STUDENT SOLUTIONS MANUAL TO ACCOMPANY LOSS MODELS

FROM DATA TO DECISIONS

Fourth Edition

STUART A. KLUGMAN • HARRY H. PANJER • GORDON E. WILLMOT

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Student Solutions Manual to Accompany

LOSS MODELS

From Data to Decisions

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CHAPTER 1

INTRODUCTION

The solutions presented in this manual reflect the authors' best attempt to provide insights and answers. While we have done our best to be complete and accurate, errors may occur and there may be more elegant solutions. Errata will be posted at the ftp site dedicated to the text and solutions manual: ftp://ftp.wiley.com/public/sci_tech_med/loss_models/

Should you find errors or would like to provide improved solutions, please send your comments to Stuart Klugman at sklugman@soa.org.

CHAPTER 2

CHAPTER 2 SOLUTIONS

2.1 SECTION 2.2

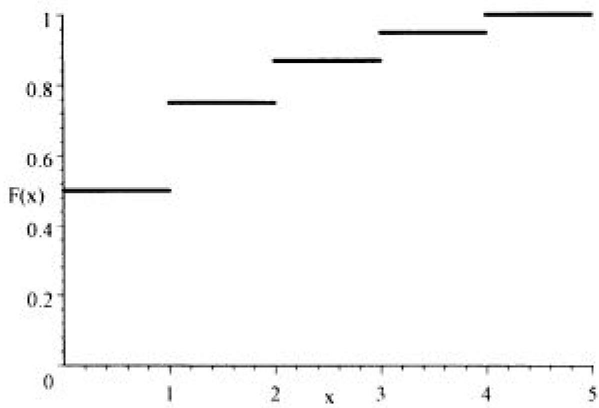
2.1

$$F_5(x) = 1 - S_5(x) = \begin{cases} 0.01x, & 0 \leq x < 50, \\ 0.02x - 0.5, & 50 \leq x < 75. \end{cases}$$

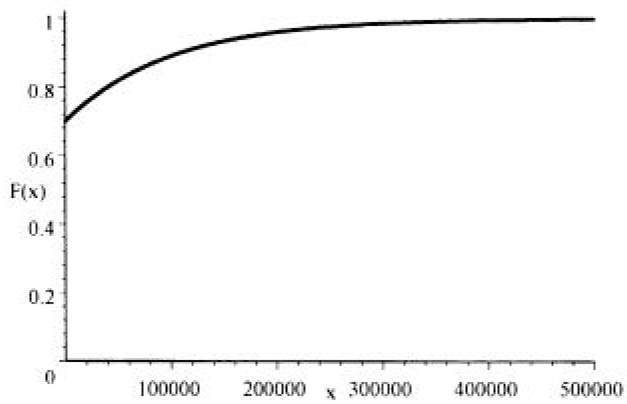
$$f_5(x) = F_5'(x) = \begin{cases} 0.01, & 0 < x < 50, \\ 0.02, & 50 \leq x < 75. \end{cases}$$

$$h_5(x) = \frac{f_5(x)}{S_5(x)} = \begin{cases} \dfrac{1}{100 - x}, & 0 < x < 50, \\ \dfrac{1}{75 - x}, & 50 \leq x < 75. \end{cases}$$

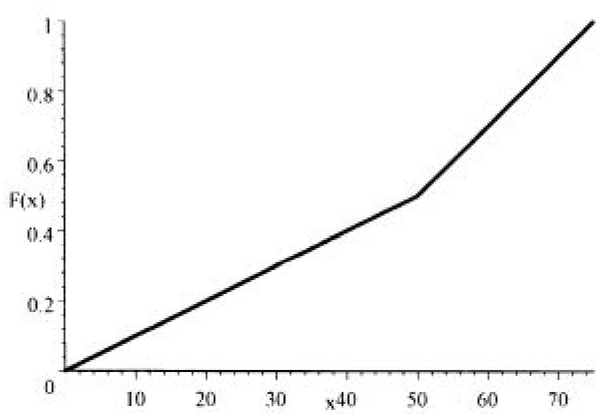
2.2 The requested plots follow. The triangular spike at zero in the density function for Model 4 indicates the 0.7 of discrete probability at zero.



