K. Ramasubramanian
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Tantrasaṅgraha of Nīlakaṇṭha Somayājī

Revised and Expanded Edition

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समर्पणम्

नानायुक्त्याक्तशोभं न मितमनधिकं तन्त्रजालाभिनीतं
ग्रासच्छायादिकार्ये बुधभृगुविषये युक्तमार्गं दिशन्तम्।
प्राणैषीत् ग्रन्थरत्नं गणितविदमलो तन्त्रसङ्गतिकाख्यं
गार्ग्यः श्रीनीलकण्ठः कृतबहुलकृतिर्भाग्यमेतद्बुधानाम्॥

व्याख्यालेख्यविशेषवृत्तिभरितं वो धीजुषां सम्मुदे
प्राकाश्यन्ननु तन्त्रसङ्गहममुं प्रीत्या नयामोऽधुना।
श्रीमन्माधवकेरलीयगणितज्योतिर्विदां संहतेः
सन्तुष्ट्यै च समार्पयाम विबुधाः हृष्टान्तरङ्गा वयम्॥

The scholarly world is indeed fortunate that Nīlakaṇṭha—an astronomer-mathematician hailing from the Garga lineage and blessed with a clear intellect—composed, among several other works, a treatise called the *Tantrasaṅgraha*, which is considered to be a gem among works [in astronomy], resplendent with a variety of *yuktis*, neither terse nor too elaborate, and which gives far more accurate procedures for solving problems involving eclipses, shadow measurements, [the application of the equation of centre] in the case of Mercury and Venus, and so on.

We are now extremely happy to bring out this treatise, the *Tantrasaṅgraha*, along with translation, annotation and detailed mathematical exposition for the intellectual delight of the scholarly community. We are also immensely pleased to dedicate this work of ours to Mādhava and the galaxy of other astronomers that this great lineage has produced.

Foreword

In the history of mathematics and astronomy in India, the Kerala school which flourished during the fourteenth–seventeenth centuries CE, has a unique position. Mādhava of Saṅgamagrāma, Parameśvara, Nīlakaṇṭha Somayājī, Jyeṣṭhadeva and Śaṅkara Vāriyar were among the luminaries of this school, which made original contributions in mathematics, formulating the infinite series for the trigonometric functions and π, that antedated similar achievements of European mathematicians by a couple of centuries. The origin of calculus also can be traced to this school.

In astronomy too, the Kerala school had significant achievements. The versatile astronomer Nīlakaṇṭha Somayājī (1444-1545) produced several works on astronomy, of which the *Tantrasaṅgraha* (about 430 verses in *anuṣṭubh* metre in eight sections or *prakaraṇas*) is a comprehensive treatise. He introduced in this elegant work a major revision of the traditional Indian planetary model, a detailed geometrical picture of which is discussed in his two small but lucid works—*Siddhāntadarpaṇa* (31 verses) and *Golasāra* (56 verses). According to Nīlakaṇṭha's geometrical picture of planetary motion, the five planets (Mercury, Venus, Mars, Jupiter and Saturn) move in eccentric orbits around the mean Sun, which in turn orbits around the Earth. Such a formulation was put forward by the European (Danish) astronomer Tycho Brahe, nearly a century later. *Tantrasaṅgraha* is also known for its other innovations introduced by Nīlakaṇṭha.

In March 2000, the Department of Theoretical Physics, University of Madras, organized a conference in collaboration with the Indian Institute of Advanced Studies, Shimla, to celebrate the 500th anniversary of *Tantrasaṅgraha*. Though the importance of this text was known to historians of Indian astronomy for quite some time, and several research papers have been published on the original ideas presented in this work, there was a great need for an accurate English translation of this seminal treatise, with detailed notes in modern notation. Profs K. Ramasubramanian and M. S. Sriram, who have the linguistic and subject expertise, have fulfilled this need admirably. As may be noted from

the current volume, every attempt has been made by the authors to make the work as self-contained as possible by giving detailed explanations as well as several explanatory appendices besides a glossary and bibliography. Historians of astronomy, both Indian and foreign, are most grateful indeed to them for their devoted efforts in bringing out this publication.

