



ADVANCED VEHICLE SCANNING METHOD

BRIDGE MODAL
PARAMETER IDENTIFICATION

HAO XU ♦ DER-SHEN YANG ♦ YEONG-BIN YANG

WILEY

Advanced Vehicle Scanning Method

Advanced Vehicle Scanning Method

Bridge Modal Parameter Identification

Hao Xu

School of Civil Engineering, Chongqing University, China

Der-Shen Yang

College of Civil Engineering, Tongji University, China

Yeong-Bin Yang

School of Civil Engineering, Chongqing University, China

WILEY

Copyright © 2025 by John Wiley & Sons, Inc. All rights reserved, including rights for text and data mining and training of artificial technologies or similar technologies.

Published by John Wiley & Sons, Inc., Hoboken, New Jersey.
Published simultaneously in Canada.

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 750-4470, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, or online at <http://www.wiley.com/go/permission>.

The manufacturer's authorized representative according to the EU General Product Safety Regulation is Wiley-VCH GmbH, Boschstr. 12, 69469 Weinheim, Germany, e-mail: Product_Safety@wiley.com.

Trademarks: Wiley and the Wiley logo are trademarks or registered trademarks of John Wiley & Sons, Inc. and/or its affiliates in the United States and other countries and may not be used without written permission. All other trademarks are the property of their respective owners. John Wiley & Sons, Inc. is not associated with any product or vendor mentioned in this book.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Further, readers should be aware that websites listed in this work may have changed or disappeared between when this work was written and when it is read. Neither the publisher nor authors shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

For general information on our other products and services or for technical support, please contact our Customer Care Department within the United States at (800) 762-2974, outside the United States at (317) 572-3993 or fax (317) 572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic formats. For more information about Wiley products, visit our web site at www.wiley.com.

Library of Congress Cataloging-in-Publication Data applied for:

Hardback ISBN: 9781394286034

Cover Design: Wiley
Cover Image: © Comezora/Getty Images

Set in 9.5/12.5pt STIXTwoText by Straive, Chennai, India

Contents

Preface	<i>xv</i>
Acknowledgments	<i>xix</i>
List of Symbols	<i>xxi</i>
List of Abbreviations	<i>xxx</i>
1 Introduction	<i>1</i>
1.1	Background <i>1</i>
1.2	Basic Concept of the VSM for Bridges <i>3</i>
1.2.1	Bridge Frequency Identification <i>3</i>
1.2.2	Bridge Mode Shape Identification <i>4</i>
1.2.3	Bridge Damping Ratio Identification <i>5</i>
1.3	Brief on the Works Conducted by Yang and Coworkers <i>5</i>
1.3.1	Vehicle and Bridge Models Used and Their Vibration Mechanisms <i>6</i>
1.3.1.1	Vehicle Models <i>6</i>
1.3.1.2	Bridge's Models and Properties <i>7</i>
1.3.2	Enhanced Methods for Bridge Frequency Identification <i>8</i>
1.3.2.1	Software-Based Approaches <i>8</i>
1.3.2.2	Hardware-Based Approaches <i>10</i>
1.3.3	Bridge Mode Shape Identification <i>11</i>
1.3.4	Bridge Damping Ratio Identification <i>12</i>
1.3.5	Bridge Damage Identification <i>13</i>
1.3.6	Extension of VSM to Railway Tracks <i>13</i>
1.4	Bridge Modal Parameter Identification by Researchers Worldwide <i>14</i>
1.4.1	Bridge Frequency Identification <i>14</i>
1.4.1.1	Vehicle and Bridge Models Used and Their Mechanism of Vibration <i>14</i>
1.4.1.2	Time-Varying Characteristics of the Vehicle–Bridge System <i>17</i>
1.4.1.3	Enhanced Methods for Bridge Frequency Identification <i>18</i>
1.4.2	Bridge Mode Shape Identification <i>20</i>
1.4.2.1	HT-Based Techniques <i>21</i>
1.4.2.2	Time–Frequency Techniques <i>21</i>
1.4.2.3	Other Mode Shape Identification Methods <i>22</i>
1.4.3	Bridge Damping Ratio Identification <i>23</i>

1.5	Bridge Damage Identification by Researchers Worldwide	24
1.5.1	Modal Parameter-Based Methods	25
1.5.1.1	Natural Frequency-Based Methods	25
1.5.1.2	Mode Shape-Based Methods	25
1.5.2	Signal Processing-Based Methods	27
1.5.3	Machine Learning-Based Methods	28
1.5.4	Other Methods	30
1.6	Pavement Roughness Identification by Researchers Worldwide	31
1.7	Vehicle Scanning Method for Railway Tracks and Bridges	32
1.7.1	Track Geometry Estimation	32
1.7.2	Identification of Dynamic Parameters of Railway Tracks and Bridges	34
1.7.3	Track Defect Detection	35
1.8	Application of Smartphone-Based IoT System in VSM	37
1.9	Conclusions and Recommendations for Future Work	39
1.9.1	Conclusions	39
1.9.2	Challenges and Recommendations	40

Part I Vehicle Scanning Method for Bridge Frequencies 43

2	Damped Scanning Vehicle for Bridge Frequencies: Theory and Experiment	45
2.1	Introduction	45
2.2	Formulation of the Analytical Theory	47
2.2.1	Dynamic Responses of the Bridge and Contact Point	47
2.2.2	Dynamic Response of the Test Vehicle	49
2.3	Calculation of Contact Response of the Damped Test Vehicle	51
2.3.1	Backward Calculation Procedure for the Contact Response	51
2.3.2	Transmissibility Between Contact Point and Vehicle Responses	53
2.4	Numerical Formulation of the Problem	54
2.4.1	Description of VBI Element for Single-DOF Vehicle	54
2.4.2	Verification of Analytical Solution	55
2.5	Parametric Study	57
2.5.1	Effect of Vehicle Damping	57
2.5.2	Effect of Test Vehicle Speed	59
2.5.3	Effect of Environmental Noise	59
2.5.4	Effect of Surface Roughness	61
2.5.4.1	Scenario 1: Bridge Without Ongoing Traffic	62
2.5.4.2	Scenario 2: Bridge with Ongoing Traffic	63
2.6	Experimental Study	65
2.6.1	Brief on the Test Bridge	65
2.6.2	Measurement by Sensors Deployed on the Bridge Surface	66
2.6.3	Design of the Test Vehicle	69
2.6.4	Measurement by the Test Vehicle in the Nonmoving State	72
2.6.5	Flat Road Test for Vehicle Frequency in the Moving State	75

