Mathematical Engineering

Alexander Kolpakov

Thin-Walled Structures with Initial Stresses

Multiscale Analysis of Classical and Non-Classical Problems

Springer

Mathematical Engineering

Today, the development of high-tech systems is unthinkable without mathematical modeling and analysis of system behavior. As such, many fields in the modern engineering sciences (e.g. control engineering, communications engineering, mechanical engineering, and robotics) call for sophisticated mathematical methods in order to solve the tasks at hand.

The series Mathematical Engineering presents new or heretofore little-known methods to support engineers in finding suitable answers to their questions, presenting those methods in such manner as to make them ideally comprehensible and applicable in practice.

Therefore, the primary focus is—without neglecting mathematical accuracy—on comprehensibility and real-world applicability.

To submit a proposal or request further information, please use the PDF Proposal Form or contact directly: Dr. Thomas Ditzinger (thomas.ditzinger@springer.com).

Indexed by SCOPUS, zbMATH, SCImago.

Alexander Kolpakov

Thin-Walled Structures with Initial Stresses

Multiscale Analysis of Classical and Non-Classical Problems

Springer

Alexander Kolpakov
Niederbipp, Switzerland

ISSN 2192-4732 ISSN 2192-4740 (electronic)
Mathematical Engineering
ISBN 978-3-032-07246-7 ISBN 978-3-032-07247-4 (eBook)
https://doi.org/10.1007/978-3-032-07247-4

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

Published 20 years ago, book *Stressed Composite Structures: Homogenized Models for Thin-Walled Heterogeneous Structures with Initial Stresses* by Kolpakov has presented a method for deriving models of thin-walled structures (plates, membranes, beams, strings) with initial stresses directly from three-dimensional of elasticity theory problem. The method developed in the book was based on the observation that for a body with initial stresses, the equations relating additional stresses and additional strains are similar to Hooke's law, but the coefficients of the equations do not possess all symmetries of the elastic constants. In thin and small diameter regions, this loss of the symmetries leads to the appearance in the limit equations the terms known in the classical theories of thin-walled structures as "additional transverse forces" or "additional moments" (Timoshenko 1961, 1983).

Over the last 20 years, the method proposed in the above book has been tested and applied to new problems of bodies and structures with initial stresses. Generalization and organization of new results has led to the appearance of the present book.

The author includes in the book the chapters devoted to uniform plates (flat plates made of homogeneous material) and uniform beams (cylindrical beams made of homogeneous material) with initial stresses. This is done in order to demonstrate the origin of the "additional transverse forces" in the asymptotic theory in its "pure" form, without regard to the inhomogeneity of the structural elements. These chapters present an alternative method to deriving the classical stability equations for uniform plates and beams.

The author includes chapters about structural elements with initial stresses greater than usually considered in classical theories (such stresses arise, for example, as a result of thermal heating). Also, the local stresses are written for all the considered cases.

As a result, this book, in couple with the book mentioned above, covers all ranges of the resulting initial stresses in plates and beams. Recall that three ranges of resulting initial stresses are distinguished: stresses corresponding to axial or in-plane forces, stresses corresponding to moments, and stresses of the order of elastic constants.

Typically, engineering structures consist of structural elements of various types, assembled into a unit by using connections (joints). Thus, analyzing a structure, one needs (1) to take into account the properties and characteristics of structural elements, and (2) to take into account the properties and structure of joints. It is obvious that the stress-strain state in the joints differs from the stress-strain state in the joined elements. However, the local stress-strain state in a joint is determined by that of all the structure. The classical structural mechanics, see, e.g., Hibbeler (2008), Ventsel and Krauthammer (2001), Washizu (1992) ignores specific properties of joints, operating by abstract notions of supports and connections. More precisely, the problems of computation of the local stress-strain states in joints and in the joined structural elements are quite freely divided between such disciplines as “Structural mechanics” (Hulse and Cain 1991) and “Mechanics of joints (connections, fasteners)” (Blake 1985; Norton 2008; Hudgins and James 2014). Currently, intensive research is being conducted to develop integrated approach(s) to calculating structural elements and connections (joints), see, e.g., Ciarlet (1990), Zhikov (2002), Zhikov and Pastukhova (2003), Gaudiello and Kolpakov (2011), Kolpakov (2011). This book treats the structural elements.

