STANISLAW RACZYNSKI

MODELING AND SIMULATION

THE COMPUTER SCIENCE OF ILLUSION

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Modeling and Simulation

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Modeling and Simulation: The Computer Science of Illusion
Stanislaw Raczynski

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The Computer Science of Illusion

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In tribute to
My Parents, Stanisław and Bronisława

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Editorial Foreword

As a first book in the series on *Computer Simulation and Modeling,* the volume entitled *Modeling and Simulation: The Computer Science of Illusion* by Stanislaw Raczynski, leaves little to be desired. Written by one of the leading contributors to the field of simulation and modeling, the book combines a lucid and authoritative exposition of the fundamentals of the mathematical modeling of systems with an insightful discussion of practical applications and a stimulating cross-referencing of the philosophy of computer simulation.

Venturing beyond the standard continuous and discrete simulation, the author highlights the importance of uncertainty in system models that is due to both the approximation of physical reality with idealized mathematical constructs, and our limited ability to measure the physical phenomena accurately. In this context, the discussion of differential inclusions provides a basis for a much richer interpretation of simulation results that emphasizes the full spectrum of the possible behaviors of systems rather than just focusing on one possible system simulation trajectory. Using the differential inclusion approach, the effect of uncertainties associated with physical systems is presented more clearly and, as such, it leads to better decision making based on simulation.

It is a credit to the author that the theoretical material is made very approachable through ample use of examples, and through the reference to the author's own simulation software system called *PASION (PAScal simulatION).* The versatility of this simulation environment is attested by its application to such diverse fields as the simulation of oscillating gas flow, the simulation of general relativity phenomena, and the simulation of dynamic interactions between terrorist and antiterrorist organizational structures.

In conclusion, Professor Raczynski has produced a text that makes a major contribution to a better understanding of the analytical and cognitive power of systems modeling and simulation. Without a doubt, *Modeling and Simulation* is essential reading for anyone interested in deploying computer simulation as an investigative tool that facilitates learning about the physical reality, be it in the engineering domain, natural sciences, or social sciences.

Andrzej Bargiela (Series Editor)
Nottingham, UK

Preface

First of all, we should define what we are talking about. Perhaps providing good definitions is the most difficult thing in any attempt to communicate something to others. The Encyclopedia Britannica defines illusion as "*A misrepresentation of a "real" sensory stimulus – that is, an interpretation that contradicts objective "reality" as defined by general agreement*". Other definitions refer to "*The use of trickery to perform feats that seem to defy conventional explanation*" (NationMaster.com). Usually, illusion is performed before an audience who are ignorant of the type of trick being used. As scientists claim to look for the truth, it might be supposed that there is no room for any trickery in scientific work. On the other hand, the existence of absolute scientific truth is rather questionable. One could say that all our perceptions from the "real world" are illusions and that it is not clear what the "real world" is at all. But these are rather philosophical questions, and this book does not deal with philosophy.

Let us observe that, in fact, what scientists are doing is showing an "ignorant audience" results from their research that frequently defy conventional explanation. Most of the fundamental discoveries over the thousands of years of human history have been facts contradicting common sense and any previously known explanation. For example, some time ago everybody "knew" that the earth was flat, until the audience accepted that it is something like a sphere. Then, a crazy idea appeared, saying that the earth is orbiting around the sun and not vice versa. Contradicting common sense is perhaps the most important driving force of new discoveries.