2.6.6	Measurement by the Test Vehicle in the Moving State	75
2.6.6.1	Scenario 1: Vehicle Moving Along the Bridge Centerline	76
2.6.6.2	Scenario 2: Vehicle Moving Along the Centerline with a Temporary Parking for 30 s	77
2.7	Concluding Remarks	79
3	Refined Detection for Bridge Frequencies: Theory and Experiment	81
3.1	Introduction	81
3.2	Contact Responses for Two Wheels of Single-Axle Vehicle	84
3.3	Brief on Test Bridge and Direct Measurement	87
3.4	Description of Self-Designed Single-Axle Test Vehicle	87
3.4.1	Properties of Single-Axle Test Vehicle	88
3.4.2	Responses of Test Vehicle Moving over Flat Road	89
3.4.3	Contact Responses of Test Vehicle Moving on Flat Road	91
3.5	Scanning Bridge's Frequencies by Test Vehicle's Rocking Motion	93
3.5.1	Scenario 1: Test Vehicle Moving Nonstop over Bridge	93
3.5.1.1	Wheel Responses of Vehicle Moving over Bridge	93
3.5.1.2	Contact Responses of Test Vehicle's Wheels Moving over Bridge	95
3.5.2	Scenario 2: Vehicle Moving over Bridge with Temporary Stop	97
3.5.2.1	Wheel Responses of Moving Vehicle with Temporary Stop on Bridge	97
3.5.2.2	Contact Responses for Test Vehicle Moving over Bridge with Temporary Stop	99
3.6	Concluding Remarks	100
4	Single-Axle Two-Mass Scanning Vehicle for Bridge Frequencies: Theory	103
4.1	Introduction	103
4.2	Analytical Formulation of the Problem	105
4.2.1	Dynamic Responses of the Bridge and Contact Point	105
4.2.2	Dynamic Responses of the Vehicle's Body and Wheel	107
4.3	Vehicle–Bridge Contact Response of Two-Mass Vehicle Model	109
4.3.1	Vehicle's Wheel Response Back-Calculated from Body Response	109
4.3.2	Contact Response Back-Calculated from Vehicle's Body and Wheel Responses	110
4.4	Numerical Simulation of the Problem	111
4.4.1	Description of Vehicle–Bridge Interaction Element	111
4.4.2	Verification of Analytical Solutions	112
4.4.3	Verification of Back-Calculation Procedure for Vehicle's Wheel Response	114
4.4.4	Verification of Back-Calculation Procedure for Contact Response	115
4.4.5	Applicability of the Contact Formula to Other Beam-Type Bridges	115
4.5	Parametric Study	117
4.5.1	Effect of Vehicle Damping	117
4.5.2	Effect of Vehicle Speed	118
4.5.3	Effect of Environmental Noise	120
4.5.4	Effect of Pavement Roughness	122
4.5.5	A More Realistic Condition	123
4.6	Concluding Remarks	126

5	Vehicle Scanning Method Enhanced by a Shaker	127
5.1	Introduction	127
5.2	Theoretical Modeling of the Problem	129
5.2.1	Dynamic Responses of the Bridge and Contact Point	130
5.2.2	Dynamic Response of the Test Vehicle	132
5.3	Dynamic Amplification Factor of the Shaker for Vehicle and Contact Responses	135
5.4	Numerical Verification	137
5.4.1	Verification of Analytical Solution	138
5.4.2	Verification of Back-Calculated Contact Response	139
5.5	Effect of the Shaker on Bridge Frequency Extraction	141
5.5.1	Effect of Shaker Frequency	141
5.5.2	Effect of Shaker Location	143
5.5.3	Effect of Vehicle Speed	144
5.6	Effects of Pavement Roughness and Environmental Noise	146
5.7	Concluding Remarks	147
6	Vehicle Scanning Method Enhanced by Amplifiers	149
6.1	Introduction	149
6.2	Analytical Formulation of the Problem	152
6.2.1	Dynamic Responses of the Bridge	152
6.2.2	Dynamic Responses of the Amplifier and Test Vehicle	153
6.3	Effect of Amplifier on the Amplifier–Vehicle–Bridge System	155
6.3.1	Dynamic Amplification Factor of Amplifier for the Bridge Response	156
6.3.2	Dynamic Amplification Factor of Vehicle for the Bridge Response	156
6.3.3	Featured Range of Amplifier Response vs. Vehicle Response	157
6.3.4	Effect of Amplifier on Vehicle’s Self-Frequency	158
6.4	Numerical Simulation of the Problem	159
6.4.1	Brief of Amplifier–Vehicle–Bridge Interaction Element	159
6.4.2	Verification of Analytical Solutions	161
6.5	Test Vehicle Set in (or Not in) Resonance	163
6.6	Effect of Amplifier on Bridge Frequency Extraction	165
6.6.1	Effect of Amplifier Mass	165
6.6.2	Effect of Amplifier Frequency	165
6.6.3	Dual Amplifiers for the Bridge Frequencies	167
6.7	Effect of Pavement Roughness	168
6.7.1	Amplifier Frequency Tuned to Vehicle Frequency	169
6.7.2	Dual Amplifiers Tuned for Different Functions	169
6.8	Concluding Remarks	171

Part II Vehicle Scanning Method for Bridge Mode Shapes and Damping Ratios 173

- 7 Theory for Scanning Bridge Mode Shapes Using a Two-Axle Vehicle 175**
 - 7.1 Introduction 175
 - 7.2 Closed-Form Solutions for Contact Responses 177
 - 7.3 Calculation of Contact Responses for Two-Axle Vehicle 179
 - 7.4 Recovery of Bridge Mode Shapes 181
 - 7.4.1 Brief on Variational Mode Decomposition 181
 - 7.4.2 Recovery of Bridge's Mode Shape by Hilbert Transform 182
 - 7.4.3 Procedure for Recovering Bridge Mode Shapes 184
 - 7.5 Numerical Verification of Back-Calculated Contact Responses 184
 - 7.6 Construction of Bridge Mode Shapes 188
 - 7.7 Parametric Study 190
 - 7.7.1 Effect of Vehicle Damping 190
 - 7.7.2 Effect of Vehicle Speed 191
 - 7.7.3 Effect of Number of Bridge Spans 192
 - 7.7.4 Effect of Environmental Noise 194
 - 7.7.5 Effect of Pavement Roughness 197
 - 7.8 Concluding Remarks 200

- 8 Formula for Determining Damping Ratio Using a Two-Axle Vehicle 201**
 - 8.1 Introduction 201
 - 8.2 Theoretical Formulation of the Problem 202
 - 8.3 Determination of Bridge Damping Ratio 204
 - 8.4 Numerical Verification 206
 - 8.4.1 Verification of Analytical Solution 206
 - 8.4.2 Verification of Back-calculated Contact Response 207
 - 8.4.3 Determination of Bridge Damping Ratio 208
 - 8.5 Effect of Pavement Roughness 210
 - 8.6 Concluding Remarks 212

- 9 Theory for Scanning Bridge Damping Ratios Using a Two-Axle Vehicle by Wavelet Transform 213**
 - 9.1 Introduction 213
 - 9.2 Analytical Formulation of the Problem 215
 - 9.2.1 Closed-form Solution for the Bridge 215
 - 9.2.2 Closed-form Solution for the Contact Responses 218
 - 9.3 Calculation of Contact Responses for Two-axle Vehicle Considering Suspension Effect 218

