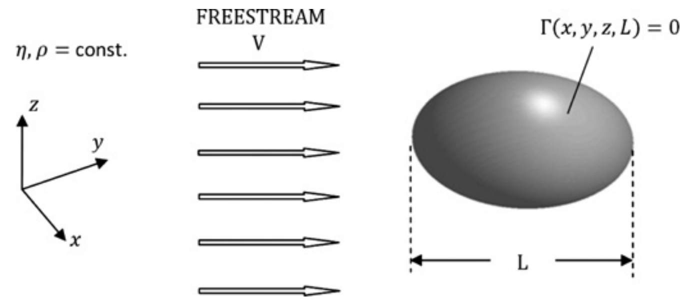

Figure 2.1 A thought experiment: incompressible flow around a body with some characteristic dimension L.

where the vorticity $\Omega = \nabla \times v$.[1] As the fluid is considered incompressible, we assume that the divergence of the velocity field is zero ($\nabla \cdot v = 0$, $\rho =$ const.) – this is true when the Mach number is small ($M \ll 1$).

In terms of the boundary conditions of (2.1), we assume that at a long distance away from the body the velocity of the flow (say, parallel with the x-axis) is V and that it vanishes on the surface of the body. The shape of the body is described in terms of the Cartesian coordinates of the space as $\Gamma(x, y, z, L) = 0$, where L is some characteristic dimension.[2] Thus, we set

$$v_x(x, y, z) = v_y(x, y, z) = v_z(x, y, z) = 0, \forall(x, y, z) \text{ satisfying } \Gamma = 0. \tag{2.2}$$

Let us now consider a new system of units to describe this flow field. If we express all lengths in terms of L and all velocities in terms of the freestream velocity V (and, implicitly, time in units of L/V), we can make the following substitutions:

$$x \to x'L, \quad y \to y'L, \quad z \to z'L, \tag{2.3}$$

as well as

$$v \to v'V \quad (v' = 1 \text{ in the freestream}) \tag{2.4}$$

and

$$t \to t' \frac{L}{V} \tag{2.5}$$

(we use the same notation for all other quantities; that is, the prime denotes a quantity expressed in the new set of units).

[1] For a complete derivation and a more detailed discussion of the line of thought set out below, see the legendary notes by Feynman *et al.* (1964).

[2] For example, a sphere of diameter L could be described in this way as $\Gamma : x^2 + y^2 + z^2 - L^2/4 = 0$.

In order to transform (2.1) to the new units, we look at vorticity first:

$$\Omega = \nabla \times v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{1}{L}\frac{\partial}{\partial x'} & \frac{1}{L}\frac{\partial}{\partial y'} & \frac{1}{L}\frac{\partial}{\partial z'} \\ v'_x V & v'_y V & v'_z V \end{vmatrix}. \tag{2.6}$$

This is a neat way of writing the curl of the velocity field, though it is a slight abuse of the determinant notation. Nonetheless, the manoeuvre is not a particularly dangerous one, so long as we are aware that the ‘products’ in the development of the determinant actually mean ‘applications of the differential operators to the terms in the third row’:

$$\Omega = \mathbf{i}\left[\frac{V}{L}\frac{\partial v'_z}{\partial y'} - \frac{V}{L}\frac{\partial v'_y}{\partial z'}\right] - \mathbf{j}\left[\frac{V}{L}\frac{\partial v'_z}{\partial x'} - \frac{V}{L}\frac{\partial v'_x}{\partial z'}\right] + \mathbf{k}\left[\frac{V}{L}\frac{\partial v'_y}{\partial x'} - \frac{V}{L}\frac{\partial v'_x}{\partial y'}\right]. \tag{2.7}$$

Thus, the vorticity expressed in terms of the new units of velocity and length can be written as

$$\Omega = \frac{V}{L}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x'} & \frac{\partial}{\partial y'} & \frac{\partial}{\partial z'} \\ v'_x & v'_y & v'_z \end{vmatrix} = \frac{V}{L}\nabla' \times v' = \frac{V}{L}\Omega'. \tag{2.8}$$

Applying the substitutions to (2.1) now, we arrive at

$$\frac{\partial \Omega'}{\partial t'} + \nabla' \times (\Omega' \times v') = \frac{\mu}{\rho V L}\nabla^2 \Omega'. \tag{2.9}$$

As a result of this normalization of the problem we now have an equation that *isolates the effect of the scale* (size) of the body on the velocity field. The scale is encapsulated in the coefficient of the right-hand-side term. This coefficient, which uniquely holds the scale information (or, rather, the inverse of this coefficient), is, of course, the *Reynolds number*:

$$Re = \frac{\rho V L}{\mu}. \tag{2.10}$$

2.1.2 Nondimensional Coefficients

This identification of the effect of scale on the flow field is especially important in experimental aerodynamics. The Reynolds number tells us how we can compensate for the difference in scale between real aircraft and their wind tunnel models: the model-scale flow field will be similar to the real flow field if we increase the flow velocity, increase the density (typically by a reduction of the temperature in *cryogenic* wind tunnels) or (less practically) decrease the viscosity. All this, of course, is only true if the Mach number stays below about 0.6 – we must not forget that the compressibility term has been omitted from (2.1) and at higher Mach numbers this would invalidate the analysis (ρ will no longer be a constant).