Sensory Neuroscience: Four Laws of Psychophysics

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Preface

Hopefully, this book may be of interest not only to psychophysicists who formally call themselves “psychophysicists,” and of whom there are very few but also to all those who use psychophysics in their activities, often not even realizing they do. Every educated person has at least an idea of what physics is, the same goes for psychology. But what is psychophysics? The name suggests a combination of both. More specifically, psychophysics is a science concerning human and animal sensory responses to physical and also chemical stimuli. Seeing is a response to light. We may see the light as red or yellow or any other color or a mixture of colors. It may appear as a short flash or a steady illumination; it may be patterned and appear as stripes, circles, squares, or any other shapes. Similarly, hearing is a response to sound that may appear as noise, musical sound, or a pure tone to pick a few examples. Tactile feeling results from pressure on the skin surface, feeling of warmth from heat, smelling from odorant substances, taste from gustatory chemicals, and so on.

If psychophysics deals so generally with sensory responses, why are there so few psychophysicists? Simply, because they tend to specialize in particular senses, like vision or hearing or taste, for example, and call themselves visual scientists, auditory scientists, and so on.

Psychophysics has wide-ranging applications in our professions – in medical diagnostics, most specifically, in ophthalmology and otology, in optometry and audiology, also in engineering, in architecture, in the arts, even in such fields as traffic control (think of traffic lights, for example).

Psychophysical processes are involved in almost everything we do. When we compare the height of a house to another by eye – that is psychophysics; when we compare the pitch of one sound to another – that is psychophysics also; even when we decide which coffee or tea we prefer, we perform a psychophysical tasting operation. In sports, whenever we throw or catch or hit a ball – subjective psychophysical measurement is involved. We can hardly make a move without running into psychophysics.

Psychophysics was discovered, or invented, if you prefer, by a nineteenth century physicist, Gustav Theodor Fechner, who was interested in the relationship between the material and the spiritual. In modern times, it has become a science of our, or even animal, sensory responses to physical or chemical stimuli, as I already stated. To a large extent, psychophysics is a quantitative science. For example, it attempts to describe numerically how fast a sensation grows with the intensity of a physical stimulus. When we increase the intensity of sound, how fast does its loudness increase, by how much do we have to increase the intensity of a light to increase its brightness by a factor of two or three or any other ratio? It also attempts to specify thresholds of detectability. What is the smallest pressure on the skin we can just notice, or the smallest perceptible concentration of an odorant or a gustatory chemical?

Psychophysics, as established by Fechner, aimed at finding pervasive quantitative rules for such relationships. In physics, which derives a tremendous power from them, such rules are called scientific laws. The first psychophysical law was established by Fechner on the basis of the observation of Ernst H. Weber that a just noticeable increment in lifted weights is directly proportional to the base weight. The observation proved to hold not only for weight but also for other physical variables as well. Fechner established yet another famous law named after him. On an intuitive insight, he decided that subjective impressions grew as logarithms of the physical stimuli that produced them. The law proved much later to be incorrect. Nevertheless, together with Weber's law, it demonstrated the importance Fechner, as a physicist, attached to scientific laws. They point to relationships of great generality from which specific relationships can be derived. In so doing, they provide a basic structure of a science.

Psychophysics has been founded in mid-nineteenth century by Gustav Theodor Fechner, a German scientist and philosopher with a strong mystical bend, who begun his scientific career as a physiologist but, subsequently, became a self-taught physicist with a consuming interest in the relationship between body and mind. Psychophysics resulted from this interest and, in practice, concerned mainly sensations, such as those of brightness and loudness or heaviness. Undoubtedly, under the influence of Fechner's physics background, it was conceived as a quantitative science structured around general laws. Two became famous, Weber's law conceived by Fechner on the basis of Weber's experiments with lifted weights, according to which a just noticeable weight difference was directly proportional to the reference weight, and Fechner's law, not based on an experiment but on an intuitive insight of Fechner's, according to which subjective magnitudes of sensations grew as logarithms of the intensity of stimuli that evoked them. The logarithmic function seemed appropriate because it reflected the subjectively slow growth of such sensations as brightness by comparison to the light intensity that produced it. Fechner, like his contemporaries, did not believe that sensations can be quantified experimentally, and I have never ceased wandering how, if this were true, he and some others before him were able to feel that sensations grew less rapidly than the corresponding physical intensities that, of course, were measured instrumentally. As discussed in this monograph, Weber's law, at least in a somewhat modified form, survived the test of time. This is not true for Fechner's logarithmic law that became replaced by a power law based on ample experimental documentation.