The authors are already well known for their studies and publications in the area of Indian mathematics and astronomy. Together with another savant, M. D. Srinivas of the Centre for Policy Studies, Chennai, they were involved in preparing a detailed explanatory notes for *Gaṇita-yuktibhāṣā* of Jyeṣṭhadeva, which was published in two volumes by the Hindustan Book Agency, New Delhi with a critical edition of the text and English translation by K. V. Sarma, an eminent scholar who published several works on Indian astronomy. A reprint of this work was also brought out recently by Springer, the noted international publishers, to make it available for the international readership. In fact, Jyeṣṭhadeva, who was a junior contemporary of Nīlakaṇṭha, at the commencement of his work states that his aim in composing the work is to explain the calculational procedures given in *Tantrasaṇgraha*. It is but fitting, therefore, that Profs Ramasubramanian and Sriram, who were involved with the production of explanatory notes of *Gaṇita-yuktibhāṣā*, are the authors of the present volume on *Tantrasaṅgraha*.

The Hindustan Book Agency has been rendering yeoman service to scholars interested in the history of mathematics, by bringing out several volumes in its series 'Culture and History of Mathematics'. I am happy that in collaboration with Springer it is publishing the present work on *Tantrasaṅgraha*, which, I am sure, will be of great value to historians of science in general and of astronomy in particular. It is my fond hope that several other timeless works of this type will emerge from the pens of these erudite authors in future.

Bangalore
March 2010

B. V. Subbarayappa
Former President, International Union of History and Philosophy of Science

Preface to the revised and expanded edition

The first edition of this book, published jointly by the Hindustan Book Agency (HBA) and Springer in 2011, received an overwhelmingly positive response from readers and scholars alike. Its encouraging reception underscored the book's value in advancing discussions in its field. However, with the first edition having been out of print for the past few years, there has been a growing demand for its rerelease. To meet this demand, we are delighted to present the second (revised and expanded) edition. In this edition, we have included necessary updates and also expanded coverage in select sections from the original edition.

As indicated in the preface to the first edition, the traditional Indian planetary model's modification by Nīlakaṇṭha in the *Tantrasaṅgraha* initially drew our attention to the work. However, this topic is only briefly addressed in the text, as it primarily serves as a *tantra* text focused on computational algorithms. Fortunately, Nīlakaṇṭha has explored his model in greater depth, along with a detailed geometrical depiction of planetary motion, in his other works. According to his geometrical framework, the planets follow eccentric orbits around the Sun, which, in turn, orbits the Earth. This quasi-heliocentric model closely resembles the one proposed by Tycho Brahe in the late 16th century, albeit derived from entirely different considerations.

In collaboration with M. D. Srinivas, we conducted a detailed analysis of this model and published our findings in the Indian journal Current Science in 1994. Since then, we have revisited and deepened our understanding of the subject. Appendix F, co-authored with M. D. Srinivas, offers an updated perspective on the traditional Indian planetary model and its revision by Nīlakaṇṭha.

Beyond the planetary model, the work introduces several significant innovations in mathematical techniques, including accurate sine table computations, the use of series expansions for sine and cosine functions, and a systematic approach to solving spherical astronomical problems.

It was noted that the first edition contained a few typographical errors, and some of the explanatory notes required editing for improved clarity. Additionally, some figures needed revision for better comprehensibility. All these aspects

have been addressed in this edition. While no major changes have been made to the content, there are notable updates to the style and format. In this edition, the typesetting of *Devanāgarī* has been done with the use of Śobhikā fonts, and all diagrams have been redrawn using the 'tikz' package. These improvements have significantly enhanced the overall presentation and visual quality of the work.