By talking about modeling and simulation (see *Chapter 1* for an attempt to define these topics) we can find elements of both science and illusion. The *simulationist* (an individual who makes simulations) uses certain simulation tools (physical or conceptual models, or computer hardware and software) to demonstrate something before an ignorant audience. This ignorance only refers to the tools (tricks) that the simulationist is using. The audience may be another scientist, the highly trained personnel of a design division or the government of a state. Obviously, the aim of the simulations performed by the simulationist should be to learn the truth (true behavior of the simulated system); otherwise, the simulationist fails to follow his professional ethics. The difference between a professional illusionist and a professional simulationist is that the former wants to neglect the truth and the latter looks for the truth. But, in any case, the result of the simulations is always an approximation and simplification of the reality, so it is some kind of an illusion. Following similar, somewhat metaphysical, considerations, we should ask the fundamental question: is the "real" world, including all of us, a part of a simulation run on a huge cosmic supercomputer? We will certainly not be able to find an answer. Perhaps, if we could prove that real time is not a continuous but a discrete variable, this might be a hint. But, reversing the question, we should consider the possibility of creating artificial, simulated beings in our computers. This is nothing new; such simulations have already been done. There is even a whole organization, *International Society of Artificial Life*, which organizes international conferences on this topic and works on the creation of artificial “living” beings inside simulation programs. These beings are rather simple compared to real living organisms. However, the level of complexity is not so important. In the future, such programs will become highly sophisticated, and the simulated creatures will be equipped with advanced artificial intelligence. The questions that may be asked are: can we shut off a program that has created a population of artificial intelligent beings? Do these beings exist? And what if they have their own conscience? Is it not arrogant for us to claim that WE are real and that the world we simulate is artificial?

The highest level of “computer science illusion” is achieved by combining simulation with *virtual reality*. But we should distinguish between those fields. Virtual reality creates visual illusions using highly advanced computer graphics and animation, while simulation is the model implementation on a computer, with or without any graphical animation output. Virtual reality creates fictitious, three-dimensional environments and allows the user to move over it. However, this is not the topic of this book. A good review of the role of virtual reality in simulation, in particular in the applications in medicine and geo-science can be found in the book of Moeller (2004).

We should clearly state what this book is NOT about. This book is not a fundamental book on computer simulation. It is assumed that the reader has basic knowledge about computer science and mathematics (mainly statistics and probability, differential equations, and the dynamics of physical systems). This text is focused on both existing methods and possible new, alternative approaches to modeling and simulation, illustrated by a series of (rather nontypical) case studies.

Looking at the annals of the publications in the field of modeling and simulation (M&S) and comparing the proceedings of simulation conferences held over the last 30 years, one can have an impression that, for more than the last three decades, the problems being discussed and the methods used are the same. For example, a common paradigm in continuous simulation consists in a strange conviction that all that is continuous can be described by differential equations. Other common practice in the field of fluid-dynamics modeling is to experiment with two-dimensional models, which are false and represent the real 3D fluid dynamics neither in quantitative, nor in qualitative aspects. If we disregard the graphical part and attractive GUIs (graphical user interface), we can see that many tools like GPSS and Simula67 (developed four decades ago) have never been surpassed. This does not mean that no progress is being made in M&S.

The simulation discipline has now expanded to include the modeling of systems that are human-centered (social, economical, commercial), thus containing a large amount of uncertainty. Those new fields of applications make M&S a dynamically expanding discipline. However, in my opinion, there is a growing gap between the new problems and the methodology. In particular, we need a stronger research effort to be made in continuous simulation methodology and numerical methods. As for the problem of uncertainty, considerable and promising efforts are being made, as mentioned in the following chapters. Another important question is model validation. In fact, we have few practical validation methods, and, in most cases, it is rather difficult to prove that the model we use is "absolutely" valid. But, looking at the System Dynamics methodology (which is 40 years old!) we observe that it is still used in many application fields, though the model developers hardly care about model validity. In my opinion, most of the recently created models are invalid, even in a simplified "input–output" sense. However, this does not mean that they are not useful. In any case, modeling and simulation is something between science and art, frequently resembling the art of illusion.

One of the aims of this book is to stimulate the reader to look for new M&S tools. Amongst the examples in the following chapters we discuss such tools as differential inclusions, semidiscrete events and a possible metric structure in the space of models.