As it was noted, the theory of thin-walled heterogeneous elastic structures with initial stresses can be constructed on the basis of the observation of the loss of symmetry in the linearized model of an elastic body with initial stresses (Washizu 1992) and the application of two-scale expansion method (Sanchez-Palencia 2000). The beginning chapter of the book contains the condensed exposition of these theories. A more detailed exposition of the general theory of elastic bodies with initial stresses can be found in the book by Washizu “Variational Methods in the Theory of Elasticity and Plasticity” (Washizu 1992) and expositions of the homogenization method can be found in the book by Sanchez-Palencia “Non-Homogeneous Media and Vibration Theory” (Sanchez-Palencia 2000). The author also recommends books (Bakhvalov and Panasenko 1989; Bensoussan et al. 1978; Cioranescu and Donato 2000; Panasenko 2005; Trabucho and Viano 1996) as referenced books on the mathematical foundations of the homogenization method and books (Andrianov et al. 2004; Andrianov and Awrejcewicz 20024; Kalamkarov and Kolpakov 1997; Kolpakov 2004; Lewinski 2000; Mityushev 1997) as referenced books on the application of the homogenization method to the engineering problems. This list is not exhaustive, other works are mentioned in the text of the book, see also the bibliography.

The structure of the book is the following.

In Chap. 1, the aims and objectives of the book are discussed, in particular, the historical background of the problems of thin-walled structural elements with initial stresses is presented in condensed form.

Chapter 2 contains examples of thin-walled structural elements in which initial stresses are significantly used and/or manifested.

In Chap. 3, we briefly present the procedures of homogenization theory that are directly used in this book.

In Chap. 4, we discuss the elastic composite body model with initial stresses, which will be used as a first-principles model throughout the book.

In Chap. 5, we make a transition from the three-dimensional elasticity theory problem to the corresponding two-dimensional problem for uniform plates (plates of constant thickness made of homogeneous material) with initial stresses. We consider the problems corresponding to in-plane resultant initial forces and moments of initial forces.

In Chap. 6, we consider plates of periodic structure with initial stresses. We consider the case corresponding to in-plane resultant initial forces.

In Chap. 7, we make a transition from the three-dimensional elasticity theory problem with initial stresses in a thin layer to the two-dimensional problem in the case when the initial stresses are of the same order as the elastic constants of the material of the plate. Although the initial stresses are of the same order as the elastic constants, they are small compared with the elastic constants in absolute value. Using this fact, we construct the first-order (linear) correctors in the problem under consideration.

In Chap. 8, we consider uniform beams with initial stresses, which are traditional and widely used structural elements.

In Chap. 9, we consider non-uniform beams of arbitrary periodic structure with initial stresses corresponding to axial resultant forces. We consider the case corresponding to the axial resultant initial force.

In Chap. 10, we make a transition from the three-dimensional problem of elasticity theory with initial stresses in a small-diameter body to a problem of beam theory when the initial stresses are of the same order as the elastic constants of the material from which the beam is made.

In Chap. 11, we compare the hypothesis of the classical theories of plates and beams with the terms of the asymptotic expansions obtained using the two-scale method and the homogenization method.

The "References" section contains a list of sources used in the book. Although this list includes over 300 references, it is far from exhaustive of the literature on thin-walled structures and homogenization.

The author thanks Prof. Dr. Sci. I. V. Andrianov (RWTH Aachen) for reading the manuscript and providing valuable comments, and Prof. Dr. Sci. V. V. Mityushev (Cracow University of Technology) for insightful discussions on random composites.

Niederbipp, Switzerland
2025

Alexander Kolpakov

References

Andrianov IV, Awrejcewicz J (2024) Asymptotic methods for engineers. CRC Press, Boca Raton, FL

Andrianov IV, Awrejcewicz J, Manevich LI (2004) Asymptotic mechanics of thin-walled structures. Springer, Berlin

Bakhvalov NS, Panasenko GP (1989) Homogenization: averaging processes in periodic media. Kluwer Academic Publication, Dordrecht

Bensoussan A, Lions J-L Papanicolaou G (1978) Asymptotic analysis for periodic structures. North-Holland Publication, Amsterdam