9.3.1	Wheel Responses Back-calculated from Car Body's Responses	219
9.3.2	Contact Responses Back-calculated from Vehicle's Body and Wheel Responses	220
9.4	Identification of Bridge Damping Ratio	221
9.4.1	Brief on the WT	221
9.4.2	Identification of Bridge Damping Ratio by the WT	222
9.5	Numerical Verification	224
9.5.1	VBI Element Used	224
9.5.2	Vehicle and Bridge Properties Used	225
9.5.3	Verification of Analytical Solution	226
9.5.4	Verification of Back-calculation Procedure for Contact Responses	227
9.6	Scanning Bridge Damping Ratio	228
9.7	Parametric Study	230
9.7.1	Effect of Vehicle Damping	230
9.7.2	Effect of Vehicle Speed	231
9.7.3	Effect of Bridge Damping Ratio	232
9.7.4	Effect of Vehicle/Bridge Mass Ratio	233
9.7.5	Effect of Number of Bridge Spans	235
9.7.6	Effect of Pavement Roughness	236
9.7.7	A More Realistic Scenario	242
9.8	Concluding Remarks	243
10	Normalized Formula for Removing Damping Effect on Mode Shape Recovery	245
10.1	Introduction	245
10.2	Theoretical Modeling of the Problem	247
10.2.1	Dynamic Responses of the Damped Bridge and Contact Point	248
10.2.2	Dynamic Response of the Moving Vehicle	251
10.2.3	Dynamic Response of the Stationary Vehicle	252
10.3	Identification of Bridge Mode Shapes with the Effect of Bridge Damping Eliminated	253
10.3.1	Instantaneous Amplitudes of Contact Responses of the Moving and Stationary Vehicles	253
10.3.2	Normalized Formula for Eliminating Bridge Damping in Mode Shape Recovery	254
10.4	Numerical Formulation of the Problem	255
10.4.1	VBI Element and Vehicle and Bridge Properties Used	255
10.4.2	Verification of Analytical Solution	257
10.4.3	Verification of Back-Calculation Procedure for Contact Responses	258
10.5	Scanning Bridge Mode Shapes with the Effect of Bridge Damping Eliminated	260
10.6	Parametric Study	261
10.6.1	Effect of Bridge Damping Ratio	261
10.6.2	Effect of Location of the Stationary Vehicle	263
10.6.3	Effect of Vehicle Damping Ratio	264
10.6.4	Effect of Vehicle Speed	264
10.6.5	Effect of Number of Bridge Spans	264
10.6.6	Effect of Pavement Roughness	266
10.7	Concluding Remarks	268

11	Recursive Formula for Removing Damping Effect on Mode Shape Recovery	269
11.1	Introduction	269
11.2	Analytical Formulation of the Problem	271
11.2.1	Closed-Form Solution for the Bridge	271
11.2.2	Closed-Form Solution for the Contact Responses	273
11.3	Eliminating the Bridge Damping Effect in Bridge Mode Shape Identification	275
11.3.1	Identification of Bridge Mode Shapes by Hilbert Transform	275
11.3.2	Identification of Bridge Mode Shapes by Wavelet Transform	277
11.4	Numerical Verification	279
11.4.1	Verification of Back-Calculation Procedure for Contact Responses	279
11.4.2	Recovering Bridge Mode Shapes by the Recursive Formula Using the HT	281
11.4.3	Recovering Bridge Mode Shapes by the Recursive Formula Using the WT	283
11.5	Parametric Study	285
11.5.1	Effect of Bridge Damping Ratio	285
11.5.2	Effect of Vehicle Damping	286
11.5.3	Effect of Vehicle Speed	288
11.5.4	Effect of Number of Bridge Spans	290
11.5.5	Effect of Pavement Roughness	290
11.6	Concluding Remarks	292

Part III Vehicle Scanning Method for Various Types of Bridges 295

12	Recovering Frequencies and Mode Shapes of Curved Bridges	297
12.1	Introduction	297
12.2	Closed-form Solutions for the Horizontal Curved Bridge and Contact Responses	300
12.2.1	Out-of-plane Vibration of the Curved Beam	301
12.2.2	In-plane Vibration of the Curved Beam	304
12.3	Calculation of Contact Responses	307
12.3.1	Vertical Contact Responses	307
12.3.2	Lateral (radial) Contact Response	308
12.4	Mode Shape Construction by the VMD-SWT	309
12.5	Numerical Modeling of the Problem	311
12.5.1	Description of the VBI Element	311
12.5.2	Verification of Analytical Solution	312
12.5.3	Verification of the Unified Formula for Contact Response	313
12.6	Numerical Verification of Mode Shape Construction	317
12.7	Parametric Study	319
12.7.1	Effect of Bridge Radius	319
12.7.2	Effect of Vehicle Speed	320
12.7.3	Effect of Pavement Roughness	322
12.8	Concluding Remarks	323

13	Recovering Damping Ratios of Curved Bridges	325
13.1	Introduction	325
13.2	Analytical Solutions for the Damped Horizontal Curved Bridge and Contact Responses	327
13.2.1	Out-of-plane Vibration of the Damped Curved Beam	329
13.2.2	In-plane Vibration of the Damped Curved Beam	333
13.3	Damping Ratio Identification	336
13.3.1	Damping Formula for Curved Bridges by Using the VMD and SWT	337
13.3.2	Procedure for Identifying Damping Ratio	339
13.4	Numerical Modeling of the Problem	339
13.4.1	Verification of Analytical Solution	339
13.4.2	Verification of the Unified Contact Formula	341
13.5	Damping Ratio Identification for the Curved Bridge by the VMD-SWT	345
13.6	Numerical Study	346
13.6.1	Effect of Vehicle Damping	346
13.6.2	Effect of Vehicle Manufacture Error	347
13.6.3	Effect of Vehicle Speed	351
13.6.4	Effect of Bridge Damping Ratio	351
13.6.5	Effect of Pavement Roughness	353
13.7	Concluding Remarks	355
14	Scanning Frequencies and Mode Shapes of Thin-Walled Girders	357
14.1	Introduction	357
14.2	Theoretical Formulation of the Problem	360
14.3	Contact Responses for the Two Wheels of Single-Axle Vehicle	365
14.4	Recovery of Bridge's Mode Shapes	366
14.4.1	Brief on the WT	366
14.4.2	Recovery of Bridge's Mode Shapes by the WT	366
14.5	Numerical Simulation of the Problem	367
14.5.1	Brief on VBI Element	368
14.5.2	Verification of Analytical Solutions	368
14.5.3	Verification of Back-Calculated Contact Response	371
14.5.4	Separating Vertical (Flexural) from Torsional-Flexural Frequencies	373
14.6	Construction of Bridge Mode Shapes	374
14.7	Parametric Study	375
14.7.1	Effect of Vehicle Damping	375
14.7.2	Effect of Eccentricity (Lateral Position) of the Vehicle	376
14.7.3	Effect of Vehicle Speed	377
14.7.4	Effect of Pavement Roughness	377
14.8	Concluding Remarks	380
15	Theory for Simultaneously Scanning Modal Properties of Thin-Walled Girders	381
15.1	Introduction	381
15.2	Theoretical Formulation of the Problem	383

15.3	Theoretical Framework for Identification of Bridge Modal Properties	388
15.3.1	Calculation of Contact Responses for the Four-Wheel Vehicle	389
15.3.2	Separating and Detecting Bridge Vertical and Torsional Frequencies	390
15.3.3	Bridge Damping Ratio Identification	391
15.3.4	Bridge Mode Shape Identification	393
15.3.5	Simultaneous Detection of Bridge Frequencies, Damping Ratios, and Mode Shapes	394
15.4	Numerical Verification	395
15.4.1	Vehicle and Bridge Properties Used	395
15.4.2	Verification of Analytical Solutions and Back-Calculation Procedure	396
15.4.3	Separating and Detecting Bridge's Vertical and Torsional-flexural Frequencies	398
15.4.4	Scanning Bridge Damping Ratios	399
15.4.5	Scanning Bridge Mode Shapes	400
15.5	Parametric Study	402
15.5.1	Effect of Bridge Damping	402
15.5.2	Effect of Vehicle Suspension Damping	404
15.5.3	Effect of Vehicle Speed	405
15.5.4	Effect of Vehicle's Eccentricity	406
15.5.5	Effect of Pavement Roughness	407
15.6	Conclusions	411
A	L'Hospital's Rule for Deriving Eq. (2.30)	413
B	VBI Element for Single-DOF Vehicle	415
C	VBI Element for Two-Axle Vehicle Used in Chapters 7 and 8	419
D	VBI Element for Two-Axle Vehicle Used in Chapters 9 and 10	421
E	Straight-Beam Approach for Vibration Analysis of Horizontal Curved Beams	423
E.1	Elastic Stiffness and Consistent Mass Matrices of the Straight Beam Element	423
E.2	Treatment of Offset between Curved Beam and Straight Beam Element	426
E.3	Transformation Matrices	427
E.4	Procedure for Calculating Dynamic Responses of Curved Beam	428
F	VBI Element Used in Chapter 14	429
G	Coefficients in Eq. (15.7) of Chapter 15	431
H	VBI Element Used in Chapter 15	433
	References	435
	Author Index	457
	Subject Index	467