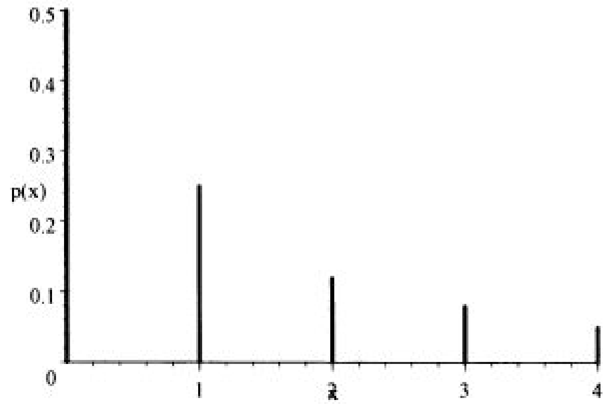
Distribution function for Model 3.



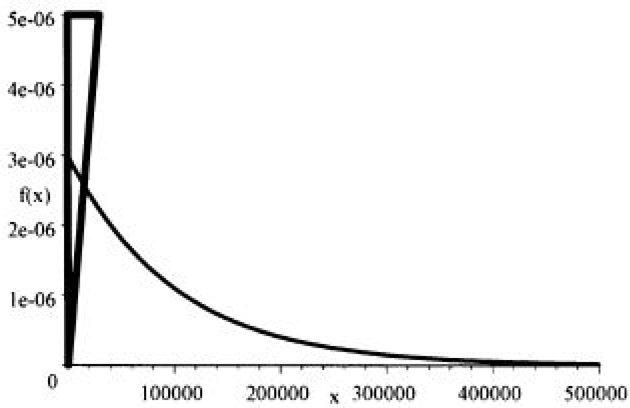
Distribution function for Model 4.



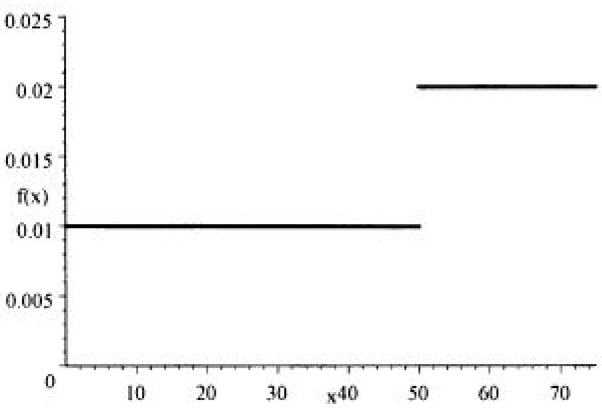
Distribution function for Model 5.



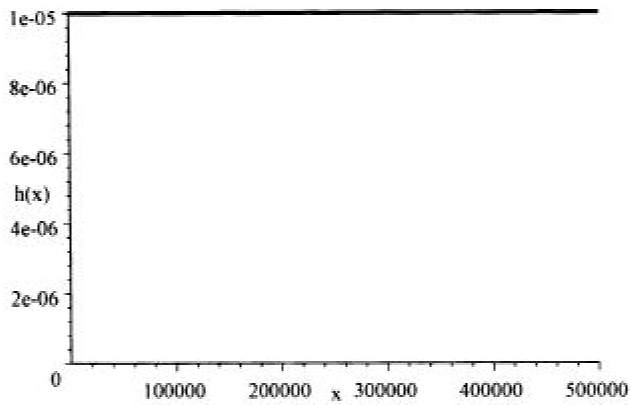
Probability function for Model 3.



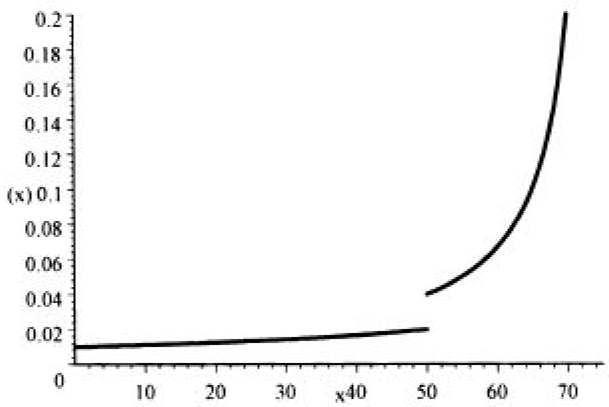
Density function for Model 4.



