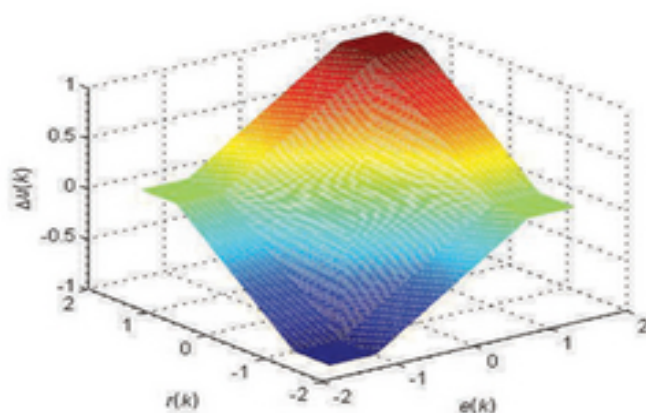
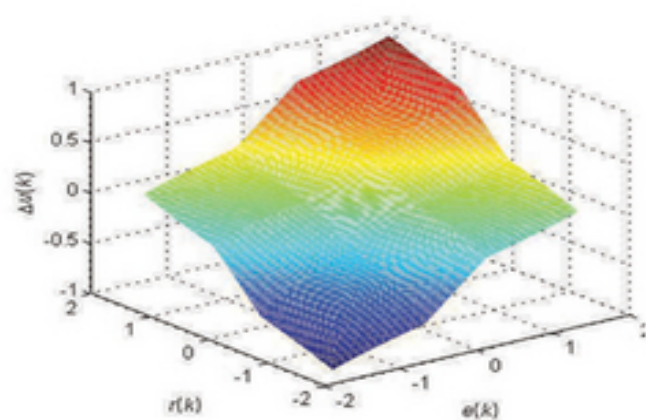




IEEE Press Series on Computational Intelligence
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INTRODUCTION TO TYPE-2 FUZZY LOGIC CONTROL

THEORY AND APPLICATIONS



JERRY M. MENDEL, HANI HAGRAS, WOEI-WAN TAN,
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Introduction to Type-2 Fuzzy Logic Control

Theory and Applications

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IEEE Press Series on Computational Intelligence
David B. Fogel, Series Editor



IEEE

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Library of Congress Cataloging-in-Publication Data:

Mendel, Jerry M., 1938-

Introduction to type-2 fuzzy logic control : theory and applications / Jerry M. Mendel,

Hani Hagra, Woei-Wan Tan, William W. Melek, Hao Ying.

pages cm

Includes bibliographical references and index.

ISBN 978-1-118-27839-0 (cloth)

1. Automatic control. 2. Fuzzy systems. I. Hagra, Hani. II. Tan, Woei-Wan. III. Melek,

William W. IV. Ying, Hao, 1958- V. Title.

TJ217.5.M46 2014

629.8'95633-dc23

2014000084

To
the memory of
Ebrahim Mamdani (1943-2010)
Founder of Fuzzy Logic Control

Preface

When Lotfi Zadeh invented fuzzy sets in 1965, he never dreamt that the field in which they would be most widely used would arguably be the one that became the most hostile to the concept of fuzziness, namely control. Perhaps this was because the word “fuzzy” in Western civilization does not have a positive connotation and suggests an abandonment of mathematical rigor, one of the cornerstones of control. Perhaps it was because some famous mathematical probabilists (incorrectly) claimed that there was no difference between a fuzzy set and subjective probability. Perhaps it was because for almost a decade, until the 1974 seminal paper by Prof. Ebrahim Mamdani, who founded the field of fuzzy logic control and to whose memory our book is dedicated, there were no substantial real-world applications for fuzzy sets. Or, perhaps, it was because after the founding of this field many exaggerated claims were made by the fuzzy logic control community that flew in the face of mathematical rigor and did not pay attention to the same metrics that were and still are the cornerstones for control and cannot be ignored.

Now, 40 years after Mamdani's seminal paper, fuzzy logic control using regular (i.e., type-1) fuzzy sets and logic has been extensively studied, applied to practical problems, and is very widely used in many real-world applications. It can and has been studied with the same level of mathematical rigor that control theorists are accustomed to, and is now considered a matured field; however, it still has some shortcomings. Its major shortcoming (in the opinions of the authors of this book) goes back to one of the earliest criticisms made about a type-1 fuzzy set, namely the unfuzziness of its membership function, that is, the

word “fuzzy” has the connotation of being uncertain. But how can this connotation be captured by a membership function that is completely certain?