Preface

Bridges are essential components of lifeline systems, serving as critical links between two sides of an area that are separated by natural or artificial barriers. Bridges constitute an essential part of transportation systems, including highways, railways, city rail systems, high-speed railways, and so on. They are engineering structures designed to provide passages for people, vehicles, and goods, enabling connectivity and facilitating transportation. Due to aging, overloading in traffic, and natural disasters, such as earthquakes, typhoons, and flooding, the health condition of a bridge may decline in various forms during its service life, e.g., the deterioration in materials, cracking in cross sections, loosening or breaking connection, support settlements, and scouring in column foundations.

To assess the health condition of a bridge, vibration-based monitoring methods have been widely adopted to diagnose the variation in modal properties. Conventionally, structural health monitoring has been carried out using the vibration data recorded by sensors directly deployed on the bridge, known as the direct measurement method. However, the health monitoring system, including the vibration sensors and data logger, is usually “tailored” for the particular bridge of concern, of which the setup and maintenance costs are generally high. In addition, the continuously generated “sealike” data cannot be digested in an efficient way. What is more, the lifespan of the electronic devices installed on a bridge may not be longer than that of the bridge to be monitored. For the huge number of bridges existing all over the world, there is an urgent need to develop economical and efficient methods that can be widely used in the health monitoring of most bridges.

In 2004, the vehicle scanning method (VSM) for bridge measurement was proposed by the senior (third) author and coworkers to circumvent the drawbacks of the direct measurement method. Such a method, originally known as the indirect measurement method, is featured by the fact of mobility, economy, and efficiency, in that no vibration sensors need to be mounted on the bridge and only a small number of sensors are required to be fixed on the vehicle. It was later renamed as the vehicle scanning method for bridges to make it self-explanatory. Over the past nearly two decades, research on various aspects of the VSM has boomed globally, including the identification of bridge frequencies, modal shapes, damping ratios, and damages.

This book intends to give a broad and systematic coverage of the VSM techniques for the identification of bridge modal parameters (frequencies, modal shapes, and damping ratios). In general, the book is divided into three parts: Part I (Chapters 2–6) is dedicated to the VSM techniques for bridge frequencies, Part II (Chapters 7–11) to the VSM techniques for bridge mode shapes and damping ratios, and Part III (Chapters 12–15) to the VSM techniques for various types of bridges. To help readers quickly engage in the

chapters of interest, each chapter will start with some concise background information, allowing readers to directly comprehend the chapter in a manner that requires minimal cross-reference to the previous chapters. This book contains a total of 15 chapters in the order of increasing complexity. The following is a summary of the content of each chapter.

In Chapter 1, the basic concept of extracting bridge modal parameters using a moving test vehicle is briefed. It then provides a comprehensive review of the current state-of-the-art research conducted globally up to roughly 2024 on the VSM. Progress in various aspects of the VSM is presented, including the identification of bridge frequencies, mode shapes, damping ratios, damages, and surface roughness, as well as applications to railways.

In Chapter 2, a more realistic theory is presented for the vehicle–bridge interaction (VBI) system considering the vehicle damping. To eliminate the overshadowing effect of vehicle frequency on the identification of bridge frequencies from the vehicle’s spectrum, the contact response is used instead, which can be calculated backwardly from the vehicle response. The transmissibility between the vehicle and contact responses is discussed. In addition, a field test is conducted to verify the theory presented.

In Chapter 3, a refined detection technique for bridge frequencies using the rocking motion of a single-axle moving vehicle is introduced. A new formula is derived for calculating the left and right contact responses of the two wheels of the single-axle test vehicle, which will be used in the spectral analysis to eliminate the vehicle’s vertical and rocking frequencies. The feasibility of the refined detection method for scanning the bridge frequencies is verified by the field test.

In Chapter 4, a theory for utilizing a single-axle, two-mass scanning vehicle to extract the bridge frequencies is presented, in which the vehicle’s suspension effect is taken into account. For the two-mass vehicle model, backward formulas are presented for computing the contact response considering the vehicle’s suspension effect.

In Chapter 5, a shaker is added to the bridge to enhance the bridge vibration for alleviating the adverse effect of vibrations induced by pavement roughness. Closed-form solutions are derived of the vehicle-shaker-bridge system to form the theoretical framework for future application of the shaker. The simple formula derived for the shaker’s dynamic amplification factor (DAF) on the vehicle and contact responses in the study can be easily used in practice.

In Chapter 6, dual-function amplifiers are proposed for use to enhance the capability of a scanning test vehicle for bridges. The DAFs of the amplifier and vehicle are presented for assessing the bridge/vehicle and vehicle/amplifier transmissibilities. Two differentially tuned amplifiers are used: one is to suppress the vehicle frequency, acting like the tuned mass damper (TMD), and the other to enlarge the amplitude of the bridge frequency of concern.

In Chapter 7, a theoretical framework for scanning the mode shapes of a bridge by a two-axle test vehicle is presented. The effect of vehicle frequencies is removed by using the contact responses, and that of pavement roughness by the residue of the front and rear contact responses of the two-axle test vehicle. Then, the contact response is processed by the variational mode decomposition (VMD) to yield the component responses and then processed by the Hilbert transform (HT) to yield the mode shapes.

In Chapter 8, a simple formula is derived for the modal damping ratio of the bridge using the correlation between the instantaneous amplitudes of the related front and rear contact responses of a two-axle scanning vehicle by the HT technique. The feasibility of the proposed damping formula is verified in the numerical study.

As a sequel to Chapter 8, a formula for determining the bridge damping ratio from two wheels of a two-axle scanning vehicle by the wavelet transform (WT) is presented in Chapter 9. This chapter improves the preceding Chapter 8 by considering the suspension effect of the two-axle vehicle, while fully utilizing the spatial correlation between the front and rear contact points in the time-space signaling.

In Chapter 10, a normalized formula for removing the damping effect in recovering the bridge mode shapes is proposed by using a moving vehicle and a stationary vehicle. The moving test vehicle is used to recover the global modal response of the bridge throughout its span length at different instants as the vehicle moves over, and the stationary vehicle is used to generate a reference response at a fixed location of the bridge for removing the damping effect.

In Chapter 11, a recursive formula for removing the damping distortion effect on bridge mode shape restoration is proposed by utilizing the spatial correlation between the front and rear contact points of a two-axle vehicle. Further, the bridge mode shapes recovered by the HT- and WT-based recursive formulas are compared, to show which technique is more effective for bridge mode shape recovery.

In Chapter 12, a procedure for recovering the frequencies and mode shapes of curved bridges is introduced. Curved bridges differ from straight bridges in that they are characterized not only by the vertical but also by radial (lateral) frequencies. The procedure for recovering vertical and radial mode shapes of the curved bridge by the VMD and synchrosqueezed wavelet transform (SWT) is presented.