As stated before, this is not a book on virtual reality, computer graphics, and games. Also, there are few comments on (physical) simulators here. Simulators are closely related to simulation, because they are "driven" by simulation software. By simulators we mean computer-controlled physical devices mainly used for training in vehicle operation, aerospace flight, military operations etc. Simulators could be a good topic for a separate book, but there is no room here to deal with them in greater detail.

Finally, you will observe that some of the topics are described with more detail and emphasis, while others are mentioned rather briefly, or not at all (like variance reduction or statistical result analysis). This is due to the fact that this book deals with the author's own research rather than with general, known facts in M&S. Most of the following chapters are closely related to some specific projects and the corresponding software.

I would also like to acknowledge the courtesy of the Journal of Artificial Societies and Social Simulation, which gave me permission to include, in Chapter 7 of this book, a reedited version of my article published in JASSS, Volume 7 Issue 2, 2004. I am also grateful to the Society for Modeling and Simulation International for the permission to use my articles published in the Transactions of the Society for Computer Simulation, vol. 13(1), vol. 15 no. 2 and vol. 5, no 1, whose new versions are included in Chapters 4, 8 and 9 of this book, respectively.

Chapter 1

Basic Concepts and Tools

1.1 MODELING AND SIMULATION: WHAT IS IT?

To put it simply, computer simulation is a process of making a computer behave like a cow, an airplane, a battlefield, a social system, a terrorist, a HIV virus, a growing tree, a manufacturing plant, a mechanical system, an electric circuit, a stock market, a galaxy, a molecule, or any other thing. This is done with a specific purpose, mainly in order to carry out some "what if?" experiments over the computer model instead of the real system.

It is known that the best investment one can make is to invest in the field of high technology and basic research, though the benefits may not be immediate or easy to evaluate. One of such fields is computer simulation software. Recall that *simulation* is used to observe the dynamic behavior of a *model* of a real or imaginary system. The subtitle of the International Journal SIMULATION (the most important publication on simulation methods and applications) is *For Understanding* (see http://www.scs.org). Indeed, when we simulate a complex system, we are able to understand its behavior at low cost. Otherwise, we would have to carry out a complicated theoretical research or to build a device (an electric heater, a building, or an airplane), and observe how it crashes to get hints for improvements in the design.

There are many definitions of computer simulation. For example, A. Alan B. Pritsker (1984) defines it as *the representation of the dynamic behavior of the system by moving it from state to state in accordance with well-defined operating rules*. Bernard P. Zeigler in his book (1976) writes: *We can therefore define simulation as the technique of solving problems by the observation of the performance*, *over the time*, *of a dynamic model of the system*. By a *system* we mean a set of components that are interrelated and interact with each other. These interrelations and interactions distinguish the system from its environment. The system is supposed to be organized in order to perform one or more functions or to achieve one or more specific goals. The commonly mentioned properties of systems include:

Aggregation, which means that systems can be grouped into categories that can be nested into larger aggregates.

Nonlinearity – A system need not be linear, i.e. it does not necessarily satisfy the principle of superposition. In other words, the behavior of a system cannot be derived from the sum of behaviors of its components. Consult Holland (1995).

Many authors define simulation in a similar way, with emphasis on the changes of the modeled system state in time. In a somewhat more general way, we can define *modeling as the relation between real systems and models, and simulation as the relation between models and computers*.

In the article of Pritsker (1979), you can find about 30 different definitions of computer simulation. Most of the authors tend to define simulation in terms of the software they develop or use. However, the common approach is to relate simulation with the changes of the model state in time. Ralph Huntsinger, past president of the Society for Computer Simulation, always says that "*Simulation is Fun*". This is true, taking into account the interdisciplinary aspect of computer simulation. A specialist in simulation must learn how planes fly, how rice grows, how shoes are produced, how AIDS is spread, how a legal process works and how galaxies are formed among many other things. This is the FUN you find dealing with computer simulation.