Density function for Model 5.



Hazard rate for Model 4.



Hazard rate for Model 5.

2.3 $f'(x) = 4(1 + x^2)^{-3} - 24x^2(1 + x^2)^{-4}$. Setting the derivative equal to zero and multiplying by $(1 + x^2)^4$ give the equation $4(1 + x^2) - 24x^2 = 0$. This is equivalent to $x^2 = 1/5$. The only positive solution is the mode of $1/\sqrt{5}$.

2.4 The survival function can be recovered as

$$\begin{aligned} 0.5 &= S(0.4) = e^{-\int_0^{0.4} A+e^{2x}dx} \\ &= e^{-Ax-0.5e^{2x}\big|_0^{0.4}} \\ &= e^{-0.4A-0.5e^{0.8}+0.5}. \end{aligned}$$

Taking logarithms gives

$$-0.693147 = -0.4A - 1.112770 + 0.5,$$

and thus $A = 0.2009$.

2.5 The ratio is

$$
\begin{aligned}
r &= \frac{\left(\dfrac{10,000}{10,000+d}\right)^2}{\left(\dfrac{20,000}{20,000+d^2}\right)^2} \\
&= \left(\frac{20,000+d^2}{20,000+2d}\right)^2 \\
&= \frac{20,000^2+40,000d^2+d^4}{20,000^2+80,000d+4d^2}.
\end{aligned}
$$

From observation or two applications of L'Hôpital's rule, we see that the limit is infinity.

CHAPTER 3

CHAPTER 3 SOLUTIONS

3.1 SECTION 3.1

3.1

$$\begin{aligned}\mu_3 &= \int_{-\infty}^{\infty}(x-\mu)^3 f(x)dx = \int_{-\infty}^{\infty}(x^3-3x^2\mu+3x\mu^2-\mu^3)f(x)dx \\ &= \mu_3' - 3\mu_2'\mu + 2\mu^3,\end{aligned}$$

$$\begin{aligned}\mu_4 &= \int_{-\infty}^{\infty}(x-\mu)^4 f(x)dx \\ &= \int_{-\infty}^{\infty}(x^4-4x^3\mu+6x^2\mu^2-4x\mu^3+\mu^4)f(x)dx \\ &= \mu_4' - 4\mu_3'\mu + 6\mu_2'\mu^2 - 3\mu^4.\end{aligned}$$

3.2 For Model 1, σ^2 = 3,333.33 – 50^2 = 833.33, σ = 28.8675.

$\mu_3' = \int_0^{100} x^3(0.01)dx = 250{,}000,\ \mu_3 = 0,\ \gamma_1 = 0.$
$\mu_4' = \int_0^{100} x^4(0.01)dx = 20{,}000{,}000,\ \mu_4 = 1{,}250{,}000,\ \gamma_2 = 1.8.$

For Model 2, σ^2 = 4,000,000 – $1{,}000^2$ = 3,000,000, σ = 1,732.05. μ_3' and μ_4' are both infinite so the skewness and kurtosis are not defined.

For Model 3, σ^2 = 2.25 – $.93^2$ = 1.3851, σ = 1.1769.

$\mu_3' = 0(0.5)+1(0.25)+8(0.12)+27(0.08)+64(0.05) = 6.57,\ \mu_3 = 1.9012,$
$\gamma_1 = 1.1663,\ \mu_4' = 0(0.5)+1(0.25)+16(0.12)+81(0.08)+256(0.05) = 21.45,$
$\mu_4 = 6.4416,\ \gamma_2 = 3.3576.$

For Model 4, σ^2 = 6,000,000,000 – $30{,}000^2$ = 5,100,000,000, σ = 71,414.