In Chapter 13, a unified theory for identifying the vertical and radial damping ratios of curved bridges in a form similar to straight bridges is proposed. By using the correlation between two connected scanning vehicles and the VMD-SWT technique, the damping formula for the vertical and radial damping ratios of curved bridges is established.

In Chapter 14, based on the kinematic hypothesis of rigid cross sections for thin-walled girders, a procedure for separating and detecting the vertical and torsional frequencies of thin-walled girders from vehicle's contact responses is introduced. By the WT technique, the vertical and torsional mode shapes can be recovered from the separated vertical and torsional contact responses.

In Chapter 15, a theoretical framework for simultaneously scanning the frequencies, damping ratios, and mode shapes of thin-walled girders by a four-wheel test vehicle is presented. The vertical and flexural modal damping ratios and mode shapes of the bridge can be extracted from the separated vertical and torsional-flexural contact responses by using the spatial correlation between the vehicle's two wheels.

In Appendixes A–H, additional materials are provided, such as L'Hospital's rule for deriving contact response formula without vehicle damping, the VBI elements used in the book, the straight-beam element formulation for the vibration analysis of horizontal curved beams, and some relevant equation coefficients. The main reason for placing this material in the appendices rather than in the main text is to avoid unnecessary disruption to the overall flow of the presentation.

The first author wishes to express his highest gratitude to his PhD adviser and third author of the book, Prof. Yeong-Bin Yang of Chongqing University, for inspiring him to pursue research that ultimately culminated in the creation of this book. Prof. Yang's strong influence and unwavering support have profoundly shaped the first author's academic journey and personal development. The first author also wishes to express his sincerest gratitude to Prof. Hui Li from the Harbin Institute of Technology for her invaluable influence and advice. Furthermore, the junior authors (i.e., the first two) wish to express sincerest gratitude to Prof. Qingshan Yang and Prof. Jianmin Hua from Chongqing University, Prof. Ying Zhou from Tongji University, Prof. Wenhui Duan from Monash University, and Prof. C.M. Wang from the University of Queensland for their valuable advice and strong support. Additionally, this book was

prepared as part of the results of our research group at Chongqing University. Numerous faculty members and graduate students have contributed directly or indirectly to the development of materials in the book, including Dr. Z.L. Wang, Dr. K. Shi, Dr. B. Zhang, Yu-Hong Liu, Jin Chen, Luo-Ke Shi, Xiao-Gang Chen, Bo-Tao Zhang.

A large portion of the research results presented in this book has been sponsored through a series of research projects granted by the National Natural Science Foundation of China, the Young Elite Scientist Sponsorship Program by the China Association for Science and Technology, the Chinese Academy of Engineering, the Chinese Society for Vibration Engineering, Chongqing Science and Technology Commission, and the China Postdoctoral Science Foundation. The continuous support from these organizations has been instrumental in sustaining the research group at Chongqing University's dedication to enhancing various aspects of the VSM techniques.

The completion of this book is deeply indebted to the unwavering support and encouragement of the authors' families, colleagues, friends, and society. In particular, special thanks are extended to the dedicated Wiley team for their invaluable efforts in bringing this work to fruition.

Chongqing, China, 2024

*Hao Xu
Der-Shen Yang
Yeong-Bin Yang*

Acknowledgments

Parts of the materials presented in this book have been revised from the papers published by the authors and their co-workers in some technical journals. Efforts have been undertaken to update, digest, and rewrite the content from each source to achieve a unified and progressive style of presentation throughout the book. In particular, the authors like to thank the copyright holders for permission to use the materials contained in the following papers:

- Xu, H., Huang, C.C., Wang, Z.L., Shi, K., Wu, Y.T., and Yang, Y.B. (2021). Damped test vehicle for scanning bridge frequencies: Theory, simulation and experiment. *J. Sound Vib.* 506(18): 116155.
- Xu, H., Liu, Y.H., Wang, Z.L., Shi, K., Zhang, B., and Yang, Y.B. (2022). General contact response of single-axle two-mass test vehicles for scanning bridge frequencies considering suspension effect. *Eng. Struct.* 270(5): 114880.
- Xu, H., Liu, Y.H., Yang, M., Yang, D.S., and Yang, Y.B. (2023). Mode shape construction for bridges from contact responses of a two-axle test vehicle by wavelet transform. *Mech. Syst. Signal Process.* 195: 110304.
- Xu, H., Liu, Y.H., Yang, M., Yang, D.S., and Yang, Y.B. (2023). Scanning and separating vertical and torsional-flexural frequencies of thin-walled girder bridges by a single-axle test vehicle. *Thin-Walled Struct.* 182: 110266.
- Xu, H., Liu, Y.H., Yang, M., Yang, D.S., and Yang, Y.B. (2023). Separating and detecting the vertical and torsional mode shapes of thin-walled girders from vehicle's contact responses by wavelet transform. *Thin-Walled Struct.* 189: 110889.
- Xu, H., Liu, Y.H., Yang, M., Yang, D.S., and Yang, Y.B. (2023). Vehicle-scanning method for bridges enhanced by dual amplifiers. *Struct. Control. Health Monit.* 2023(6): 1–19.
- Xu, H., Liu, Y.H., Chen, J., Yang, D.S., and Yang, Y.B. (2024). Novel formula for determining bridge damping ratio from two wheels of a scanning vehicle by wavelet transform. *Mech. Syst. Signal Process.* 208: 111026.
- Xu, H., Yang, D.S., Chen, J., Wang, C.H., and Yang, Y.B. (2024). Novel recursive formula for removing damping distortion effect on bridge mode shape restoration using a two-axle scanning vehicle. *Eng. Struct.* 308: 117914.
- Xu, H., Chen, X.Y., Chen, J., Shi, L.K., Yang, D.S., Wang, Z.L., and Yang, Y.B. (2025). Review on vehicle scanning method for bridges from 2004 to 2024. *Int. J. Struct. Stab. Dyn.* 25: 2530003.
- Yang, Y.B., Xu, H., Zhang, B., Xiong, F., and Wang, Z.L. (2020). Measuring bridge frequencies by a test vehicle in non-moving and moving states. *Eng. Struct.* 203: 109859.

- Yang, Y.B., Huang, C.C., Xu, H., Wang, M.H., Wang, Z.L., and Shi, K. (2022). Frequency extraction for bridges with rough surface by a moving test vehicle enhanced by a shaker. *Eng. Struct.* 266: 114598.
- Yang, Y.B., Xu, H., Wang, Z.L., and Shi, K. (2022). Using vehicle-bridge contact spectra and residue to scan bridge's modal properties with vehicle frequencies and road roughness eliminated. *Struct. Control. Health Monit.* 29(8): e2968.
- Yang, Y.B., Xu, H., Wang, Z.L., Shi, K., and Wu, Y.T. (2022). Refined detection technique for bridge frequencies using rocking motion of single-axle moving vehicle. *Mech. Syst. Signal Process.* 162: 107992.
- Yang, Y.B., Liu, Y.H., and Xu, H. (2023). Recovering mode shapes of curved bridges by a scanning vehicle. *Int. J. Mech. Sci.* 253: 108404.
- Yang, Y.B., Chen, J., and Xu, H. (2024). Normalized formula for removing damping effect in recovering bridge mode shapes using a moving and a stationary vehicle. *J. Sound Vib.* 573: 118219.
- Yang, Y.B., Liu, Y.H., Xu, H., Guo, D.Z., and Zhou, Z.Y. (2024). Unified theory for identifying vertical and radial damping ratios of curved bridges by two connected scanning vehicles. *Eng. Struct.* 315: 118356.
- Yang, Y.B., Yang, M., Liu, D.H., Liu, Y.H., and Xu, H. (2024). Bridge damping formula based on instantaneous amplitudes of vehicle's front and rear contact responses by Hilbert transform. *Int. J. Struct. Stab. Dyn.* 24(15): 2471006.
- Yang, Y.B., Liu, Y.H., Xu, H., Liu, Y.Z., and Guo, D.Z. (2025). Straight-beam approach for vibration analysis of horizontal curved beams. *Int. J. Struct. Stab. Dyn.* 25: 2571002.
- Yang, Y.B., Shi, L.K., Xu, H., Chen, J., and Liu, Y.H. (2025). General theory for separate scanning of vertical and torsional modal properties of thin-walled girders by a moving four-wheel test vehicle. *J. Sound Vib.* DOI: 10.1016/j.jsv.2025.119189.

