Gero Brockschnieder

Asymptotics of Cubic Number Fields with Bounded Second Successive Minimum of the Trace Form

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Abstract (English)

We present a new way of investigating totally real algebraic number fields of degree 3. Instead of making tables of number fields with restrictions only on the field discriminant and/or the signature as described by Pohst, Martinet, Diaz y Diaz, Cohen, and other authors, we bound not only the field discriminant and the signature but also the second successive minima of the trace form on the ring of integers $\mathcal{O}_K$ of totally real cubic fields K. With this, we eventually obtain an asymptotic behaviour of the size of the set of fields which fulfill the given requirements. This asymptotical behaviour is only subject to the bound X for the second successive minima, namely the set in question will turn out to be of the size $\mathcal{O}(X^{5/2})$.

We introduce the necessary notions and definitions from algebraic number theory, more precisely from the theory of number fields and from class field theory as well as some analytical concepts such as (Riemann and Dedekind) zeta functions which play a role in some of the computations. From the boundedness of the second successive minima of the trace form of fields we derive bounds for the coefficients of the polynomials which define those fields, hence obtaining a finite set of such polynomials. We work out an elaborate method of counting the polynomials in this set and we show that errors that arise with this procedure are not of important order. We parametrize the polynomials so that we have the possibility to apply further concepts, beginning with the notion of minimality of the parametrization of a polynomial. Considerations about the consequences of allowing only minimal pairs (B, C) (as parametrization of a polynomial $f(t) = t^3 + at^2 + bt + c$) to be of interest as well as a bound for the number of Galois fields among all fields in question and their importance in the procedure of counting minimal pairs, polynomials, and fields finally lead to the proof that the number of fields K with second successive minimum $M_2(K) \leq X$ divided by the size of the suitably "cut back" set of polynomials tends to 1 if X tends to infinity, particularly because the number of fields with more than one related minimal pair (B, C) is of negligible order.

A considerable amount of work accounts for the computational investigation of the theory, namely we show how fast the convergence of the above-mentioned limit actually is by computing the value of the fraction for several values of X. Computational results are presented as comprehensive tables and, as a vivid representation, as graphs.