$\mu_3' = 0^3(0.7) + \int_0^{\infty} x^3(0.000003)e^{-.00001x}dx = 1.8\times 10^{15},$
$\mu_3 = 1.314\times 10^{15},\ \gamma_1 = 3.6078.$
$\mu_4' = \int_0^{\infty} x^4(0.000003)e^{-.00001x}dx = 7.2\times 10^{20},\ \mu_4 = 5.3397\times 10^{20},$
$\gamma_2 = 20.5294.$

For Model 5, σ^2 = 2,395.83 – 43.75^2 = 481.77, σ = 21.95.

$\mu_3' = \int_0^{50} x^3(0.01)dx + \int_{50}^{75} x^3(0.02)dx = 142{,}578.125,\ \mu_3 = -4{,}394.53,$
$\gamma_1 = -0.4156.$
$\mu_4' = \int_0^{50} x^4(0.01)dx + \int_{50}^{75} x^4(0.02)dx = 8{,}867{,}187.5,\ \mu_4 = 439{,}758.30,$
$\gamma_2 = 1.8947.$

3.3 The Standard deviation is the mean times the coefficient, of Variation, or 4, and so the variance is 16. From (3.3) the second raw moment is $16 + 2^2 = 20$. The third central moment is (using Exercise 3.1) $136 – 3(20)(2) + 2(2)^3 = 32$. The skewness is the third central moment divided by the cube of the Standard deviation, or $32/4^3 = 1/2$.

3.4 For a gamma distribution the mean is $\alpha\theta$. The second raw moment is $\alpha(\alpha + 1)\theta^2$, and so the variance is $\alpha\theta^2$. The coefficient of Variation is $\sqrt{\alpha\theta^2}/\alpha\theta = \alpha^{-1/2} = 1$. Therefore $\alpha = 1$. The third raw moment is $\alpha(\alpha + 1)(\alpha + 2)\theta^3 = 6\theta^3$. From Exercise 3.1, the third central moment is $6\theta^3 – 3(2\theta^2)\theta + 2\theta^3 = 2\theta^3$ and the skewness is $2\theta^3/(\theta^2)^{3/2} = 2$.

3.5 For Model 1,

$$e(d) = \frac{\int_d^{100}(1-0.01x)dx}{1-0.01d} = \frac{100-d}{2}.$$

For Model 2,

$$e(d) = \frac{\displaystyle\int_d^{\infty}\left(\frac{2,000}{x+2,000}\right)^3 dx}{\left(\dfrac{2,000}{d+2,000}\right)^3} = \frac{2{,}000+d}{2}.$$

For Model 3,

$$e(d) = \begin{cases} \dfrac{0.25(1-d)+0.12(2-d)+0.08(3-d)+0.05(4-d)}{0.5} \\ \quad = 1.86 - d, & 0 \leq d < 1, \\ \dfrac{0.12(2-d)+0.08(3-d)+0.05(4-d)}{0.25} = 2.72 - d, & 1 \leq d < 2, \\ \dfrac{0.08(3-d)+0.05(4-d)}{0.13} = 3.3846 - d, & 2 \leq d < 3, \\ \dfrac{0.05(4-d)}{0.05} = 4 - d, & 3 \leq d < 4. \end{cases}$$

For Model 4,

$$e(d) = \frac{\int_d^\infty 0.3e^{-0.00001x}dx}{0.3e^{-0.00001d}} = 100{,}000.$$

The functions are straight lines for Models 1, 2, and 4. Model 1 has negative slope, Model 2 has positive slope, and Model 4 is horizontal.

3.6 For a uniform distribution on the interval from 0 to *w*, the density function is $f(x) = 1/w$. The mean residual life is

$$\begin{aligned} e(d) &= \frac{\int_d^w (x-d)w^{-1}dx}{\int_d^w w^{-1}dx} \\ &= \frac{\left.\dfrac{(x-d)^2}{2w}\right|_d^w}{\dfrac{w-d}{w}} \\ &= \frac{(w-d)^2}{2(w-d)} \\ &= \frac{w-d}{2}. \end{aligned}$$

The equation becomes

$$\frac{w-30}{2} = \frac{100-30}{2} + 4,$$

with a solution of $w = 108$.