Importantly, in 1975 Zadeh introduced more general kinds of fuzzy sets in which their membership function grades are themselves fuzzy. The two most widely studied of these are *interval-valued fuzzy sets* and *type-2 fuzzy sets*. For the former, the membership grade is a uniformly weighted interval of values, whereas for the latter the membership grade is a nonuniformly weighted interval of values. Obviously, interval-valued fuzzy sets are a special case of type-2 fuzzy sets and are therefore called by many (as we do in this book) *interval type-2 fuzzy sets*.

Why should using type-2 fuzzy sets be of interest to the fuzzy logic control community? This question is answered in great detail in this book, but two short answers are: (1) they are more robust to system uncertainties and can provide better control system performance than type-1 fuzzy sets; and (2) there is now more than a critical mass of papers that have been published that demonstrate these improvements for many real-world applications.

Because of the lack of basic calculation methods for type-2 fuzzy sets in their early days, type-2 fuzzy logic controllers (T2 FLCs) did not emerge until fairly recently. Things have changed a lot during the past decade, so that type-2 fuzzy logic control (which is still an emerging field) now has the attention of the fuzzy systems community, and, as a result of this, the number of publications on it is growing quickly.

Recall that the central themes of any control methodology, fuzzy or conventional, are (1) to analyze various aspects of a control system and (2) to design a control system to achieve given user specifications. This book focuses on both topics for T2 FLCs and type-2 fuzzy logic control systems. The analysis includes (1) the mathematical

structure of some T2 FLCs, (2) stability of type-2 fuzzy logic control systems, and (3) robustness of the type-2 fuzzy logic control systems.

This book, the first one entirely on T2 FLC, shows how to design type-2 fuzzy logic control systems based on a variety of choices for the T2 FLC components and also demonstrates how to apply type-2 fuzzy logic control theory to applications. It has been written by five of the leading experts on type-2 fuzzy sets, systems, and control, with the help of six contributors. It will be useful to any technical person interested in learning type-2 fuzzy logic control theory and its applications, from students to practicing engineers.

This is an introductory book that provides theoretical, practical, and application coverage of type-2 fuzzy logic control, and uses a coherent structure and uniform mathematical notations to link chapters, which are closely related, reflecting the book's central themes—analysis and design of type-2 fuzzy logic control systems. It has been written with an educational focus rather than a pure research focus. Each chapter includes worked examples, and most refer to their computer codes (programs) accessible through the book's common website, and outline how to use them at some high level. It is a self-contained reference book suitable for engineers, researchers, and college graduate students who want to gain deep insights about type-2 fuzzy logic control.

The book begins with an easy-to-read chapter meant to whet the reader's appetite so that he or she will read on; it explains what the differences are between a type-1 fuzzy set and a type-2 fuzzy set, and a T2 FLC and a T1 FLC, and, it provides many real-world applications in which T2 FLCs have shown marked improvements in performance over T1 FLCs. Chapter 2 provides all of the background material

that is needed about type-2 fuzzy sets so that you can read the rest of the book; its main emphasis is on interval type-2 fuzzy sets because at present they are the most widely used type-2 fuzzy sets in type-2 fuzzy logic control. Chapter 3 is about Mamdani and TSK interval T2 FLCs. Chapter 4 examines the analytical structure of various interval type-2 fuzzy PI and PD controllers. Chapter 5 is about ways to simplify interval type-2 fuzzy PI and PD controllers. Chapter 6 is about the rigorous design of interval type-2 TSK fuzzy controllers. Chapter 7 provides each of the five authors with an opportunity to look into the future of type-2 fuzzy logic control. The book's appendix describes Java-based software that will let the reader examine type-1, interval type-2, and even general type-2 FLCs. All references (which are very extensive) have been integrated into one list that is at the end of the book.

The book's software can be downloaded by means of the following procedure: Software for Examples 4.1 and 4.6 and the examples in Chapter 6 can be accessed at <http://booksupport.wiley.com>, and software for Appendix A, that supports T1, IT2 and GT2 FLCs, is available at <http://juzzy.wagnerweb.net>.

In addition to the five authors, six of their (former) graduate students contributed to this book, to whom the authors are greatly appreciative. Their names are listed in the Contributors List. More specifically, Christian Wagner contributed to Chapters 2, 3 and 7, and prepared the entire Appendix; Xinyu Du and Haibo Zhou contributed to Chapter 4; Maowen Nie and Dongrui Wu contributed to Chapter 5; and Mohammad Biglarbegian contributed to Chapter 6.