List of Symbols

The following is a list of symbols used throughout this book. All the symbols are defined at the place where they first appear in the text.

A	cross-sectional area of bridge
A_{cf}	instantaneous amplitude of bridge component response extracted from front contact response of two-axle vehicle
A_{cr}	instantaneous amplitude of bridge component response extracted from rear contact response of two-axle vehicle
A_m	instantaneous amplitude of bridge component response extracted from contact response of moving vehicle
A_s	instantaneous amplitude of bridge component response extracted from contact response of stationary vehicle
a	dilatation or scale parameter for wavelet stretch
a_F	distance from vehicle's center to front-axle
a_R	distance from vehicle's center to rear-axle
b	time parameter for wavelet translation
b_l	distance from vehicle's center to left-axle
b_r	distance from vehicle's center to rear-axle
c	damping coefficient of bridge
c_{ba}	axial damping coefficient of curved beam
c_{br}	radial damping coefficient of curved beam
c_{bv}	vertical damping coefficient of curved beam
$c_{b\theta}$	torsional damping coefficient of curved beam
c_m	damping coefficient of moving vehicle
c_r	lateral damping coefficient of vehicle
c_s	damping coefficients of stationary vehicle
c_{sf}	suspension damping coefficient for front wheel of two-axle vehicle
c_{sr}	suspension damping coefficient for rear wheel of two-axle vehicle
$c_{s,Fl}$	damping coefficient for front left suspension of four-wheel vehicle
$c_{s,Fr}$	damping coefficient for front right suspension of four-wheel vehicle
$c_{s,Rl}$	damping coefficient for rear left suspension of four-wheel vehicle

$c_{s,Rr}$	damping coefficient for rear right suspension of four-wheel vehicle
c_v	vehicle damping coefficient
c_{vf}	damping coefficient for front axle of two-axle vehicle in Chapters 7 and 8
c_{vl}	damping coefficient for left wheel of single-axle two-degree-of-freedom (DOF) vehicle in Chapter 3
c_{vr}	damping coefficient for rear axle of two-axle vehicle in Chapters 7 and 8
c_{vr}	damping coefficient for right wheel of single-axle two-DOF vehicle in Chapter 3
c_w	wheel damping coefficient
c_{wf}	damping coefficient for front wheel of two-axle vehicle
c_{wr}	damping coefficient for rear wheel of two-axle vehicle
$c_{w,Fl}$	damping coefficient for front left wheel of four-wheel vehicle
$c_{w,Fr}$	damping coefficient for front right wheel of four-wheel vehicle
$c_{w,Rl}$	damping coefficient for rear left wheel of four-wheel vehicle
$c_{w,Rr}$	damping coefficient for rear right wheel of four-wheel vehicle
c_y	vertical damping coefficient of thin-walled girder
c_z	lateral damping coefficient of thin-walled girder
c_θ	torsional damping coefficient of thin-walled girder
$[C_b]$	damping matrix of beam element
$[C_{bf}]$	damping matrix of beam element f associated with front contact point
$[C_{br}]$	damping matrix of beam element r associated with rear contact point
DAF	dynamic amplification factor of shaker
DAF_a	dynamic amplification factor of amplifier for bridge response
DAF_v	dynamic amplification factor of vehicle for bridge response
d	distance from front axle to rear axle of two-axle vehicle
d	distance from left wheel to right wheel of single-axle two-DOF vehicle in Chapters 3 and 14
d_f	distance from vehicle's center of gravity to front axle of two-axle vehicle
d_l	distance from center of vehicle axle's gravity to left wheel of single-axle two-DOF vehicle in Chapter 3
d_r	distance from vehicle's center of gravity to rear axle of two-axle vehicle
d_r	distance from center of vehicle axle's gravity to right wheels of single-axle two-DOF vehicle in Chapter 3
E	Young's modulus
e_{cl}	eccentricity of left contact point to bridge center
e_{cr}	eccentricity of right contact point to bridge center
e_f	beam element subject to front wheel of two-axle vehicle
e_r	beam element subject to rear wheel of two-axle vehicle
e_{Fl}	eccentricity of front left wheel from bridge center line
e_{Fr}	eccentricity of front right wheel from bridge center line
e_{Rl}	eccentricity of rear left wheel from bridge center line
e_{Rr}	eccentricity of rear right wheel from bridge center line
F_c	vehicle-bridge contact force
F_{cl}	vehicle-bridge contact force for left wheel of single-axle two-DOF vehicle

F_{cr}	vehicle–bridge contact force for right wheel of single-axle two-DOF vehicle
F_{cm}	contact force of moving vehicle
F_{cs}	contact force of stationary vehicle
F_{cvr}	centrifugal force generated by vehicle moving along a horizontal curve beam
F_{cr1}	centrifugal force generated by front (1) vehicle moving along a horizontal curve
F_{cr2}	centrifugal force generated by rear (2) vehicle moving along a horizontal curve
F_G	minimum spacing of kernel function’s movement in frequency domain of Gabor transform
F_r	radial contact force
F_s	shaker force
F_v	vehicle–bridge contact force used in Chapter 5
f_a	cyclic frequency of amplifier
$f_{b,n}$	n th vertical cyclic frequency of bridge
$f_{bl,n}$	n th left-shifted cyclic frequency of bridge, i.e., $f_{b,n} - \frac{mv}{2L}$
$f_{br,n}$	n th right-shifted cyclic frequency of bridge, i.e., $f_{b,n} + \frac{mv}{2L}$
$f_{br,1}$	fundamental radial cyclic frequency of curved beam
$f_{brD,1}$	fundamental radial damped cyclic frequency of curved beam
$f_{bv,n}$	n th out-of-plane (vertical) cyclic frequency of curved beam
$f_{bvD,n}$	n th out-of-plane (vertical) damped cyclic frequency of the curved beam
$f_{by,n}$	n th vertical cyclic frequency of thin-walled girder
$f_{bz,n}$	n th lateral cyclic frequency of thin-walled girder
$f_{b\theta,n}$	n th torsional cyclic frequency of thin-walled girder
$\tilde{f}_{bz,n}, \tilde{f}_{bz,n}$	torsional–flexural cyclic frequency of thin-walled girder that is closer to lateral frequency $f_{bz,n}$
$\tilde{f}_{b\theta,n}, \tilde{f}_{b\theta,n}$	torsional–flexural cyclic frequency of thin-walled girder that is closer to torsional frequency $f_{b\theta,n}$
f_m	vertical cyclic frequency of moving vehicle
f_p	pitching cyclic frequency of four-wheel vehicle’s body
f_r	rotational cyclic frequency of two-axle vehicle
f_r	rocking cyclic frequency of single-axle two-DOF vehicle in Chapters 3, 14, and 15
f_s	vertical cyclic frequency of stationary vehicle
f_v	vertical cyclic frequency of vehicle
$f_{v,D}$	damped vertical cyclic frequency of vehicle
$f_{w,Fl}$	cyclic frequency of front left wheel of four-wheel vehicle
$f_{w,Fr}$	cyclic frequency of front right wheel of four-wheel vehicle
$f_{w,Rl}$	cyclic frequency of rear left wheel of four-wheel vehicle
$f_{w,Rr}$	cyclic frequency of rear right wheel of four-wheel vehicle
f_o	lateral cyclic frequency of vehicle
G	shear modulus
$G()$	Gabor transform
$G_p()$	Gabor coefficient for torsional response of front ($p = F$) or rear ($p = R$) contact point
g	acceleration of gravity
$H()$	Heaviside step function