Foundation of psychophysics preceded by about a decade the establishment of the laboratory of experimental psychology by Wilhelm Maximilian Wundt and partially inspired it. The laboratory gave rise to several branches of psychology. Psychophysics evolved to become one of them; a branch that has been both pervasive with respect to the others and, at the same time, distinct because of its strong interactions with physics and chemistry, and because of its many applications outside psychology. Some of the most famous psychophysicists begun their research careers as physicists or engineers. Among the best known applications have been those to medical diagnostics, most specifically, in ophthalmology and otology, in optometry and audiology, others to engineering, architecture, the arts, even to such fields as traffic control – think, for example, of the colors of traffic lights.

Psychophysical research is often performed in tandem with physiological research, in particular, with neurophysiological one. Psychophysics attempts to determine what our and animal sensory systems do, physiology, how they do it.

Most recently, psychophysics and physiology together have become essential parts of environmental sciences by telling us how our environments affect us.

Psychophysical processes are involved in almost everything we do. When we compare the height of a house to another by eye – that is psychophysics; when we compare the pitch of one sound to another – that is psychophysics also; even when we decide which coffee or tea we prefer, we perform a psychophysical tasting operation. In sports, whenever we throw or catch or hit a ball – subjective psychophysical measurement is involved. We can hardly make a move without running into psychophysics.

I have watched psychophysics evolve during four to five decades. At and participated actively in its evolution, at first, as a young electrical engineer who chose to work in otology on diagnostic procedures rather than pursue a traditional engineering career. The place was the Department of Oto-Rhino-Laryngology at the Medical School of the University of Basel, Switzerland. During that time, I discovered what is now called "forward masking," a phenomenon according to which a preceding tone decreases the audibility of a following tone for a short span of time, and that only recently experienced a wave of research popularity. I discovered that, in the absence of direct acoustic interference, a sound in one ear masked in a strongly time-dependent and frequency-dependent fashion the audibility of sound in the contralateral ear. A phenomenon I called "central masking" because it had to take place in central neural interaction. I also discovered that the size of just noticeable differences in tone intensity depended on loudness rather than directly on the intensity – a revolutionary, paradoxical appearing finding, inconsistent with the foundations of stimulus-oriented psychophysics. It led to a diagnostic method concerning inner ear disorders.

A doctoral dissertation written at that time provided me with the degree of a Science Doctor and brought me to Harvard's Psychoacoustic Laboratory where I spent 6 years. The laboratory belonged to the Department of Experimental Psychology, and I suddenly found myself, surrounded by psychologists. It is in this ambiance that, together with several coworkers, I demonstrated an unexpectedly strong dependence of the measured threshold of audibility for pure tones on practice and motivation of the listeners. On this occasion, taking off from the automated audiometer of Georg v. Békésy's (Nobel, 1960) I devised a "criterion-free" automated method of threshold determination. The method, based on somewhat different statistics, was later reinvented twice, once in hearing and once in vision. All these methods now form the class of adaptive methods for automatic, criterion-free threshold determination.

My work on practice and motivation and the criterion-free method fitted right into the "theory of signal detectability," which burst upon psychophysics around that time and provided a mathematical foundation for criterion-free signal detection. Mainly through the efforts of David M. Green, it revolutionized the part of psychophysics, especially auditory psychophysics, which dealt with thresholds of detectability.