The authors gratefully acknowledge material quoted from books or journals published by Elsevier, IEEE, John Wiley & Sons, Mancy Publishing (www.maney.co.uk/journals/irs and www.ingentaconnect.com/content/maney/ias) and Pearsons

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Chapter 1

Introduction

1.1 Early History of Fuzzy Control

Fuzzy control (also known as fuzzy logic control) is regarded as the most widely used application of fuzzy logic and is credited with being a well-accepted methodology for designing controllers that are able to deliver satisfactory performance in the face of uncertainty and imprecision (Lee, 1990; Sugeno, 1985); Feng, 2006). In addition, fuzzy logic theory provides a method for less skilled personnel to develop practical control algorithms in a user-friendly way that is close to human thinking and perception, and to do this in a short amount of time. Fuzzy logic controllers (FLCs) can sometimes outperform traditional control systems [like proportional-integral-derivative (PID) controllers] and have often performed either similarly or even better than human operators. This is partially because most FLCs are nonlinear controllers that are capable of controlling real-world systems (the vast majority of such systems are nonlinear) better than a linear controller can, and with minimal to no knowledge about the mathematical model of the plant or process being controlled.

Fuzzy logic controllers have been applied with great success to many real-world applications. The first FLC was developed by Mamdani and Assilian (1975), in the United Kingdom, for controlling a steam generator in a laboratory setting. In 1976, Blue Circle Cement and SIRA in Denmark developed a cement kiln controller (the first industrial application of fuzzy logic), which went into operation in 1982 (Holmblad and Ostergaard, 1982). In the 1980s, several important industrial applications of fuzzy logic

control were launched successfully in Japan, including a water treatment system developed by Fuji Electric. In 1987, Hitachi put a fuzzy logic based automatic train operation control system into the Sendai city's subway system (Yasunobu and Miyamoto, 1985). These and other applications of FLCs motivated many Japanese engineers to investigate a wide range of novel applications for fuzzy logic. This led to a “fuzzy boom” in Japan, a result of close collaboration and technology transfer between universities and industry.

According to Yen and Langari (1999), in 1988, a large-scale national research initiative was established by the Japanese Ministry of International Trade and Industry (MITI). The initiative established by MITI was a consortium called the Laboratory for International Fuzzy Engineering Research (LIFE). In late January 1990, Matsushita Electric Industrial (Panasonic) named their newly developed fuzzy-controlled automatic washing machine the fuzzy washing machine and launched a major commercial campaign of it as a *fuzzy* product. This campaign turned out to be a successful marketing effort not only for the product but also for fuzzy logic technology (Yen and Langari, 1999). Many other home electronics companies followed Panasonic's approach and introduced fuzzy vacuum cleaners, fuzzy rice cookers, fuzzy refrigerators, fuzzy camcorders (for stabilizing the image under hand jittering), fuzzy camera (for smart autofocus), and other applications. As a result, consumers in Japan recognized the now en-vogue Japanese word “fuzzy,” which won the gold prize for a new word in 1990 (Hirota, 1995). Originating in Japan, the “fuzzy boom” triggered a broad and serious interest in this technology in Korea, Europe, the United States, and elsewhere. For example, Boeing, NASA, United Technologies, and other aerospace companies developed FLCs for space and aviation applications (Munakata and Jani, 1994).

Today FLCs are used in countless real-world applications that touch the lives of people all over the world, including white goods (e.g., washing machines, refrigerators, microwaves, rice cookers, televisions, etc.), digital video cameras, cars, elevators (lifts), heavy industries (e.g., cement, petroleum, steel), and the like.

While this book focuses on type-2 fuzzy logic control, it will also provide background material about type-1 fuzzy logic control. Indeed, before we can explain what type-2 fuzzy logic control is we must briefly explain what type-1 fuzzy sets, type-1 fuzzy logic control, and type-2 fuzzy sets are. In this chapter we do this from a high-level perspective without touching on the mathematical aspects in order to give a feel for the nature of fuzzy sets and their applications. Later chapters in this book provide rigorous treatments of mathematical underpinnings of the subjects just mentioned.

1.2 What Is a Type-1 Fuzzy Set?

Suppose that a group of people is asked about the temperature values they associate with the linguistic concepts Hot and Cold. If *crisp sets* are employed, as shown in [Fig. 1.1a](#), then a threshold must be chosen above which temperature values are considered Hot and below which they are considered Cold. Reaching a consensus about such a threshold is difficult, and even if an agreement can be reached—for example, 18°C —, is it reasonable to conclude that 17.99999°C is Cold whereas 18.00001°C is Hot?