The first step in any modeling and simulation (M&S) task is to construct a system model. As Bratley, Bennet, and Schrage (1987) say in their book: *A model is a description of some system intended to predict what happens if certain actions are taken*. This is perhaps one of the most essential definitions of what a model of a real system is, although it needs some additional explanations. Many years ago, A. Rosenblueth and N. Wiener pointed out that modeling is one of the central needs of scientific reasoning. When we deal with models we must take into account many factors, like the level of simplification, experimental frame, model validity, tractability, credibility, and the aim of modeling among others. Bernard P. Zeigler (1976) gives some basic concepts on modeling. First of all, we must define the *model components* as elemental parts of the model, like clients in a bank, ships entering a harbor, cars on a street etc. Each component is described by the descriptive variables that include input, state, and output variables. The set of all descriptive variables in a model forms the *experimental frame*. The same real system may have several different experimental frames. Each experimental frame results in the corresponding simplified model, as shown in Figure 1.1 (following Zeigler, 1976).

Here the *basic model* is that which exactly reflects the behavior of the real system. Such a model normally does not exist.

Note that the aim of the modeling task, as well as the technical limitations (the computer on which we want to run the resulting simulation program) reduce the number of possible simplified models. This helps us select the appropriate simplification. If there is more than one simplification that satisfies these criteria, we must apply other selection rules (e.g. modeling cost).

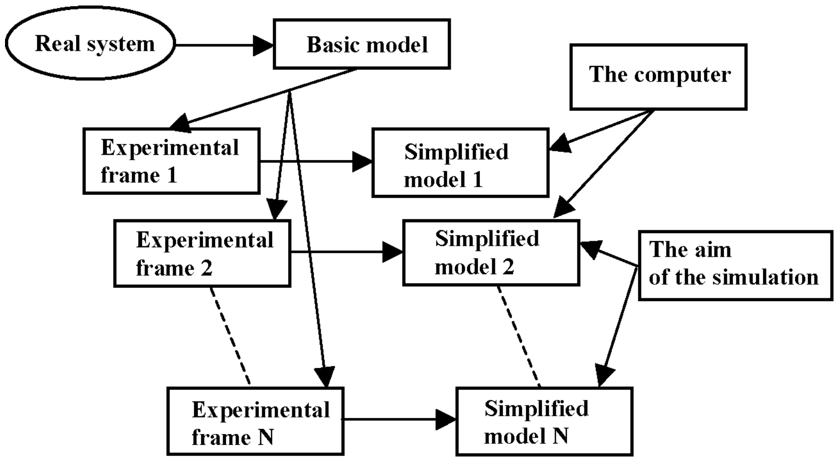

Figure 1.1 Real system, basic model, experimental frames and simplified models.

If no model exists satisfying our aim and technical limitation, then no simulation is possible. Remember that looking for something that does not exist is simply a waste of time. Also note that the same real (or imaginary) system can have several different experimental frames and several simplified models. For example, while modeling an electric circuit, the common experimental frame is the set of the voltages and currents on the corresponding circuit components. But, for the same circuit, someone can define the experimental frame as the set of all voltages and currents, power dissipated on each element, the temperature of each integrated circuit and of the printed-circuit plate, as well as the intensity of the electromagnetic field produced by the circuit. The first frame suggests the use of an appropriate package for circuit simulation, while the second one implies the use of a circuit simulation package, as well as sophisticated heat transfer and electromagnetic wave software.

1.2 VALIDITY, CREDIBILITY, TRACTABILITY, AND VERIFICATION

The concepts of *model validity*, *verification*, *credibility*, and *tractability* are of great importance in any M&S task.