Blake A (1985) Design of mechanical joints. CRC Press, Boca Raton, FL

Ciarlet G (1990) Plates and junctions in elastic multi-structures. An asymptotic analysis. Macon, Paris
Cioranescu D, Donato P (2000) An introduction to homogenization. Oxford University Press, Oxford
Gaudiello A, Kolpakov AG (2011) Influence of non degenerated joint on the global and local behavior of joined rods. Int J Engg Sci 49(3):295–309
Hibbeler RC (2008) Structural analysis. 7th Ed. Prentice-Hall, Englewood Cliffs, NJ
Hudgins A, James B (2014) Fatigue of threaded fasteners. Adv Mater Proces Tech Art 172(8):18–22
Hulse R, Cain J (1991) Structural mechanics. MacMillan Publication, London
Kalamkarov AL, Kolpakov AG (1997) Analysis, design and optimization of composite structures. John Wiley & Sons, Chichester
Kolpakov AG (2004) Stressed composite structures: homogenized models for thin-walled nonhomogeneous structures with initial stresses. Springer, Berlin
Kolpakov AG (2011) Influence of non degenerated joint on the global and local behavior of joined plates. Int J Engg Sci 49(11):1216–1231
Lewinski T, Telega JJ (2000) Plates, laminates and shells: asymptotic analysis and homogenization. World Scientific, Singapoure
Mityushev V (1997) A functional equation in a class of analytic functions and composite materials. Demost Math 30:63–70
Norton RL (2008) Design of machinery. 4th Ed. McGraw-Hill Higher Education, Boston, MA
Panasenko GP (2005) Multi-scale modeling for structures and composites. Springer, Berlin
Sanchez-Palencia E (2000) Non-homogeneous media and vibration theory. Springer, Berlin
Timoshenko S (1961) Theory of elastic stability. 2nd Ed. McGraw-Hill, New York
Timoshenko S (1983) History of strength of materials (with a brief account of the history of elasticity and theory of structures). Dover, New York
Trabucho L, Viano JM (1996) Mathematical modeling of rods. In Ciarlet PG, Lions J-L editors, Handbook of numerical analysis, volume IV, pp 487–974, Amsterdam, Elsevier
Ventsel E, Krauthammer Th (2001) Thin plates and shells. theory, analysis, and applications. Marcel Dekker, New York
Washizu K (1992) Variational methods in elasticity and plasticity. Pergamon Press, Oxford
Zhikov VV (2002) Homogenization of elasticity problems on singular structures. Izvestiay: Math 66(2):299–365
Zhikov VV, Pastukhova SE (2003) Homogenization for elasticity problems on periodic networks of critical thickness. Russian Acad Sci: Sb Math 194:697–732

Competing Interests The author have no conflicts of interest to declare that are relevant to the content of this book.

Contents

Chapter 1
Introduction

The mechanics of structures with initial stresses is a traditional part of structural mechanics. It is closely related to the problem of stability (buckling) of structures and structural elements. Note that the theory of stability of beams arose earlier than the theory of elastic bodies with initial stresses. The basic concepts of elastic stability of structures go back to works by Euler (1759) (beams) and Bryan (1891, 1888) (plates). Later, Hill (1958), Marguerre (1938), Prager (1947), Timoshemko and Goodier (1951), Trefftz (1930), Washizu (1992), and other researchers demonstrated that the problem of deformation of solids with initial stresses is related to variational principles and nonlinear problems in elasticity.

The engineering analysis of stability (buckling) problems for thin plates and beams with initial stresses is based on the computation of additional normal force arising as a result of additional deformation of stressed structural element. Ab alternative approach is based on the computation of work of initial stresses on the additional displacements (Amabili 2008; Ventsel and Krauthammer 2001).

In the 1950s, composite materials began to be widely used in engineering, especially in aeronautics and spacecraft. It quickly became apparent that traditional computations and design methods developed for homogeneous materials and uniform structures were not applicable to composite structural elements. This initiated intensive research in the field of composite material and structure theory (Broutman and Krock 1974; Kelly and Rabotnov 1988), in particular, research on the buckling of composite structures (Boscolo et al. 2013; Zhang et al. 2024; Ghorbani et al. 2024; Turvey and Marshall 2012; Miller et al. 2011; Lopez Jimenez and Triantafyllidis 2013). This research continues to this day. The main directions of the research are the theory of stability of complex/nonhomogeneous thin-walled structures (Kesavan 1979; Ciarlet and Kesavan 1981; Mignot et al. 1980; Wang and Abdalla 2002; Marcal and Yamagata 2016), design of materials and structures possessing required properties (Alaire 2002; Bendsøe and Sigmund 2004; Haug et al. 1986; Kolpakov and Rakin 1986; Foraboschi 2013): (in particular, unusual properties: negative Poisson's ratio Almgren 1985; Kolpakov 1985; Lakes 1987; Novikov and Wojciechowski

A. Kolpakov, *Thin-Walled Structures with Initial Stresses*, Mathematical Engineering,
https://doi.org/10.1007/978-3-032-07247-4_1

1999, negative thermal expansions Kolpakov and Rakin 1986; Lakes 1996; Miller et al. 2009; Kelly et al. 2005, etc.), vibration of complex structures (Kaplunov et al. 2002; Kolpakov 2001, 2005; Pichugin and Rodgerson 2004; Wozniak et al. 2004). The lists of the problems and the references are not exhausting. Additional references may be found in the cited literature.