3.7 From the definition,

$$e(\lambda) = \frac{\int_\lambda^\infty (x-\lambda)\lambda^{-1}e^{-x/\lambda}dx}{\int_\lambda^\infty \lambda^{-1}e^{-x/\lambda}dx} = \lambda.$$

3.8

$$\begin{aligned}\mathrm{E}(X) &= \int_0^\infty xf(x)dx = \int_0^d xf(x)dx + \int_d^\infty df(x)dx + \int_d^\infty (x-d)f(x)dx \\ &= \int_0^d xf(x)dx + d[1-F(d)] + e(d)S(d) = \mathrm{E}[X \wedge d] + e(d)S(d).\end{aligned}$$

3.9 For Model 1, from (3.8),

$$\mathrm{E}[X \wedge u] = \int_0^u x(0.01)dx + u(1-0.01u) = u(1-0.005u)$$

and from (3.10),

$$\mathrm{E}[X \wedge u] = 50 - \frac{100-u}{2}(1-0.01u) = u(1-0.005u).$$

From (3.9),

$$\mathrm{E}[X \wedge u] = -\int_{-\infty}^0 0\,dx + \int_0^u 1 - 0.01x\,dx = u - \frac{0.01u^2}{2} = u(1-0.005u).$$

For Model 2, from (3.8),

$$\mathrm{E}[X \wedge u] = \int_0^u x\frac{3(2{,}000)^3}{(x+2{,}000)^4}\,dx + u\frac{2{,}000^3}{(2{,}000+u)^3} = 1000\left[1 - \frac{4{,}000{,}000}{(2{,}000+u)^2}\right],$$

and from (3.10),

$$\mathrm{E}[X \wedge u] = 1{,}000 - \frac{2{,}000+u}{2}\left(\frac{2{,}000}{2{,}000+u}\right)^3 = 1{,}000\left[1 - \frac{4{,}000{,}000}{(2{,}000+u)^2}\right].$$

From (3.9),

$$\begin{aligned}\mathrm{E}[X \wedge u] &= \int_0^u \left(\frac{2{,}000}{2{,}000+x}\right)^3 dx = \left.\frac{-2{,}000^3}{2(2{,}000+x)^2}\right|_0^u \\ &= 1{,}000\left[1 - \frac{4{,}000{,}000}{(2{,}000+u)^2}\right].\end{aligned}$$

For Model 3, from (3.8),

$$
\mathrm{E}[X \wedge u] = \begin{cases} 0(0.5) + u(0.5) = 0.5u, & 0 \le u < 1, \\ 0(0.5) + 1(0.25) + u(0.25) = 0.25 + 0.25u, & 1 \le u < 2, \\ 0(0.5) + 1(0.25) + 2(0.12) + u(0.13) \\ = 0.49 + 0.13u, & 2 \le u < 3, \\ 0(0.5) + 1(0.25) + 2(0.12) + 3(0.08) + u(0.05) \\ = 0.73 + 0.05u, & 3 \le u < 4, \end{cases}
$$

and from (3.10),

$$
\mathrm{E}[X \wedge u] = \begin{cases} 0.93 - (1.86 - u)(0.5) = 0.5u, & 0 \le u < 1, \\ 0.93 - (2.72 - u)(0.25) = 0.25 + 0.25u, & 1 \le u < 2, \\ 0.93 - (3.3846 - u)(0.13) = 0.49 + 0.13u, & 2 \le u < 3, \\ 0.93 - (4 - u)0(.05) = 0.73 + 0.05u, & 3 \le u < 4. \end{cases}
$$

For Model 4, from (3.8),

$$
\begin{aligned} \mathrm{E}[X \wedge u] &= \int_0^u x(0.000003)e^{-0.00001x}dx + u(0.3)e^{-0.00001u} \\ &= 30{,}000[1 - e^{-0.00001u}], \end{aligned}
$$

and from (3.10),

$$
\mathrm{E}[X \wedge u] = 30{,}000 - 100{,}000(0.3e^{-0.00001u}) = 30{,}000[1 - e^{-0.00001u}].
$$