Model *validity* is one of the central problems in modeling and simulation. Before discussing this concept, recall the concept of the system input and output variables. Roughly speaking, by the input we mean all external signals like electric excitations, control signals, or orders that come from the environment. The output variables are those values that we want to observe, measure, store, print, or plot as the result of a simulation run. The concept of input and output comes from the problems of signal processing, automatic control, and similar fields. However, not all systems must be causal, *and the input and output concept may not work.* For example, an electric resistor treated as an isolated, stand-alone system has no input and no output. One can define the input signal as the voltage applied to the resistor and the output as the resulting current. But the same resistor in another circuit (context) can have forced current (connected to a current source) as the input signal, the resulting variable being the voltage (model output). A new approach to M&S of physical systems rejects the input–output concept, which means that from this point of view, physical systems are not causal (see Cellier, 1993). Other aspects of model descriptive variables and formal model definition will be discussed in the section devoted to the DEVS (Discrete Event Specification) formalism.

But now let us come back to causal systems with input and output signals well defined. Consider a real dynamic system and its model. Let S be the operation of modeling (passing from a real system to its model). By $x(t)$ we will denote the system state in the time instant t, by $y(t)$ the system output, by f the state transition function that maps the state $x(t)$ and input over $[t, t + h]$ into the new state $x(t + h)$. The same letters with suffix s denote the corresponding items for the model. Our model is said to be valid if and only if the diagram of Figure 1.2 commutes.

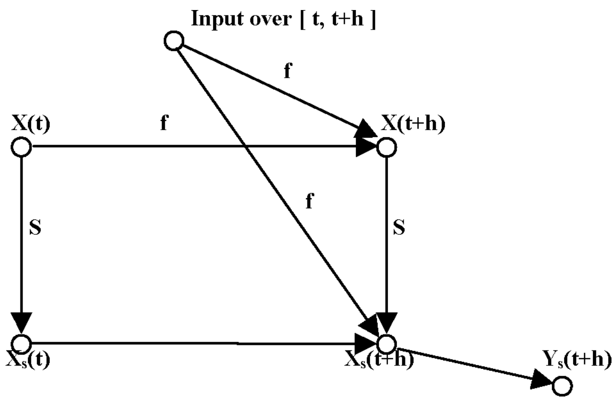

Figure 1.2 Model validity.

In other words, starting with $x(t)$ we must obtain the same model output $y_s(t + h)$ separately from the way we choose. This must be satisfied for *any* possible initial state and input.

The above definition of model validity is somewhat difficult for practical applications. A more practical concept is the *input–output* or *external* validity that can be illustrated by the scheme shown on Figure 1.3.

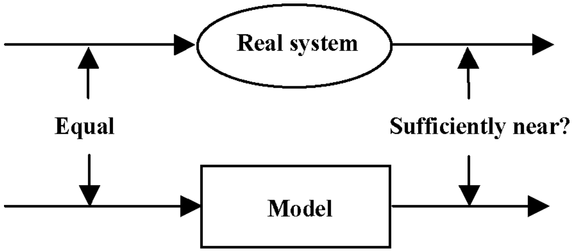

Figure 1.3 Validity “input-output”.

The model is supposed to be *I/O valid* if the outputs from the model and from the real system are “sufficiently” near. What “sufficiently” means is the individual judgment of the modeler.

Obviously, the above property must be satisfied for a long time interval, and perhaps for future model inputs. As we do not know what input signals will affect our model, the only way to check the I/O validity is to compare the model behavior and the real system output using some historic data.

Observe that according to the first definition (Figure1.2), *any approximation of a real continuous system by a model with discrete time is invalid*. Indeed, in a real system, a disturbance may come within the time period of discretization, as it is undetected by the discrete-time model. Even if the integration step of our numerical method is small, the input signals may have high frequencies, and the aliasing phenomenon may result in completely wrong results. Another source of error is the time-discretization itself that implies its own errors. The most direct way to make the discrete-time model more exact is to decrease the sampling time for the discrete model. However, there are limitations for such actions, mainly caused by the computer resolution and limitations of a numerical method applied for trajectory integration. In any case, the modeler must look for a reasonable compromise and not forget that what he or she is creating is nothing more than an illusion. Another useful and simple test that may be used to qualify a model as invalid is to prove that the model does not explain certain known properties of phenomena that we observe in the real system.