This book is devoted to a particular problem—the construction of the low-dimensional and homogenized models of elastic composite thin-walled structural elements with initial stresses. The analysis is carried out in the framework of a linear (linearized Washizu 1992) approach. Both uniform and nonhomogeneous plates and beams are considered in the book.

No doubt, there are many interesting and important problems outside the frame of our consideration, for example, a local loss of stability (Li et al. 2015; Sun et al. 2023) or substantially non-linear problems of loss of stability (Chakraborty et al. 2024a, b; Lal and Markad 2021; Hu et al. 2021; Goriely et al. 2008; Machado 2008).

Historically, the basic concepts of structural mechanics were developed for uniform structural elements: flat plates, beams and membranes, cylindrical beams and strings, etc. made of homogeneous materials. When dealing with composite plates and beams or structural elements of complex geometry (such as ribbed plates or lattice masts), one finds that many concepts of classical theory cannot be transferred to the mechanics of heterogeneous structure. To illustrate this point, we discuss the following two concepts of classical theory.

1. *Cross-section*. The notion of cross-section plays a important role in the classical theory of beams and plates. In particular, the classical notions of the resultant forces and moments in a plate or beam assume the integration over cross-section. In an inhomogeneous plate/beam, the geometry and/or material characteristics of the cross-section change from one to another adjacent section, see Fig. 1.1. In Fig. 1.1, three cross-sections: $S(y_1)$, $S(y_2)$, and $S(y_3)$ of a plate of a complex structure are displayed. These cross-sections are very different. This example demonstrated that we are usually unable to introduce either a standard cross-section or a representative section for plates or beams of complex geometry.
2. *Neutral plane (axis)*. Considering a non-flat plate, we have two possibilities to introduce neutral surfaces. The first is to introduce a neutral plane as the median surface of the actual plate. In this case, the neutral plane may be a curved surface. With respect to this curved neutral surface, one can write the formulas of the classical theory. Note that this procedure is not possible for an arbitrary plate. For example, it is possible for a simple corrugated plate, but it is impossible for the plate displayed in Fig. 1.1, which consists of a corrugated plate connected with a carrier plane. An alternative way is to introduce a flat neutral plane for the plate as a whole. It may happen that a large part of such neutral plane will be located outside the physical plate, as shown in Fig. 1.1. In addition, it will be necessary to predict the position of the neutral plane. It is not easy to do for the plate displayed in Fig. 1.1.

As a result, we arrive at the conclusion that it would be useful to have a theory of thin-walled inhomogeneous structures developed on the basis of three-dimensional

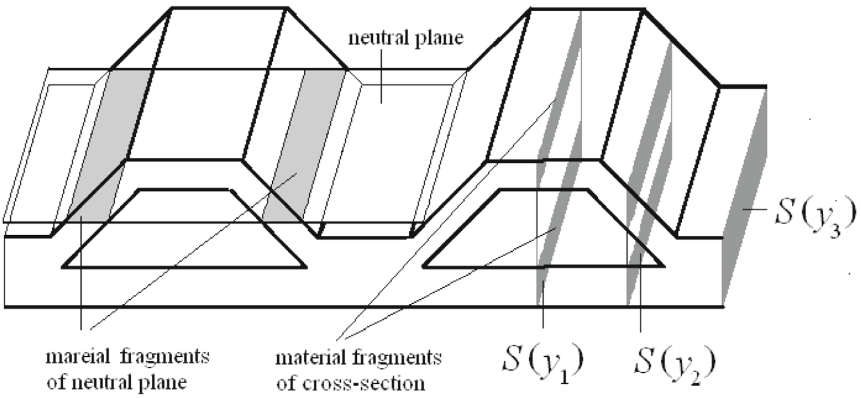

Fig. 1.1 A complex geometry plate

elasticity theory with no simplifying assumptions (in particular, with no a priori hypothesis). The need for such a theory was noted not only in connection with problems of stability, but also in connection with other problems with thin-walled structures, see e.g., Ball (2002), Ciarlet (1990), Kalamkarov (1992), Kaplunov et al. (1998), Smirnov and Tovstik (2001).