The theory of signal detectability did not have much effect on measurement of suprathreshold events. Such an effect was provided by a discovery made by Stanley Smith Stevens, the director of the Psychoacoustic Laboratory. He found that people without any special training were able to express ratios between the subjective magnitudes of their sensations in numbers. For example, they were able to tell how many times a given sound was louder than another sound, or, a given light brighter than another light. From their numerical responses, Stevens was able to construct functions relating the sensation magnitudes to stimulus magnitudes. He found that, almost invariably, the functions conformed to power functions. Through extensive experimentation, he determined that the generality of this phenomenon was so extensive that it deserved to be regarded as a scientific law. The law is now generally known in psychophysics as Stevens' Power Law.

Soon after the establishment of Stevens' law, my research tenure at Harvard was over, and I accepted a Faculty position at Syracuse University. Here, following up on my experience in diagnostic otolaryngology and at Harvard's Psychoacoustic Laboratory, I organized successively the Bioacoustic Laboratory and the Laboratory of Sensory Communication that advanced to become the Institute for Sensory Research at the departmental level. The Institute, as well as its precursor pioneered multidisciplinary research on human and animal senses. Its multidisciplinary faculty included the disciplines of anatomy, neurophysiology, and psychophysics in three sense modalities: hearing, touch, and vision. The multisensory nature of the Institute promoted intersensory comparisons, which led to the discovery of some fundamental intersensory generalities.

This book has resulted from the experience I acquired during my academic career at in all three places, Basel University, and Harvard University, but mainly, at the Institute for Sensory Research at Syracuse University. It also resulted from the conviction that scientific laws form the backbone of a science. Thanks in part to the multisensory nature of the Institute, the laws I have been able to propose apply, with some exceptions, to most if not all human senses. No laws applying exclusively to one or another sense modality have been included.

At the end of this preface, I want to abandon psychophysics for a more personal subject and express my most sincere thanks to two persons who contributed in two different but essential ways to this book. As custom dictates, I first thank

Nicole Sanpetrino, my graduate assistant, for her dedicated and excellent help in the graphics, the editing, and the indexing of this book. Without her help, my task of putting this book together would have been infinitely more difficult. Last but not the least, I thank my wife, Marie, for her inspiration, constructive criticism, and patiently putting up with me while I was being engulfed by this book. Before all, I thank her for keeping me all together during the arduous task of writing.

Syracuse, New York *Josef J. Zwislocki*

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Introduction

Scientific laws are defined here as quantitative, invariant relationships of broad generality. Newton's law $F = am$, where F means a force applied to the mass, m, to produce an acceleration, a, and Einstein's $E = mc^2$, where E means energy, m, mass, and c, the speed of light are classical examples. The laws constitute the backbone of a science from which other relationships can be derived, and the effects of variables producing deviations from the relationships defined by the laws studied. For example, Newton's law applies to objects falling in vacuum. When an object falls in air due to the effect of gravity, its acceleration is decreased by air resistance. To establish a law empirically, all the variables not specified in the law must be eliminated or their effects determined and accounted for.

Here, the concept of scientific laws is applied to psychophysics, a science established by Gustav Theodor Fechner, a nineteenth-century German physicist and physiologist, to deal with the relationship between the spiritual world and the material world. More down to earth, psychophysics may be defined nowadays as a science of quantitative relationships between psychological variables and the physical variables that elicit them. "Physical" is used here as a generic term including "chemical." Some of the relationships are so intimate that before sufficient instrumentation was developed, the only way people knew about the physical events was through their senses. This is probably the reason why, even today, the same word is used for the physical light as for the sensation of it. The same is true for sound and some other physical variables. We have to be clear in specifying whether we mean the physical variable or its sensation. When we say: "the light is bright" we really mean our sensory impression, not the physical quantity that we can know only through inference. The inference may be quite inaccurate and depend on context variables. Optical illusions are well known.