In the preparation of the revised edition, we received immense help from some of the project staff and research scholars working at the Cell for Indian Science and Technology in Sanskrit (CISTS), IIT Bombay. The help received was both in terms of proof-reading as well as redrawing all the diagrams using the 'tikz' package. In particular, we would like to acknowledge Mandadi Varuneshwar Reddy, Sreelekshmy C. J. and Ashwini Devaraj. The countless hours they would have spent in the process of getting this revised edition by meticulously working on the earlier volume is deeply appreciated. We would also like to record our gratitude to all those involved in making funds available to support the project staff through the Science and Heritage Initiative (SandHI) at IIT Bombay. Our special thanks go to Jainendra Jain and Devendra Jain of the HBA, New Delhi, for publish-ing the first edition of this work in collaboration with Springer, London, and urging us to prepare the current edition. Though HBA requested us to submit the manuscript more than a couple of years ago, there has been significant delay from our part. We deeply appreciate their patience in bearing with us the extraordinary delay on our part.

August 15, 2024

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Preface to the first edition

Tantrasaṅgraha composed in 1500 CE by the Kerala astronomer Nīlakaṇṭha Somayājī, has long been recognized as an important Indian text in astronomy. It is a comprehensive text which discusses all aspects of mathematical astronomy such as the computation of the longitudes and latitudes of planets, various diurnal problems, the determination of time, eclipses, the visibility of planets etc. There are two critical editions of the Sanskrit text, by S. K. Pillai in 1958 and K. V. Sarma in 1977, which between them include the commentaries *Laghu-vivṛti* in prose for the entire text, and *Yukti-dīpikā* in verses for the first four chapters, both of which are composed by Śaṅkara Vāriyar. The need has long been felt for an English translation of the work, with detailed explanatory notes in modern notation, so that the work is accessible to a larger audience. It is with this objective that we began a project on *Tantrasaṅgraha*, funded by the Indian National Science Academy (INSA), in 2000.

Meanwhile, along with M. D. Srinivas (Centre for Policy Studies, Chennai), we were involved in preparing detailed explanatory notes for *Gaṇita-yukti-bhāṣa* (GYB) of Jyeṣṭhadeva, edited and translated by K. V. Sarma, and published in 2008 by the Hindustan Book Agency and reprinted in 2009 by Springer. Though the work on GYB caused delay in the publication of the present work, it was very rewarding as GYB gives detailed explanations of most of the algorithms in *Tantrasaṅgraha*, and provides valuable insights on many topics covered in that work.

Scholars in the area of the history of astronomy in general, and Indian astronomy in particular, form the natural readership for this work. However, keeping the larger readership—anyone wanting to know the methods of Indian astronomy—in mind, we have attempted to make it as self-contained as possible, so that any motivated person with a sound background in mathematics at the final school (+2, as it is termed in India) level and interested in spherical astronomy will find it useful. We have also included a glossary of frequently occurring Sanskrit terms and several appendices that should serve to clarify many concepts relevant to the topics in the main text.

The modification of the traditional Indian planetary model by Nīlakaṇṭha in *Tantrasaṅgraha* is what attracted us to the work initially. But this topic is dealt with all too briefly in it, as it is a *Tantra* text devoted mainly to computational algorithms. However, Nīlakaṇṭha has discussed his model extensively, along with his geometrical picture of planetary motion, in other works. In collaboration with M. D. Srinivas, we had made an incisive study of this model and published a paper on it in the Indian journal *Current Science* way back in 1994. Since then we have had occasions to study this in more detail. Appendix F, on the traditional Indian planetary model and its revision by Nīlakaṇṭha Somayājī, of which M. D. Srinivas is a co-author, reflects our current understanding on this subject.

We are deeply indebted to Prof. M. D. Srinivas—our collaborator on the different aspects of studies on Indian astronomy and mathematics that we have been doing for almost two decades now—for meticulously going through the entire manuscript and offering several valuable suggestions. We would also like to acknowledge the suggestions given by the two anonymous referees for improving the manuscript. We are grateful to (the late) Prof. K. V. Sarma with whom we have had an extensive collaboration, especially during the preparation of *Yuktibhāṣā*, and who has been a source of great inspiration for us. We would like to thank Profs C. S. Seshadri and R. Sridharan of the Chennai Mathematical Institute for their continued support and encouragement. Our special thanks go to Prof. B. V. Subbarayappa, Bangalore, the doyen of the history of science in India, for having readily agreed to write the Foreword to this work.