The first problem one encounters when analyzing a composite plate or a plate of complex structure is the problem of approximating such a plate with a suitable homogeneous plate. The reason for searching for an approximation is that a composite plate is formed from many constituent elements. A direct solution for many elements is impossible even on a modern computer. For example, the number of simple (basic) elements that form a plate with a honeycomb core varies from 10^4 to 10^6. Solving a problem of joint deformation of so many elements is difficult (often impossible) even with a modern computer. Direct numerical analysis of large systems may not give us an exact solution, even if we use a powerful computer. For example, numerical analysis of the strength of a composite in the key paper (Babŭshka et al. 1999) did not detect the energy localization effect between the closely placed particles (Kolpakov 2007; Kolpakov and Kolpakov 2007, 2010). It was because the finite elements mesh was regular and not condensed in the necks between closely placed particles. Obviously, one could have made the mesh finer in the necks if one had known about the (at that time unknown) localization effect. Note that the localization effect was discovered later numerically (Kolpakov 2007; Rakin 2014).

Returning to the two examples above, we can indicate the way of resolution to the problems by using the homogenization theory (the justifications and details of solutions will be presented below, see also Kolpakov 2004).

1. *Cross-section*. The resultant forces and moments in the plate or beam are determined by integration over the three-dimensional periodicity cell or representative cell of the plate or beam. In this case, the mechanical concept of a cross-section is absent altogether. It does not lead to a contradiction with classical theory

because for uniform (classical) plate the periodicity cell is cylinder with the base coinciding with the cross-section.

2. *Neutral plane/axis*. It was demonstrated (Kolpakov 2004) that there exist the "invariant" forms of the plate/beam model which do not depend (have the same form) on translation of the coordinate system in the direction normal to the plate plane or beam axis.

If we decide that it would be useful to have a theory of fine heterogeneous structures developed on the basis of three-dimensional elasticity theory without any a priori hypotheses, we are faced with the question:

(1) what can be adopted as the first-principles model(s) and
(2) what methods can be used to investigate the adopted model(s) to achieve the goal.

The author takes the linearized model of an elastic body with initial stresses (Washizu 1992) as the starting (the first principle) model. A weighty argument for this choice is that all classical models of structural elements subjected to initial stresses: composite solids, plates and membranes, beams and strings; can be derived from that model (it was done in Kolpakov 2004). All the derived models coincide in form with the corresponding engineering models, when the corresponding engineering model exists.

The author considers the asymptotic homogenization method as suitable for solving the second problem. The possibility of examining a homogeneous solid instead of examining the original inhomogeneous composite solid, was one of the principal results of the mathematical theory of homogenization, see Bensoussan et al. (1978), Oleinik et al. (1962), Jikov et al. (1994). Now, the asymptotic method is successfully applied to many problems in the theory of composite materials and complex structures. The advantage of asymptotic homogenization is that it uses three-dimensional elasticity models as the starting point, analyzes these models without any simplified hypothesis, and it can be applied to all types of structural elements: solids, plates, rods (see Caillerie 1984; Trabucho and Viano 1996; Kolpakov 1991). On the basis of the homogenization theory, engineering methods for analysis of composite materials and structures were developed, see, e.g., Annin et al. (1990), Kalamkarov (1992), Kalamkarov and Kolpakov (1997), Oskay and Pal (2010). The homogenization theory does not assume a direct full-size solution to the original problem. In the homogenization theory, it is sufficient to solve so-called periodicity cell problems (or local perturbation problems Gaudiello and Kolpakov 2011; Kolpakov 2011, if joints are considered).

It should be noted that the linearized elasticity problem, in the form as it was formulated in Washizu (1992) was used as a starting point for the analysis of bodies subjected to initial stresses, including the problem of the stability theory, for a long time (Timoshenko 1961; Biot 1939; Green et al. 1952; Machado and Cortinez 2007). The disadvantage of earlier approaches was that the linearized elasticity model was used in the couple with the Kirchhoff-Love hypothesis. Note that most works based at the linearized elasticity model (Washizu 1992) were devoted to volumetric bodies, see e.g., Guz (1999), Guz (1986), Pan'kov (2022).