Before the relationships between the psychological variables and their underlying physical variables could be determined, methods and instrumentation for independent measurement of both had to be established. This has been accomplished for the physical variables under a satisfactory number of circumstances. What about the measurement of psychological variables? It remains controversial, and the dispute, begun for good in the nineteenth century, goes on. The fundamental question continues to be: can subjective variables, such as brightness or loudness or pressure or sweetness or all the other sensations we can imagine, be measured? Most people seem to think in a mystical way that they are part of our inner world that is not accessible to others, therefore, not measurable. For this reason, most of the psychophysics pursued today avoids specifying the magnitudes of these variables and limits itself to addressing human observers as null instruments. Experiments are limited to questions of detectability – did an event occur or not? In a pair of unequal events, was the greater or smaller event presented? In classical psychophysics that preserves some elements of subjectivity, the questions become – did you perceive the event; which event appeared the greater? Still, specification of the absolute magnitudes of the events is avoided.

In psychophysics that restricts the observer, or subject, to a null device; the determination of sensory characteristics occurs indirectly by measuring one stimulus variable as a function of another. For example, the threshold of audibility is determined as a function of sound frequency, the threshold of visibility as a function of the wavelength of light, the vibration detectability as a function of vibration duration, and so forth. It is also possible to measure magnitudes of different stimuli producing equal subjective magnitudes; for example, sound intensities at two different sound frequencies that produce equal loudnesses. All such measurements have proven to be useful. Nevertheless, they are limited to threshold values, or to subjective magnitudes relative to other subjective magnitudes specified indirectly in terms of stimulus values that produce them. Such stimulus-oriented psychophysics provides only an incomplete image of our sensory functioning that most often occurs at suprathreshold stimulus values not referred to specific reference standards.

The situation begun to change cautiously in mid-twentieth century when S.S. Stevens demonstrated more convincingly than his predecessors that mutually consistent ratio measurements of loudness and brightness were possible. The initial experiments were performed in hearing. An observer was given a reference standard consisting of a tone at a predetermined intensity and a number to express its subjective loudness magnitude. He was instructed to assign numbers to subsequently presented tones in proportion to their loudness magnitudes relative to the standard. The numbers proved to follow a power function. Repetition of the experiment on several other observers produced similar responses. An analogous result was obtained when light flashes were substituted for the tonal stimuli. The subjective brightness magnitudes, as expressed by assigned numbers, followed a similar power function. Stevens decided that he may have found a general principle for the relationships between sensory stimulus intensities and the subjective sensation magnitudes they evoked. Because the relationships followed power functions, he designated the principle as the Power Law. The Power Law has been confirmed by many experimenters in many experiments performed in many sense modalities. Next to the Weber Law that withstood the test of almost two centuries, it is the best documented general relationship of psychophysics. Because it may be considered as the answer to Fechner's fundamental question of the relationship between the "spiritual" and the "material," to use Fechner's language, Stevens regarded the Power Law as *the* Psychophysical Law. In this monograph, the view is accepted that the Power Law is the most fundamental law of psychophysics, and I designate it as the First Law of Psychophysics. Nevertheless, additional laws are possible and may have considerable usefulness.

Before the Power Law was firmly established, methods of measuring psychological quantities had to be developed. The original method introduced by Stevens to measure loudness and brightness, which he called "magnitude estimation," proved partially misguided and produced the right results somewhat by lucky coincidence. Stevens and his coworkers soon discovered that the exact functions relating the psychological magnitudes to the underlying stimulus magnitudes depended on the designated reference standards, and the "best" power functions were obtained when the observers were allowed to choose the standards themselves. This discovery suggested that the observers did not obey strictly the rules of ratio scaling, which allow the reference standards to be entirely arbitrary, but, to some degree, attached to numbers absolute values.

A more systematic investigation of the effects of reference standards was performed by Rhona P. Hellman, a graduate student of mine, and myself. The investigation led us to the conclusion that experimental observers do not use numbers in a relative way, depending on chosen units, but rather in an absolute way. In other words, probably because of the way they are used in everyday life and the way children use them when learning them, they acquire absolute subjective values. Children learn numbers by counting objects – pebbles or pencils, for example. As a consequence, coupling of numbers to perceived objects occurs early in life according to the rules of numerosity where numbers have absolute values. These values appear to be extrapolated to continua. When asked to assign numbers to subjective impressions of line length or to loudness, adults and children produce the same absolute functions within the range of the numerals they know. If numbers acquire absolute subjective values, magnitude estimation (ME) becomes a matching operation. The subjective values of numbers are matched to the subjective values of whatever variable is being scaled.