Invalid models are often results of wrong assumptions. Creating a simplified model, we idealize the real system, which may result in physically wrong models.

Consider a simple example of an electric circuit shown on Figure 1.4.

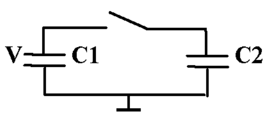

Figure 1.4 An electric circuit.

Let the initial voltage for the capacitor C_1 be equal to V, and initial voltage for C_2 equal to zero. After closing the switch, the two voltages obviously become equal to each other and equal to $V/2$, provided $C_1 = C_2 = C$. Now, consider a simplified model of the circuit (Model 1). We assume that all elements are "ideal", which means that the capacitors have no internal resistance or inductance, the same assumption being taken for the conductors ($R = 0$). Now calculate the initial energy of the circuit. It is equal to $(CV^2)/2$. After closing the circuit, the total energy is $2(C(V/2)^2/2) = (CV^2)/4$. Where did half of the initial energy go? Our model cannot explain this. In other words, *the model is invalid*. The very simple reason is that it is too simplified. Now, consider Model 2, where the connecting wires have resistance R. One could expect that Model 1 is a limit case of a sequence of models of type Model 2, with R approaching zero. Unfortunately, this is not the case. It is easy to show that *half of the energy is always being dissipated in the wire resistance during the transient process*. Moreover, this energy does not depend on the value of the resistance. This means that Model 1 is *not* the limit of a sequence of models (Model 2), with R approaching zero(does not converge to Model2). In more "mathematical" terms, we can say that in the space of all models of type 2, Model 1 (point of models space) with $R = 0$ is a discontinuity point (or singularity). Also note that the model validity depends on the actual experimental frame Modeling. The first model of our circuit is valid if the experimental frame only includes the (static) voltages. However, if we add the system energy to the experimental frame, this model becomes invalid and we must look for another one. Here we only talk about the energy of the electric field and the dissipated energy. In a more complete model we should also take into account the energy of the electromagnetic field. In fact, if we neglect the wire resistance and take into account the inductance of the loop (with ANY value different from zero), than we will see that the circuit will always oscillate, exchanging the energy between electric and electromagnetic fields.

Other remarks on validity can be found in Section 3.1.

Model *credibility* is another important aspect of M&S. Rather, this is the question of the relation between the target user and the model implementation. The model can be valid and well implemented, but this does not mean that the user believes in it. If he/she does not, the whole project is a waste of time, because the user will not use it to solve his problems (design, decision making etc.). A good way to make the model credible for the user is to involve him in the process of model building. If the team that works on the model includes one or more people from the company the model is created for, it is more likely that the model and the resulting simulation experiments will be credible and accepted by the company.

The *model verification* is rather a technical problem, related to the computer implementation. Verification in simulation tasks is nearly the same as testing in any other software development process. Sometimes testing is even more costly and time-consuming than the model development. To perform good testing in any complex software project, it is recommended the test team be separated from the developer team. The goals of the two teams are opposite: a tester wants to find bugs in the software, while the developer wants to prove that the software is error free. The interactions between the developer and the tester team result in improving the software quality. Remember the law of Murphy! If a complex software package has been well tested, so that the probability of any malfunctioning is nearly zero, and if it contains a small bug, then the first thing the user will do is to introduce data or action that activates this "improbable" erroneous functioning.

The technical limitations are mainly imposed by available hardware. It might appear that the rapid growth of the computing capabilities of our tools implies that what is impossible to implement now, will be quite possible in the near future. However, some problems are so complicated that they are *computationally intractable*, whatever the computer speed will be in the future. It is well known in operations research that some problems that have a nice mathematical description cannot be solved using the hardware we actually have. The same occurs with models prepared to be run on a computer as a simulation task. Some models are computationally intractable, which means that they are too time-consuming or expensive to be realized.