H_{cf}	instantaneous amplitude for front contact response by Hilbert transform
H_{cr}	instantaneous amplitude for rear contact response by Hilbert transform
\mathcal{H}	Hilbert transform
$h_{a,1}$	homogeneous solution of general solution for axial displacement of curved beam
$h_{r,1}$	homogeneous solution of general solution for radial displacement of curved beam
I	moment of inertia of beam
$I_{v\theta}$	mass moment of inertia about vehicle's z_v -axle in Chapter 15
$I_{v\varphi}$	mass moment of inertia about vehicle's x_v -axle in Chapter 15
I_y	moment of inertia of bridge about y -axle
I_z	moment of inertia of bridge about z -axle
I_α	polar moment of inertia per unit length
J	torsional moment of inertia of bridge
J_v	mass moment of inertia of vehicle
k_a	stiffness of amplifier
k_m	suspension stiffness of moving vehicle
k_s	suspension stiffness of stationary vehicle
k_r	lateral stiffness of vehicle
k_{sf}	front suspension stiffness of two-axle vehicle
k_{sr}	rear suspension stiffness of two-axle vehicle
$k_{s,Fl}$	stiffness for front left suspension of four-wheel vehicle
$k_{s,Fr}$	stiffness for front right suspension of four-wheel vehicle
$k_{s,Rl}$	stiffness for rear left suspension of four-wheel vehicle
$k_{s,Rr}$	stiffness for rear right suspension of four-wheel vehicle
k_v	suspension stiffness of vehicle
k_v	wheel stiffness in Chapters 12–14
k_{vf}	stiffness for front wheel of two-axle vehicle in Chapters 7 and 8
k_{vl}	stiffness for left wheel of single-axle two-DOF vehicle in Chapter 3
k_{vr}	stiffness for rear wheel of two-axle vehicle in Chapters 7 and 8
k_{vr}	stiffness for right wheel of single-axle two-DOF vehicle in Chapter 3
k_w	wheel stiffness
k_{wf}	stiffness for front wheel of two-axle vehicle
k_{wr}	stiffness for rear wheel of two-axle vehicle
$k_{w,Fl}$	stiffness for front left wheel of four-wheel vehicle
$k_{w,Fr}$	stiffness for front right wheel of four-wheel vehicle
$k_{w,Rl}$	stiffness for rear left wheel of four-wheel vehicle
$k_{w,Rr}$	stiffness for rear right wheel of four-wheel vehicle
$[K_b]$	stiffness matrix of beam element
$[K_{bf}]$	stiffness matrix of beam element f associated with front contact point
$[K_{br}]$	stiffness matrix of beam element r associated with rear contact point
$[K_v]$	stiffness matrix of vehicle system
L	span length of bridge
l	length of beam element
M_m	mass of moving vehicle in Chapter 10

M_s	mass of stationary vehicle in Chapter 10
m	mass-per-unit length of bridge
m_a	amplifier mass
m_v	vehicle body mass
m_w	wheel mass
m_{wf}	front wheel's mass of two-axle vehicle
m_{wr}	rear wheel's mass of two-axle vehicle
$m_{w,Fl}$	front left wheel mass of four-wheel vehicle
$m_{w,Fr}$	front right wheel mass of four-wheel vehicle
$m_{w,Rl}$	rear left wheel mass of four-wheel vehicle
$m_{w,Rr}$	rear right wheel mass of four-wheel vehicle
$[M_b]$	mass matrix of beam element
$[M_{bf}]$	mass matrix of beam element f associated with front contact point
$[M_{br}]$	mass matrix of beam element r associated with rear contact point
$[M_v]$	mass matrix of vehicle system
n_a	selected specific frequency in Gabor transform
n_G	modulation parameter of kernel function in frequency domain of Gabor transform
n_s	spatial frequency
$\{N\}_c$	cubic Hermitian interpolation function evaluated at contact point
$\{N_i\}$	cubic Hermitian interpolation function evaluated at contact point of element i
$\{N_t\}$	interpolation function evaluated at contact point for vertical direction
$\{N_v\}$	interpolation function evaluated at contact point for rotational direction
P_d	displacement power spectral density of roughness
$p_{a,1}$	particular solution of general solution for axial displacement of curved beam
$p_{r,1}$	particular solution of general solution for radial displacement of curved beam
p_f	front axle load of two-axle vehicle
p_{Fl}	load of front left wheel of four-wheel vehicle
p_{Fr}	load of front right wheel of four-wheel vehicle
p_{Rl}	load of rear left wheel of four-wheel vehicle
p_{Rr}	load of rear right wheel of four-wheel vehicle
p_r	rear axle load of two-axle vehicle
p_s	load amplitude of shaker
$q_{a,n}$	n th generalized modal coordinate for axial displacement of curved beam
$q_{b,n}$	n th vertical modal displacement of bridge
$q_{by,n}$	n th vertical modal displacement of thin-walled girder
$q_{bz,n}$	n th lateral modal displacement of thin-walled girder
$q_{b\theta,n}$	n th torsional modal displacement of thin-walled girder
$q_{r,n}$	n th modal coordinate for radial displacement of curved beam
$q_{v,n}$	n th modal coordinate for vertical displacement of curved beam
$q_{\theta,n}$	n th modal coordinate for torsional displacement of curved beam
$\{q_{bf}\}$	displacement of beam element subject to front wheel of two-axle vehicle
$\{q_{br}\}$	displacement of beam element subject to rear wheel of two-axle vehicle
R	radius of curvature of curved beam