Because Stevens' method of ME appeared to produce biased results, being asymmetrical, he introduced a complementary method in which numbers were given by the experimenter, and the observers had to find matching sensation magnitudes that they produced by manipulating appropriate instrumental controls. He called this method "magnitude production" (MP). In the methods of scaling subjective magnitudes developed by myself, Hellman, and several other coworkers, the numbers are assumed to have absolute subjective values. Consequently, we call what started as ME, "absolute magnitude estimation" (AME), and what started as MP, "absolute magnitude production" (AMP). The designations conserve Stevens' tradition but are not completely accurate because both are regarded as matching operations. The methods have opened a wide world of subjective magnitudes to measurement in spite of objections by staunch conservatives that they do not constitute legitimate measurements. The mutual consistency of the results they produce belies the objections.

Sensation magnitudes almost generally follow power functions of adequate stimulus variables, except at very low values of these variables. As thresholds of detectability of the variables are approached, their subjective magnitudes converge on direct proportionality to the stimulus intensity or a related variable. Line length squared would be an example of such a variable. For sufficient lengths, the subjective line length tends to be directly proportional to the physical line length. As surprising as it may appear, this is no longer true for very short thin lines that become somewhat difficult to see. According to measurements of N.M. Sanpetrino, my Graduate Assistant, the subjective line length then becomes proportional to the square of the physical line length. The phenomenon can be explained by the physiological noise that is added to the visual line image. In agreement with the theory of signal detectability, such a process can be expected to take place near the threshold of detectability of all sensory stimuli. It would be consistent with a linear relationship of subjective magnitudes to stimulus intensity. Because I was able to ascertain empirically such a relationship for many sensory modalities and because of the likely generality of the underlying physiological process, I am suggesting the relationship as the Second Law of Psychophysics.

Additivity of subjective magnitudes is introduced in this monograph as the third law. Its demonstration is essential for the understanding of the function of a sensory system. It signals linear processing. Additivity is also essential in validating the scales of measurement obtained by more direct means, such as AME. Such scales can be constructed by simply adding quantities defined as units. Linear summation of two units produces a quantity equal to two units. Linear summation of two doubled quantities produces a quadrupled quantity, and so forth. Early attempts at establishing the functional relationship between loudness and sound intensity were based on such an additive process on the assumption of additivity. Much of the chapter concerning the additivity is dedicated to the demonstration that it does take place in several, perhaps all sense modalities under appropriate conditions. The demonstration is complicated by the fact that, according to physiological evidence, the summation process is preceded by more peripheral neural processes that may introduce nonlinear interactions. Nevertheless, existence of additivity has been demonstrated with scientific certainty in hearing, touch and vision. The situation in chemical senses had to be left unresolved.

The fourth and last law included in this monograph concerns detectability of intensity increments. In its classical nineteenth-century form of Weber's law, according to which the just detectable intensity increments or, more generally, magnitude increments are directly proportional to the base intensity, or magnitude, it is probably the oldest law of psychophysics. The law is often expressed as the Weber fraction consisting of the ratio between the just noticeable increment and the base magnitude. The fraction tends to have a constant value, except at very low stimulus values, where it rapidly increases. In hearing, for pure tones, and in vibrotaction, for any stimuli, the value tends to decrease slowly as the base intensity is increased. The phenomenon is referred to as the "near miss to Weber's law."

In more recent times, paradoxical-like properties of Weber's law have been discovered in hearing. When measured by means of just detectable intensity increments or the difference between two intensity increments, Weber's fraction has been shown not to depend on the rate of growth of loudness with stimulus intensity but only on the loudness itself. When measured as the standard deviation of the variability of loudness matches between two tones with loudness magnitudes increasing according to two different functions, it has been found to depend on the slopes of the functions in a predictable but complicated way. Counterintuitively, it depended not only on the slope of the loudness function of the ear in which the sound intensity was varied but also on the slope of the loudness function of the contralateral ear. Somewhat unexpectedly, I found it possible to describe the differential intensity sensitivity in all its methodological variations by one simple mathematical equation. I suggest the equation as an expression of a General Law of Differential Intensity Sensitivity.