In spite of the contradiction in the Kirchhoff-Love hypothesis (the main one of which is that the thickness of the plate does not change during deformation despite the Poisson effect Timoshenko and Woinowsky-Krieger 1959) they provide us with two-dimensional models, which are confirmed by centuries of engineering practice. A similar situation takes place for Euler-Bernoulli beam model. One meets here the situations where contradictory hypotheses lead to the correct final results. Note, that the fact that a wrong premise can lead to correct results is well-known from the course of logic (Walicki 2011). It is also known from a logic course that the incorrect premise can lead to both the correct and the incorrect result. Therefore, the argument "the theory contains contradictions, but it works", generally, cannot be accepted.

Once one wants to develop a consistent theory of thin-walled structures, the solution comes from the use of asymptotic methods. An early example of the application of asymptotic methods to problems with no initial stresses are the papers (Gol'denveizer 1961, 1962, 1963) devoted to the "asymptotic integration of the equations of the theory of elasticity". An early example of the application of asymptotic methods to the problem with initial stress is the paper (Alekseev 2002). In Alekseev (2002), an expansion in the Legendre series was used. The proposal to use a power series for the same purpose was expressed in Washizu (1992). The mentioned above approaches have a significant limitation, they are suitable for homogeneous plates of constant thickness, but not for non-homogeneous plates. Note that the use of asymptotic methods in engineering was, if not blocked, then restricted. There were weighty reasons for this restriction. On the one hand, conflicting hypotheses of Kirchhoff-Love, still lead to correct results for uniform plates. On the other hand, asymptotic methods are beyond the standard courses of mathematics for engineers. The problem became actual when the use of composite plates and plates of complex structure was started.

Historically, the first works in which a rigorous model of composite plate was derived from a three-dimensional elasticity theory problem by using a modification of the homogenization method were (Caillerie 1982, 1984). They were followed by works by other authors, see, e.g., Babŭshka and Li (1991), Destuynder (1981), Mielke (1995), Nazarov (2002). Note that the computations based on the asymptotic homogenization method were more compact and rigorous rather then the "asymptotic integration" computations. It is because the asymptotic homogenization method uses the methods of functional analysis and modern methods of partial differential equations (Bensoussan et al. 1978; Duvaut 1976; Jikov et al. 1994).

For plates with initial stresses, the homogenization method was used for the first time for the analysis of two-dimensional models (Kolpakov 1987; Mignot et al. 1980). The results of these studies are valid if the characteristic dimension of inhomogeneities is large as compared to the thickness of the plate. This condition is not fulfilled for many structures used in practice, for example, for lattice plates (Pshenichnov 1993) or perforated plates (Andrianov et al. 1995), but not for plates made of composite materials or plates with rapidly varying thickness. In the last cases, the three-dimensional elasticity problem should be taken as the original model.

Analysis of the inhomogeneous thin-walled structural elements subjected to initial stresses on the basis of the asymptotic homogenization method as applied to a three-dimensional elasticity theory problem started in 1990s. In Kolpakov (1992), Kolpakov (2001), asymptotic analysis was applied to three-dimensional composite solids of small diameter with initial stresses, and a one-dimensional model for a inhomogeneous beam with initial stresses was obtained. In Kolpakov (1995), Kolpakov and Sheremet (2004), the homogenization method was applied to a body of small thickness, and a two-dimensional model for a inhomogeneous plate with initial stresses was obtained. In Kolpakov and Sheremet (1997) two-dimensional membrane model and one-dimensional string model were derived from a three-dimensional elasticity theory problem. The results of the mentioned investigations corresponding to the classical thin-walled structures were summarized in Kolpakov (2004).

The new approach to inhomogeneous structures subjected to initial stresses leads to some significant changes in the concepts of thin-walled structures.

It should be noted that the analysis of problems in the mechanics of composites using the homogenization method requires knowledge of mathematics beyond what is typically covered in traditional textbooks on structural mechanics and the strength of materials. While the level of mathematics used in the book does not exceed the level of modern university mathematics courses, the reading of the book may require some mathematical efforts from an engineer. As a generous reward for some mathematical effort, the reader will gain access to a thorough theory of composite materials and structures with initial stresses. The examples given in the book will show readers that the behavior of composite materials and structures is not always similar to that of homogeneous materials and structures. However, the book also demonstrates that for homogeneous materials and structures, an approach based on homogenization theory leads to classical models. In addition to the fact that homogenization is a mathematically rigorous method, this will give the reader another reason to trust the methods and models presented in the book.