3.10 For a discrete distribution (which all empirical distributions are), the mean residual life function is

$$
e(d) = \frac{\sum_{x_j > d}(x_j - d)p(x_j)}{\sum_{x_j > d} p(x_j)}.
$$

When *d* is equal to a possible value of *X*, the function cannot be continuous because there is jump in the denominator but not in the numerator. For an exponential distribution, argue as in Exercise 3.7 to see that it is constant. For the Pareto distribution,

$$
\begin{aligned}
e(d) &= \frac{\mathrm{E}(X) - \mathrm{E}(X \wedge d)}{S(d)} \\
&= \frac{\frac{\theta}{\alpha-1} - \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{\theta+d}\right)^{\alpha-1}\right]}{\left(\frac{\theta}{\theta+d}\right)^{\alpha}} \\
&= \frac{\theta}{\alpha-1}\frac{\theta+d}{\theta} = \frac{\theta+d}{\alpha-1},
\end{aligned}
$$

which is increasing in d. Only the second statement is true.

3.11 Applying the formula from the solution to Exercise 3.10 gives

$$\frac{10{,}000 + 10{,}000}{0.5 - 1} = -40{,}000,$$

which cannot be correct. Recall that the numerator of the mean residual life is E(X)–E($X \wedge d$). However, when $\alpha \leq 1$, the expected value is infinite and so is the mean residual life.

3.12 The right truncated variable is defined as $Y = X$ given that $X \leq u$. When $X > u$, this variable is not defined. The kth moment is

$$\mathrm{E}(Y^k) = \frac{\int_0^u x^k f(x)dx}{F(u)} = \frac{\sum_{x_i \leq u} x_i^k p(x_i)}{F(u)}.$$

3.13 This is a single parameter Pareto distribution with parameters $\alpha = 2.5$ and $\theta = 1$. The moments are $\mu_1 = 2.5/1.5 = 5/3$ and $\mu_2 = 2.5/.5 - (5/3)^2 = 20/9$. The coefficient of Variation is $\sqrt{20/9}/(5/3) = 0.89443$.

3.14 $\mu = 0.05(100) + 0.2(200) + 0.5(300) + 0.2(400) + 0.05(500) = 300$.

$\sigma^2 = 0.05(-200)^2 + 0.2(-100)^2 + 0.5(0)^2 + 0.2(100)^2 + 0.05(200)^2 = 8{,}000$.

$\mu_3 = 0.05(-200)^3 + 0.2(-100)^3 + 0.5(0)^3 + 0.2(100)^3 + 0.05(200)^3 = 0$.

$\mu_4 = 0.05(-200)^4 + 0.2(-100)^4 + 0.5(0)^4 + 0.2(100)^4 + 0.05(200)^4 = 200{,}000{,}000$.

Skewness is $= \gamma_1 = \mu^3/\sigma^3 = 0$. Kurtosis is $\gamma_2 = \mu_4/\sigma^4 = 200{,}000{,}000/8{,}000^2 = 3.125$.

3.15 The Pareto mean residual life function is

$$e_X(d) = \frac{\int_d^\infty \theta^a (x+\theta)^{-\alpha} dx}{\theta^\alpha (x+d)^{-\alpha}} = (d+\theta)/(\alpha-1),$$

and so $e_X(2\theta)/e_X(\theta) = (2\theta + \theta)/(\theta + \theta) = 1.5$.

3.16 Sample mean: 0.2(400) + 0.7(800) + 0.1(1,600) = 800. Sample variance: $0.2(-400)^2 + 0.7(0)^2 + 0.1(800)^2 = 96{,}000$. Sample third central moment: $0.2(-400)^3 + 0.7(0)^3 + 0.1(800)^3 = 38{,}400{,}000$. Skewness coefficient: $38{,}400{,}000/96{,}000^{1.5} = 1.29$.

3.2 SECTION 3.2

3.17 The pdf is $f(x) = 2x^{-3}$, $x \geq 1$. The mean is $\int_1^\infty 2x^{-2}dx = 2$. The median is the solution to $.5 = F(x) = 1 - x^{-2}$, which is 1.4142. The mode is the value where the *pdf* is highest. Because the pdf is strictly decreasing, the mode is at its smallest value, 1.

3.18 For Model 2, solve $p = 1-\left(\frac{2{,}000}{2{,}000+\pi_p}\right)^3$ and so $\pi_p = 2{,}000[(1 - p)^{-1/3} - 1]$ and the requested percentiles are 519.84 and 1419.95.