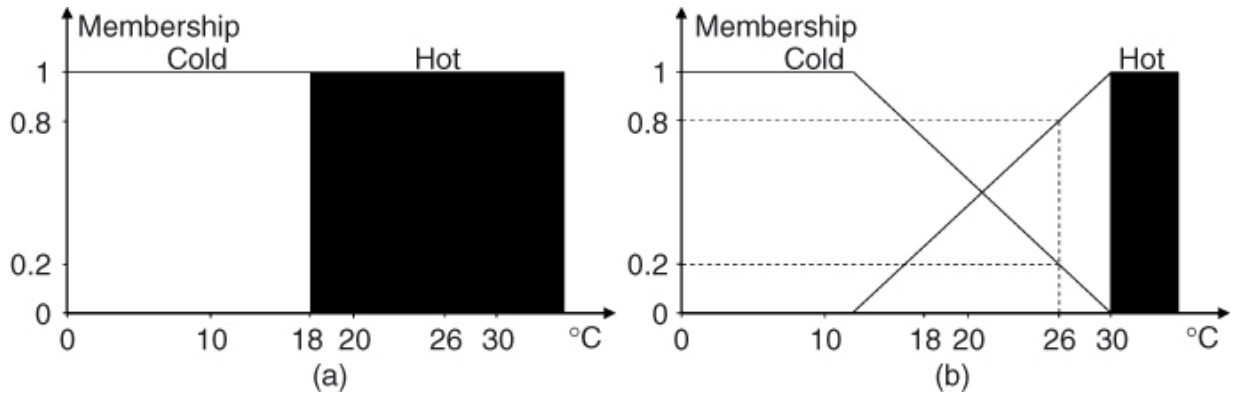


Figure 1.1 Representing Cold and Hot using (a) crisp sets, and (b) type-1 fuzzy sets.

On the other hand, Hot and Cold can be represented as *type-1 fuzzy sets* (T1 FSs) whose membership functions (MFs) are shown in [Fig. 1.1b](#). Note that, prior to the appearance of type-2 fuzzy sets, the phrase *fuzzy set* was used instead of the phrase *T1 fuzzy set*. Even today, in many publications that focus only on T1 FSs, such sets are called fuzzy sets. In this book we shall use the phrase *type-1 fuzzy set*. Returning to [Fig. 1.1b](#), observe that no sharp boundaries exist between the two sets and that each value on the horizontal axis may simultaneously belong to more than one T1 FS but with different degrees of membership. For example, 26°C, which is in the crisp Hot set with a membership value of 1.0 ([Fig. 1.1a](#)), is now in that set to degree 0.8, but is also in the Cold set to degree 0.2 ([Fig. 1.1b](#)).

Type-1 FSs provide a means for calculating intermediate values between the crisp values associated with being absolutely true (1) or absolutely false (0). Those values range between 0 and 1 (and can include them); thus, it can be said that a fuzzy set allows the calculation of shades of gray between white and black (or true and false). As will be seen in this book, the smooth transition that occurs between T1 FSs gives a good decision response for a type-1

fuzzy logic control system in the face of noise and other uncertainties.

1.3 What Is a Type-1 Fuzzy Logic Controller?

With the advent of type-2 fuzzy sets and type-2 fuzzy logic control, it has become necessary to distinguish between *type-2 fuzzy logic control* and all earlier fuzzy logic control that uses type-1 fuzzy sets (the distinctions between such fuzzy sets are explained in Section 1.4). We refer to fuzzy logic control that uses type-1 fuzzy sets as *type-1 fuzzy logic control*. When it does not matter whether the fuzzy sets are type-1 or type-2, we just use *fuzzy logic control* or *fuzzy control*.

Fuzzy logic control aims to mimic the process followed by the human mind when performing control actions. For example, when a person drives (controls) a car, he/she will not think:

If the temperature is *10 degrees Celsius* and the rainfall is *70.5 mm* and the road is *40% slippery* and the distance between my car and the car in front of me is *3 meters*, then I will depress the acceleration pedal only *10%*.

Instead, it is much more likely that he/she thinks:

If it is *Cold* and the rainfall is *High* and the road is *Somewhat Slippery* and the distance between my car and the car in front of me is *Quite Close*, then I will depress the acceleration pedal *Slightly*.

So, in systems controlled by humans, the control cycle starts by a person converting a physical quantity (e.g., a distance) from numbers into words or perceptions (e.g., *Quite Close distance*). The input words (or perceptions)