References

Alaire G (2002) Shape optimization by homogenization method. Springer, Berlin

Alekseev AE (2002) Linearized equations of nonlinear elastic deformation of thin plates. J Appl Mech Tech Phys 43(1):133–139

Almgren RF (1985) An isotropic three-dimensional structure with Poisson's ratio = –1. J Elast 15:427–430

Amabili M (2008) Nonlinear vibrations and stability of shells and plates. Cambridge University Press, Cambridge

Andrianov IV, Shevchenko VV, Kholod EG (1995) Asymptotic methods in the statics and dynamics of perforated plates and shells with periodic structures. Tech Mech 15(2):141–157

Annin BD, Kalamkarov AL, Kolpakov AG (1990) Analysis of local stresses in high modulus fiber composite. In: Localized damage computer-aided assessment and control, Vol 2. Comp. Mech. Publ., Southampton, pp 131–144

Babushka I, Anderson B, Smith P, Levin K (1999) Damage analysis of fiber composites. Part I. Statistical analysis on fiber scale. Comput Method Appl Mech Eng 172:27–77

Babushka I, Li L (1991) Hierarchic modelling of plates. Comput Struct 91:419–430

Ball JM (2002) Some open problems in elasticity. In: Newton P, Holmes P, Weinstein A (eds) Geometry mechanics and dynamics. Springer, New York, pp 3–59

Bendsøe MP, Sigmund O (2004) Topology optimization. Springer, Berlin

Bensoussan A, Lions J-L, Papanicolaou G (1978) Asymptotic analysis for periodic structures. North-Holland Publication, Amsterdam

Biot MA (1939) Non linear theory of elasticity and the linearized case for a body under initial stress. Philos Mag 27:89–115

Boscolo M, Fazzolari FA, Banerjee JR (2013) Buckling of composite plate assemblies using higher order shear deformation theory-an exact method of solution. Thin-Walled Struct 71:18–34

Broutman LJ, Krock RH (eds) (1974) Composite materials. In: Eight volumes. Academic, New York, London

Bryan GH (1891) On the stability of a plane plate under thrusts in its own plane, with applications to the “buckling” of the sides of a ship. Proc Lond Math Soc 22:54–67

Bryan GH (1888) On the stability of elastic systems. Proc Cambridge Philos Soc 6:199–210

Caillerie D (1982) Plaques elastiques minces a structute periodique de periode et depaisseur comparable. C R Acad Sci Ser 2 294(3):159–162

Caillerie D (1984) Thin elastic, periodic plates. Math Method Appl Sci 6:159–191

Chakraborty S, Dash S, Dey T (2024b) Non-linear stability characteristics of randomly distributed cnt-reinforced fiber composite beams under thermo-mechanical loadings. Mech Based Des Struct Mach Int J 52(9):7021–7044

Chakraborty S, Naskar S, Dey T, Kumar R, Mukhopadhyay T (2024a) Nonlinear stability of curved multiphase composite panels: Influence of agglomeration in randomly distributed carbon nanotubes with nonuniform in-plane loads. J Aerosp Eng 37(3)

Ciarlet G (1990) Plates and junctions in elastic multi-structures. An asymptotic analysis. Macon, Paris

Ciarlet PG, Destuynder P (1979) A justification of two-dimensional plate model. J Mecan 18:315–344

Ciarlet PG, Kesavan S (1981) Two-dimensional approximations of three-dimensional eigenvalue problems in plate theory. Comput Meth Appl Mech Eng 26(2):145–172

Destuynder P (1981) Unu Theorie Aymptotiques des Plaques Mincea en Elasticite Lenearie. Masson, Paris

Duvaut G (1976) Analyse fonctionnelle et mecanique des milieux continus—application a l’etude des materiaux composites elastiques a structure periodique—homogeneisation. In: Koiter W (ed) Proceedings 14th IUTAM congress. North Holland, Amsterdam, pp 119–132

Euler L (1759) Sur la force des colonnes. Histoire de l’Academie Royal des sciences et belle-lettres avec les Memoires. Tires des Registres de cette Academie 13:252–282

Foraboschi P (2013) Layered plate with discontinuous connection: exact mathematical model. Comput Part B: Eng 47:365–378

Gaudiello A, Kolpakov AG (2011) Influence of non degenerated joint on the global and local behavior of joined rods. Int J Eng Sci 49(3):295–309

Ghorbani A, Mirzaali MJ, Roebroek T, Coulais C, Bonn D, van der Linden E, Habibi M (2024) Suppressing torsional buckling in auxetic meta-shells. Nat Commun 15(6999)

Gol’denveizer AL (1961) Theory of elastic thin shells. Pergamon, Oxford

Gol’denveizer AL (1962) Derivation of an approximate theory of bending of a plate by the method of asymptotic integration of the equations of the theory of elasticity (in Russian). Prikl Math Mekh 26:668–686