At the end of this introduction, a crucial caveat must be added. All the mathematical theories used in this monograph are based on the assumption of linear interactions. Nonlinear interactions are excluded. Nevertheless, most sensation magnitudes are compressed functions of underlying stimulus intensities, the powers of the Power Law functions have exponents different from unity, usually, smaller than one. Every physiologist must know that the process of generating neural action potentials is nonlinear, and the rate at which the potentials, usually called "spikes," occur is a saturating function of stimulus intensity. Yet, several researchers have been able to show for hearing that the compression originates in the cochlear mechanics, and I pointed out that, for the most part, it is likely due to a kind of automatic gain control (AGC) that produces negligible distortions of waveforms of sound. AGC is used generally in radio communication. If it produced nonnegligible wave distortions, telecommunication would become impossible. The same goes for the auditory system.

Information transmission through neural spikes can be linear if it occurs through modulation of the spike rate. The overall output of the auditory nerve for pure-tone stimuli has been demonstrated to parallel the loudness function. This suggests overall linear processing above the level of the auditory nerve. The processing does not have to be linear in detail and, according to physiological evidence, it certainly is not. But the nonlinearities have to cancel each other in the summated neural output to produce what is called a "quasi-linear" process.

In the sense of touch, the subjective sensation of pressure is nearly directly proportional to the depression of the skin surface, so that here the problem of compression does not arise. Deviations from linearity in the sensation magnitudes observed in the chemical senses are less substantial than in hearing, but their physiological mechanisms are not clearly specified. In vision, the substantial compression evident in the brightness functions of luminous targets seen on a black background has not been analyzed in this monograph in terms of the theory of linear signal processing, except for small signals, where linear approximations are possible.

Importantly, in all instances, the theoretical results have been validated by empirical confirmation.

Chapter 1
Stevens' Power Law

1.1 Definition and Genesis

According to the power law, sensation magnitudes grow as power functions of stimulus intensities that produce them. The law was first proposed by S.S. Stevens for light and sound. It was announced in a 1953-paper presented before the National Academy of Sciences (USA) (cit. Stevens, 1975). Subsequently, Stevens suggested it as a general law describing quantitatively the relationships between human sensations as well as other subjective impressions and the physical stimuli that evoke them (rev. Stevens, 1975). According to the proposed law, the relationships approximate power functions of the form

$$\psi = k\phi^{\theta} \tag{1.1}$$

with ψ symbolizing the sensation magnitude, ϕ, the magnitude of the physical stimulus, θ the power exponent, and k, a dimensional constant.

The genesis of the law has a stormy history. The question of the relationship between the physical world surrounding us and its representation in our minds has haunted scientists for centuries, but did not mature to a quantitative science until Gustav Theodor Fechner, the physicist becoming philosopher, established the science of Psychophysics in 1860 (English translation, 1966). Before he did, he postulated in 1850 (cit. Stevens, 1975) on an intuitive insight that sensation magnitudes were related to the magnitudes of physical stimuli by logarithmic functions. The logarithmic "formula," as Fechner called it, had an important antecedent. Already in 1738, the famous Swiss mathematician, D. Bernoulli, came to the conclusion that the subjective value of money increased as the logarithm of the amount of money (Bernoulli, translation, 1954). He observed that the subjective value increased much more slowly than the objective one, an impression that was in agreement with the strongly compressed logarithmic function.