For Model 4, the distribution function jumps from 0 to 0.7 at zero and so $\pi_{0.5} = 0$. For percentile above 70, solve $p = 1 - 0.3e^{-0.00001\pi_p}$, and so $\pi_p = -100{,}000 \ln[(1 - p)/0.3]$ and $\pi_{0.8} = 40{,}546.51$.

For Model 5, the distribution function has two specifications. From $x = 0$ to $x = 50$ it rises from 0.0 to 0.5, and so for percentiles at 50 or below, the equation to solve is $p = 0.01\pi_p$ for $\pi_p = 100p$. For $50 < x \leq 75$, the distribution function rises from 0.5 to 1.0, and so for percentiles from 50 to 100 the equation to solve is $p = 0.02\pi_p - 0.5$ for $\pi_p = 50p + 25$. The requested percentiles are 50 and 65.

3.19 The two percentiles imply

$$\begin{aligned} 0.1 &= 1-\left(\frac{\theta}{\theta+\theta-k}\right)^\alpha, \\ 0.9 &= 1-\left(\frac{\theta}{\theta+5\theta-3k}\right)^\alpha. \end{aligned}$$

Rearranging the equations and taking their ratio yield

$$\frac{0.9}{0.1} = \left(\frac{6\theta-3k}{2\theta-k}\right)^\alpha = 3^\alpha$$

Taking logarithms of both sides gives ln 9 = α ln 3 for α = ln 9/ln 3 = 2.

3.20 The two percentiles imply

$$\begin{aligned} 0.25 &= 1 - e^{-(1{,}000/\theta)^\tau}, \\ 0.75 &= 1 - e^{-(100{,}000/\theta)^\tau}. \end{aligned}$$

Subtracting and then taking logarithms of both sides give

$$\begin{aligned} \ln 0.75 &= -(1{,}000/\theta)^\tau, \\ \ln 0.25 &= -(100{,}000/\theta)^\tau. \end{aligned}$$

Dividing the second equation by the first gives

$$\frac{\ln 0.25}{\ln 0.75} = 100^\tau.$$

Finally, taking logarithms of both sides gives τ ln 100 = ln[ln 0.25/ln 0.75] for τ = 0.3415.

3.3 SECTION 3.3

3.21 The sum has a gamma distribution with parameters α = 16 and θ = 250. Then, Pr(S_{16} > 6,000) = 1 – Γ(16; 6,000/250) = 1 – Γ(16;24). From the Central Limit Theorem, the sum has an approximate normal distribution with mean $\alpha\theta$ = 4,000 and variance $\alpha\theta^2$ = 1,000,000 for a Standard deviation of 1000. The probability of exceeding 6,000 is 1 – Φ[(6,000 – 4,000)/1,000] = 1 – Φ(2) = 0.0228.

3.22 A single claim has mean 8,000/(5/3) = 4,800 and variance

$$2(8{,}000)^2/[(5/3)(2/3)] - 4{,}800^2 = 92{,}160{,}000.$$

The sum of 100 claims has mean 480,000 and variance 9,216,000,000, which is a Standard deviation of 96,000. The probability of exceeding 600,000 is approximately

$$1 - \Phi[(600{,}000 - 480{,}000)/96{,}000] = 1 - \Phi(1.25) = 0.106.$$

3.23 The mean of the gamma distribution is 5(1,000) = 5,000 and the variance is $5(1{,}000)^2$ = 5,000,000. For 100 independent claims, the mean is 500,000 and the variance is 500,000,000 for a Standard deviation of 22,360.68. The probability of total claims exceeding 525,000 is

$$1 - \Phi[(525{,}000 - 500{,}000)/22{,}360.68] = 1 - \Phi(1.118) = 0.13178.$$

3.24 The sum of 2,500 contracts has an approximate normal distribution with mean 2,500(1,300) = 3,250,000 and Standard deviation $\sqrt{2,500}$(400) = 20,000. The answer is Pr(X > 3,282,500) $\doteq$ Pr[Z > (3,282,500 – 3,250,000)/20,000] = Pr(Z > 1.625) = 0.052.