A modeling and simulation task is intractable if its computational complexity increases faster than exponentially or factorially with the number of its descriptive variables. The computational complexity can roughly be defined as the minimal cost of guaranteeing that the computed answer to our question (simulation task) is within a required error threshold.

A mostly cited example of an intractable problem is the salesman problem, namely, the simulation of all possible routes of a salesman that must visit various cities. The problem is to find the route with the shortest distance possible. This problem can be treated with many suboptimal algorithms known in the operation research field. However, we can only obtain a suboptimal solution even with a small number of cities, without knowing if it is really the optimal one. Other examples in continuous system simulation can be found in fluids dynamics applied to problems, like the reentry of a space shuttle to the atmosphere of the earth (modeling the airflow around the craft, and its thermodynamics).

For more discussion on intractability, consult Traub, and Wozniakowski (1994) or Werschulz (1991).

1.3 SYSTEM STATE AND CAUSAL SYSTEMS

Let $U\{s, t\}$ be the input to the system over the interval $[s, t]$, where s and t are two time instants, s $X(t) = F(t, U\{s, t\}, s, X(s))$

In other words, it is necessary that the system state at a moment t can be calculated using some past state in time instant s and the input function over the interval between the two moments of time. For example, the state of a system that has one spring, one mass, and one damper linked together is given by the mass position and velocity. All other descriptive variables of this system are parameters, inputs (e.g. external forces), or some output functions defined by the modeler.

In the case of an electric circuit composed by any number of resistors and one capacitor, the state is a (scalar) value of the capacitor voltage. In this case all the currents in the resistors can be calculated provided we know the initial capacitor voltage and the external excitations (input signals). The system state may be a scalar, a vector, or an element of a more certain abstract space. For example, the state of the model describing the changes of the temperature distribution inside a piece of metal belongs to the space of all differentiable functions of three variables defined in the region occupied by the modeled body.

Now, let us define what we mean by the phrase “equivalent to”, applied to functions. Obviously, two functions equal to each other in all points of a given interval are equivalent. In general, we treat two input signals as equivalent if they produce the same output. For example, if the system is an integrator, two input signals that are equal to each other on $[t_o, t_1]$, except a set of points of total measure zero, are equivalent. Another example is a sampled data system (e.g. a digital controller with an A/D converter at the input) with sampling period T. Two different input signals that coincide only at $t = 0T, 2T, 3T$ ... are equivalent to each other because the system cannot observe the values in time instants other than the sampling moments.

Consider a dynamic system whose state is X. Let us suppose that we can define input and output signals for this system as U and Y, respectively. The system is said to be *causal* if and only if for every t_1

$U_1(t)$ equivalent to $U_2(t)$ over an interval $[t_o, t_1]$, implies that $Y_1(t_1) = Y_2(t_1)$

where $Y_1(t_1)$ and $Y_2(t_1)$ are two outputs at the time instant $t = t_1$, obtained with inputs U_1 and U_2, respectively, t_o is a fixed initial time instant, and $X(t_o)$ is fixed. The output signal is supposed to be an algebraic function of the system state. We suppose that the system state in t_o is the same for the two possible trajectories.

An example of a noncausal system is as follows:

$$Y(t) = \int_{-\infty}^{\infty} p(t-s)U(s)ds$$

where $p(t)$ is a function different from zero everywhere. In this case the value of Y at the time instant t depends not only on the past, but also on the future values of p over the time. If we replace p by the Dirac impulse at zero, then the system becomes causal (in this case simply $Y(t) = U(t)$).

1.4 CLASSIFICATION OF DYNAMICAL SYSTEMS

To correctly select an M&S method and software we must know what kind of system we will be working with. The general classification given below can help us in this task. This is a known classification that frequently appears in texts on automatic control and physical system dynamics. Recall the notion of the equivalence class of input signals. As stated in the previous section, the signals belonging to the same class of equivalence produce the same effect. So, the number of classes of equivalence is the number of possible final system states. Now, we can classify