Fechner's formula received some experimental support. On the request, of another physicist, J.A.F. Plateau, J. Delboeuf (1873; cit. Stevens, 1975) let some

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painters mix white and black paints in various proportions to obtain equal appearing contrast steps or intervals. The result followed roughly a logarithmic function. Furthermore, an interval scale of stellar brightness created in antiquity, around 150 BC by the astronomer Hipparchus for classification of stars was found to agree roughly with a logarithmic scale when physical photometry became possible (Jastrow, 1887; cit. Stevens, 1975). Fechner found his formula to be consistent with the experiments of E.H. Weber (1834; cit. Marks, 1974) who had determined that difference limens (DLs) or just noticeable differences (JNDs) between sensory stimulus magnitudes were directly proportional to stimulus magnitudes – a quite general relationship that became known as Weber's fraction, or law. Assuming that the JND steps had equal subjective magnitudes, and integrating the relationship, Fechner recovered the logarithmic formula (cit. Stevens, 1975). To be mathematically correct the derivation should take the form:

$$\Delta\psi = a\frac{\Delta\phi}{\phi} \tag{1.2}$$

After integration,

$$\psi = a\log\left(\frac{\phi}{\phi_0}\right) \tag{1.3}$$

where ψ means the subjective (psychological) magnitude, as before, ϕ the physical magnitude with ϕ_0 as its reference value, and a, a dimensional constant.

Buoyed by these and other similar results, Fechner's formula became regarded as a psychophysical law that reigned supreme for almost a century and even invaded neurophysiology (e.g. Matthews, 1931; Hartline and Graham, 1932). Communication engineers devised a logarithmic scale based on a unit called decibel (dB) to match what they thought would be the loudness function. A difference of 10 dB meant an intensity ratio of 10, that of 20 dB, one of 100, that of 30 dB one of 1,000, and so forth. Because sound intensity is proportional to the square of sound pressure, a sound pressure ratio of 10 is equivalent to 20 dB.

Unexpectedly, the logarithmic decibel scale proved to be mortal to Fechner's law. When the decibels of sound intensity were doubled, for example, from 40 to 80 dB, the subjective loudness did not double as it should have, were it proportional to the logarithm of the intensity, but increased much more (Churcher, 1935; cit. Marks, 1974). In addition, when equal numbers of intensity JNDs were added up at two different sound frequencies, the resulting loudness magnitudes did not appear to be equal, as could be easily verified by a direct loudness match (Newman, 1933; cit. Marks, 1974). The loudness grew faster than predicted by the logarithmic function. This conclusion was confirmed by many other experiments discussed extensively by Marks (1974).

The significance of the mentioned subjective impressions goes far beyond demonstrating that Fechner's logarithmic law cannot be true. They suggest that sensations have quantifiable magnitudes. This insight is consistent with Bernouilli's and Fechner's decisions to use highly compressive functions in describing the growth of the subjective magnitudes they experienced, rather than a linear one. Other scientists, who may not have used logarithmic functions for similar purposes, used compressive functions, nevertheless. They all must have been able to gauge subjectively the rate of growth of their sensation magnitudes.

The above observation makes the outcry of some prominent psychologists and philosophers against the attempts of Fechner and a few others to quantify sensation magnitudes appear hollow. To quote from James (1890), W. Wundt, the father of experimental psychology stated: “How much stronger or weaker one sensation is than another, we are never able to say.” Of course, Wundt thought of numerical ratios which may not be explicit in the feeling that loudness grows with sound intensity more rapidly than according to a logarithmic function. Nevertheless, the logarithmic function is expressed numerically. Again, quoting after James (1890), the famous philosopher, Carl Stumpf, stated: “One sensation cannot be a multiple of another. If it could, we ought to be able to subtract the one from the other, and to feel the remainder by itself. Every sensation presents itself as an indivisible unit.” Stumpf’s statement was later parodied by two British physicists, Richardson and Ross (1930), who wrote: “One mountain cannot be twice as high as another. If it could, we ought to be able to subtract the one from the other and to climb up the remainder by itself. Every mountain presents itself as an indivisible lump.” They went on to produce a numerical scale of loudness. The method they used may be regarded as a precursor of the method Stevens subsequently worked out in great detail and called “magnitude estimation.” Their result suggested that loudness was related to sound intensity by a power function rather than a logarithmic function.

Perhaps Richardson and Ross were the first to come up with an empirical power-function relationship between a physical variable and the subjective impression it evoked. According to Stevens (1975), the idea of the power function relationship may have been first conceived by a young eighteenth century mathematician, Gabriel Cramer, however, whose work was cited by Bernoulli. Cramer, like Bernoulli was interested in the subjective value of money.