Our heart-felt thanks are due also to Profs S. Balachandra Rao of Bangalore, and V. Srinivasan of Hyderabad (currently with the University of Madras) for their constant and vociferous support to us over all the years in all our work on Indian astronomy. We also thank Profs P. M. Mathews, G. Bhamati, M. Seetharaman, S. S. Vasan, K. Raghunathan, A. S. Vytheeswaran, R. Radhakrishnan, and Dr Sekhar Raghavan associated with the Department of Theoretical Physics, University of Madras, for their kind and active interest in our work over the years. We would like to acknowledge the keen interest expressed by Swami Atmapriyananda, Belur Math, in promoting studies in Indian astronomy and mathematics. We are indeed grateful to Profs David Mumford of Brown University and Manjul Bhargava of Princeton University for their kind encouragement and enthusiastic support. It is a pleasure to thank Profs S. M. R. Ansari, A. K. Bag, Jitendra Bajaj, V. Balakrishnan, A. V. Balasubramanian, Rajendra Bhatia, S. G. Dani, Sinniruddha Dash, Amartya Datta, P. C. Deshmukh, P. P. Divakaran, Raghavendra Gadagkar, George Joseph, Rajesh Kocchar, S. Madhavan, Madhukar Mallayya, N. Mukunda, Roddam Narasimha, M. G. Narasimhan, Jayant Narlikar, C. K. Raju, Sundar Sarukkai, B. S. Shylaja, Navjyoti Singh, S. P. Suresh, T. Trivikraman, Mayank Vahia, Padmaja Venugopal and K. Vijayalakshmi, as well as Profs Mohammad Bagheri, Subhash Kak, Agathe Keller, Francois Patte, Kim Plofker,

T. R. N. Rao, S. R. Sarma and Michio Yano for their kind interest in our work in Indian astronomy and mathematics in general, and this work in particular.

One of the authors (Ramasubramanian) would like to acknowledge the unstinting support and encouragement received from the former Director of IIT Bombay, Prof. Ashok Misra, and other IIT Bombay fraternity members, particularly Profs. S. D. Agashe, Rangan Banerjee, Jayadeva Bhat, S. M. Bhave, Amitabha Gupta, Devang Khakhar, Malhar Kulkarni, Ravi Kulkarni, H. Narayanan, Prabhu Ramachandran, Krithi Ramamritham, H. S. Shankar, G. Sivakumar, A. K. Suresh, Sivaramakrishnan and Jugal Verma. Special thanks are also due to K. Mahesh, U. K. V. Sarma, R. Venketeswara Pai, Dinesh Mohan Joshi and Vanishri Bhat for their devoted assistance at different stages of the preparation of the manuscript. In fact, we are particularly indebted to Vanishri for spending countless hours in setting a uniform style for all the figures that appear in the book and U. K. V. Sarma for carefully proof reading the entire text.

This work is the outcome of a project sanctioned by INSA, New Delhi, during October 2000–March 2004. We would also like to place on record our gratitude to the Sir Dorabji Tata Trust and the National Academy of Sciences, India, for their financial assistance by way of projects, which was extremely useful in offering fellowship to the project staff as well as in the production of the manuscript in a camera-ready form. We are deeply indebted to INSA for the financial support as well as for readily granting the permission to publish the work. Our special thanks go to Jainendra Jain and Devendra Jain of the Hindustan Book Agency, New Delhi, for graciously coming forward to publish this work in collaboration with Springer, London.

Finally, the authors are grateful to the copy-editor(s) for going through the manuscript meticulously, and making valuable comments and suggestions.

विकृति-आश्वयुजशुक्लदशमी, कल्यब्द ५१११ **K. Ramasubramanian**

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October 17, 2010 **M. S. Sriram**

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