r_c	roughness elevation for contact point of single-DOF vehicle
r_{cf}	roughness elevation for front contact point of two-axle vehicle
r_{cl}	roughness elevation for left contact point of single-axle two-DOF vehicle
r_{cr}	roughness elevation for rear contact point of two-axle vehicle
r_{cr}	roughness elevation for right contact point of single-axle two-DOF vehicle in Chapters 12–14
r_p	polar radius of gyration
S_n	speed parameter
$S_{r,1}$	speed parameter for radial vibration of curved beam
S_{vl}	left acceleration sensor mounted on vehicle's axle
S_{vr}	right acceleration sensor mounted on vehicle's axle
T	travel time of test vehicle ($=L/v$)
T_c	torque acting at shear center of bridge
T_G	minimum spacing of kernel function's movement in time domain of Gabor transform
T_u	reallocated wavelet coefficients by synchrosqueezed wavelet transform
t	time
t_F	entry time of front two wheels of four-wheel vehicle
t_f	entry time of front wheel of two-axle vehicle
t_R	entry time of rear two wheel of four-wheel vehicle
t_r	entry time of rear wheel of two-axle vehicle
u	vertical displacement of beam
u_A	vertical displacement of beam element at node A
u_B	vertical displacement of beam element at node B
u_a	axial displacement of curved beam
u_c	vertical displacement of contact point
u_{cf}	vertical displacement for front contact point of two-axle vehicle
u_{cl}	vertical displacement for left contact point of single-axle two-DOF vehicle
u_{cr}	vertical displacement for rear contact point of two-axle vehicle
u_{cr}	vertical displacement for right contact point of single-axle two-DOF vehicle in Chapters 3, 13, and 14
u_{cr}	radial displacement of contact point in Chapter 12
u_{cr1}	radial displacement of front (1) vehicle's contact point
u_{cr2}	radial displacement of rear (2) vehicle's contact point
u_{cvl}	vertical displacement for left contact point of single-axle two-DOF vehicle in Chapter 12
u_{cvr}	vertical displacement for right contact point of single-axle two-DOF vehicle in Chapter 12
$u_{c,Fl}$	displacement of front left contact point of four-wheel vehicle
$u_{c,Fr}$	displacement of front right contact point of four-wheel vehicle
$u_{c,Rl}$	displacement of rear left contact point of four-wheel vehicle
$u_{c,Rr}$	displacement of rear right contact point of four-wheel vehicle
$u_{cv1,n}^l$	vertical displacements of front (1) vehicle's left contact point
$u_{cv2,n}^l$	vertical displacements of rear (2) vehicle's left contact point
$u_{cv1,n}^r$	vertical displacements of front (1) vehicle's right contact point

$u_{cv2,n}^r$	vertical displacements of rear (2) vehicle's right contact point
u_m	displacement of moving vehicle's contact point in Chapter 10
u_r	radial displacement of curved beam
u_s	displacement of stationary vehicle's contact point in Chapter 10
u_s	vertical displacement of beam due to shaker in Chapter 5
$u_{s,c}$	vertical displacement of contact point due to shaker in Chapter 5
u_v	vertical displacement of beam due to moving vehicle in Chapters 5 and 13
$u_{v,c}$	vertical displacement of contact point due to moving vehicle in Chapter 5
u_y	vertical displacement of thin-walled girder
u_z	lateral displacement of thin-walled girder bridge
\hat{u}_k^n	k th modal function in frequency domain for n th iteration in variational mode decomposition
$\{u_b\}$	beam element displacement
V	vehicle speed in Chapters 14 and 15
v	vehicle speed in Chapters 2–13
W	wavelet transform
$W_{cf,n}$	n th wavelet coefficient of front contact acceleration
$W_{cr,n}$	n th wavelet coefficient of rear contact acceleration
W_{cy}	wavelet coefficient of vertical response
W_{ck}	wavelet coefficient of torsional-flexural response
W_u	wavelet coefficient in Chapters 12 and 13
x	beam axis
x_m	coordinate of moving vehicle on beam in Chapter 10
x_s	coordinate of stationary vehicle on beam in Chapter 10
x_s	location of shaker in Chapter 5
y_a	displacement of amplifier
y_m	displacement of moving vehicle in Chapter 10
y_r	radial displacement of vehicle
y_s	displacement of stationary vehicle in Chapter 10
y_s	vehicle body's vertical displacement caused by shaker in Chapter 5
y_v	vehicle body's vertical displacement
y_w	vehicle wheel's vertical displacement
y_{wf}	vertical displacement of front wheel of two-axle vehicle
y_{wr}	vertical displacement of rear wheel of two-axle vehicle
$y_{w,Fl}$	vertical displacement of front left wheel of four-wheel vehicle
$y_{w,Fr}$	vertical displacement of front right wheel of four-wheel vehicle
$y_{w,Rl}$	vertical displacement of rear left wheel of four-wheel vehicle
$y_{w,Rr}$	vertical displacement of rear right wheel of four-wheel vehicle
$\{z_b\}$	displacement of bridge
$\{z_v\}$	displacement of vehicle body
$\{z_w\}$	displacement of vehicle wheels
α	quadratic penalty term of variational mode decomposition in Chapter 7
α	shape factor of bridge in Chapters 14 and 15

$\alpha_{v,a}$	vehicle/amplifier mass ratio m_v/m_a
β	subtended angle of curved bridge
$\beta_{bl,n}$	frequency ratio between n th left-shifted bridge frequency $\omega_{bl,n}$ and vehicle frequency ω_v , i.e., $\omega_{bl,n}/\omega_v$
$\beta_{br,n}$	frequency ratio between n th right-shifted bridge frequency $\omega_{br,n}$ and vehicle frequency ω_v , i.e., $\omega_{br,n}/\omega_v$
$\beta_{bl,n,a}$	frequency ratio between n th left-shifted bridge frequency $\omega_{bl,n}$ and amplifier frequency ω_a , i.e., $\omega_{bl,n}^2/\omega_a^2$
$\beta_{br,n,a}$	frequency ratio between n th right-shifted bridge frequency $\omega_{br,n}$ and amplifier frequency ω_a , i.e., $\omega_{br,n}^2/\omega_a^2$
$\beta_{d,n}$	frequency ratio between n th driving frequency $\omega_{d,n}$ and vehicle frequency ω_v , i.e., $2\omega_{d,n}/\omega_v$
$\beta_{d,n,a}$	frequency ratio between n th driving frequency $\omega_{d,n}$ and amplifier frequency ω_a , $(2\omega_{d,n})^2/\omega_a^2$
$\beta_{s,bn}$	frequency ratio between shaker frequency ω_s and n th bridge frequency $\omega_{b,n}$, i.e., $\omega_s/\omega_{b,n}$
$\beta_{v,a}$	vehicle/amplifier frequency ratio, i.e., ω_v^2/ω_a^2
$\Gamma(\cdot)$	kernel function
γ	dual frame window
Δn_s	sampling interval of spatial frequency
Δt	time step
$\Delta_{s,n}$	n th vertical modal static deflection of bridge caused by shaker in Chapter 5
$\Delta_{st,n}$	n th vertical modal static deflection of bridge caused by vehicle
$\Delta_{stm,f}$	n th vertical modal static deflection of bridge caused by front axle load of two-axle vehicle
$\Delta_{stm,r}$	n th vertical modal static deflection of bridge caused by rear axle load of two-axle vehicle
$\Delta_{stm,n}$	n th vertical modal static deflection of bridge caused by moving vehicle in Chapter 10
$\Delta_{sts,n}$	n th vertical modal static deflection of bridge caused by stationary vehicle in Chapter 10
$\Delta_{v,n}$	n th vertical modal static deflection of bridge caused by vehicle in Chapter 5
δ	Dirac's delta function
∂_t	partial derivative with respect to time t
ε	preset tolerance in variational mode decomposition
ζ	distance between vehicle's two wheels
ζ	distance between left and right sensors mounted on the vehicle's axle in Chapters 12 and 13
η	offset between bridge's shear center S and centroid C
η	distance of front and rear vehicle in Chapter 13
θ	torsional displacement of bridge
θ_A	rotational angle of beam element at node A
θ_B	rotational angle of beam element at node B
θ_v	rotation of two-axle vehicle
θ_v	rocking angle of sing-axle vehicle in Chapters 3 and 12–14
λ	Lagrangian multiplier
ξ	polar radius of gyration of bridge