The decade following Richardson and Ross’ paraphrase witnessed an explosion of experiments on loudness quantification, spurred by developments in electronics and communication engineering. Sound intensity and, with it, loudness became easy to vary. Most experiments followed ratio procedures in which the observers were asked to produce loudnesses that were subjectively twice, three times, four times and so on louder than a given standard loudness. Fractionation, in which the loudness had to be made 1/2, 1/3 or smaller than the standard was also employed. Many of the results were summarized by Churcher (1935) and used by Stevens (1936) to construct a loudness scale – the so-called *sone* scale. One sone served as a unit and was defined as the loudness of a 1,000 Hz. tone presented binaurally 40 dB above its threshold. The resulting function clearly departed form a logarithmic one and resembled a power function instead.

The ratio procedures, which required an intensity adjustment by the observer, were tedious and time consuming. Stevens looked for a more efficient method. He stumbled upon one almost accidentally in 1953, a year that may become almost as important for psychophysics as the year 1850 in which Fechner conceived of quantifying sensation magnitudes (cit. Stevens, 1975). Stevens’ discovery had such an impact on psychophysics that I would like to honor it here by quoting verbatim his anecdotal description of it (Stevens, 1975).

> What I wanted was a method that would tell me the overall form of the scale, from a weak stimulus to a strong one. I expressed that idea to a colleague who objected that he had no loudness scale in his head from which he could read such values directly. That was a novel thought, however, and I persuaded him to explore it with me.
>
> I turned on a very loud tone at 120 decibels, which made my colleague jump, and which we agreed would be called 100. I then turned on various other intensities in irregular order, and for each stimulus he called out a number to specify the loudness. I plotted the numbers directly on a piece of graph paper in order to see immediately what course was being followed by the absolute judgments, as I first called them. The experiment seemed to work so well that I proceeded to enlist other observers. The plots of the magnitude estimations, as I now call them, that were produced by the first half-dozen observers are shown in Fig. 7 (Fig. 1.1 here). Each observer's estimations were plotted on a separate graph, I had no assurance that it would be proper to average the data from different observers. The general agreement among the responses of the first few observers persuaded me that I had probably hit upon a promising method, and that the potential of the procedure ought to be explored seriously.

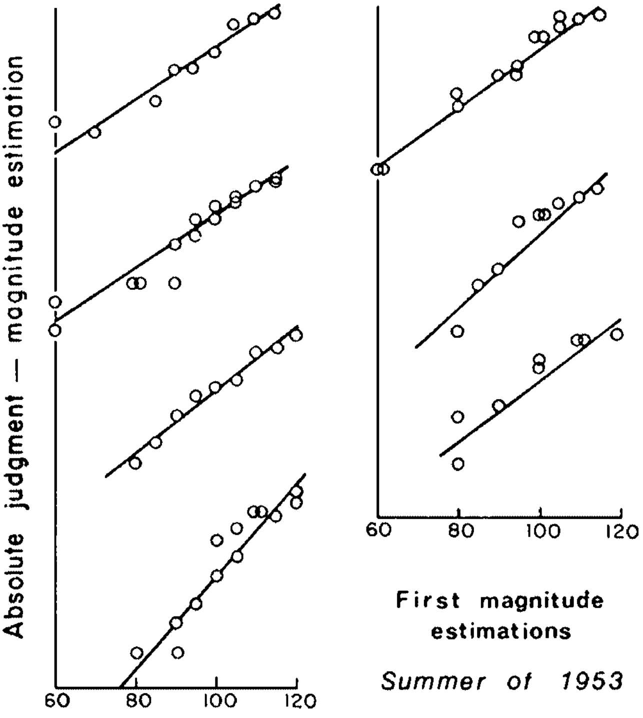

Fig. 1.1 The results of the first magnitude-estimation experiment performed in 1953. The data points indicate single estimates given by individual observers relative to a reference sound pressure level of 120 dB assigned the number 100. Reproduced from Stevens (1975) with permission from John Wiley & Sons, Inc.