Gol’denveizer AL (1963) Derivation of an approximate theory of shells by means of asymptotic integration of the equations of the theory of elasticity (in Russian). Prikl Math Mekh 27:593–608

Goriely A, Vandiver R, Destrade M (2008) Nonlinear Euler buckling. Proc R Soc A 464(2099):3003–3019

Green AE, Rivlin RS, Shield RT (1952) General theory of small elastic deformations superposed on finite elastic deformations. Proc R Soc Ser A 211(1104):128–154

Guz AN (1986) Elastic waves in bodies with initial stresses. In: Two volumes. Naukova Dumka, Kiev (in Russian)
Guz AN (1999) Fundamentals of the three-dimensional theory of stability of deformable bodies. Springer, Berlin
Haug EJ, Choi KK, Komkov V (1986) Design sensitivity analysis of structural systems. Academic, Orlando
Hill R (1958) A general theory of uniqueness and stability in elastic-plastic solids. J Mech Phys Solids 6(3):236–249
Hu Ch-F, Li Zh, Hu Q-S (2021) On non-linear behavior and buckling of arch-beam structures. Eng Struct 239:112214
Jikov VV, Kozlov SM, Oleinik OA (1994) Homogenization of differential operators and integral functionals. Springer, Berlin
Kalamkarov AL, Kolpakov AG (1997) Analysis, design and optimization of composite structures. Wiley, Chichester
Kalamkarov AL (1992) Composite and reinforcement of elements of construction. Wiley, Chichester
Kaplunov JD, Kossovich LY, Nolde EV (1998) Dynamics of thin walled elastic bodies. Academic, New York
Kaplunov JD, Nolde EV, Rogerson GA (2002) Short-wave motion of a pre-stressed plate. IMA J Appl Math 67:383–399
Kelly A, McCartney LN, Clegg WJ, Stearn RJ (2005) Controlling thermal expansion to obtain negative expansivity using laminated composites. Comput Sci Tech 65(1):47–59
Kelly A, Rabotnov YN (eds) (1988) Handbook of composites. North-Holland, Amstredam
Kesavan S (1979) Homogenization of elliptic eigenvalue problems. Parts I and II. Appl Math Optim 5:157–167 and 197–216
Kolpakov AG (1995) Asymptotic problem stability of beams. Buckling under torsion. Appl Mech Tech Phys 6:133–141
Kolpakov AG (2004) Stressed composite structures: homogenized models for thin-walled nonhomogeneous structures with initial stresses. Springer, Berlin
Kolpakov AA, Kolpakov AG (2010) Capacity and transport in contrast composite structures: asymptotic analysis and applications. CRC Press, Boca Raton, FL
Kolpakov AG, Sheremet IG (2004) 2-D model of stresses plate derived from the 3-D elasticity theory model. In: Kienzler R, Ott I, Altenbach H (eds) Theories of plates and shells. Critical review and new applications, volume 16 of Lecture notes in applied and computational mechanics. Springer, Berlin, pp 97–106
Kolpakov AG (1985) On determination of the average characteristics of elastic frameworks. J Appl Math Mech 49:739–745
Kolpakov AG (1987) Averaging in problems of the bending and oscillation of stressed inhomogeneous plate. J Appl Math Mech 51(1):44–49
Kolpakov AG (1991) Calculation of the characteristics of thin elastic rods of periodic structure. J Appl Math Mech 57:440–448
Kolpakov AG (1992) On problem of beam with initial stress. Appl Mech Tech Phys 6:139–144
Kolpakov AG (2001) On the calculation of rigidity characteristics of the stressed constructions. Int J Solids Struct 38(15):2469–2485
Kolpakov AG (2005) Effect of inflation of initial stresses on the homogenized characteristics of composite. Mech Mater 37(8):840–854
Kolpakov AA (2007) Numerical verification of existence of the energy concentration effect in high-contrast high-filled composite material. J Eng Phys Thermophys 4:812–819
Kolpakov AG (2011) Influence of non degenerated joint on the global and local behavior of joined plates. Int J Eng Sci 49(11):1216–1231
Kolpakov AA, Kolpakov AG (2007) Asymptotic of capacity of a system of closely placed bodies. Dokl Phys 415(2):188–192
Kolpakov AG, Rakin SI (1986) On problem of sintsesys of composite material of unidirectional structure with prescribed characteristics. J Appl Mech Tech Phys 6:143–150