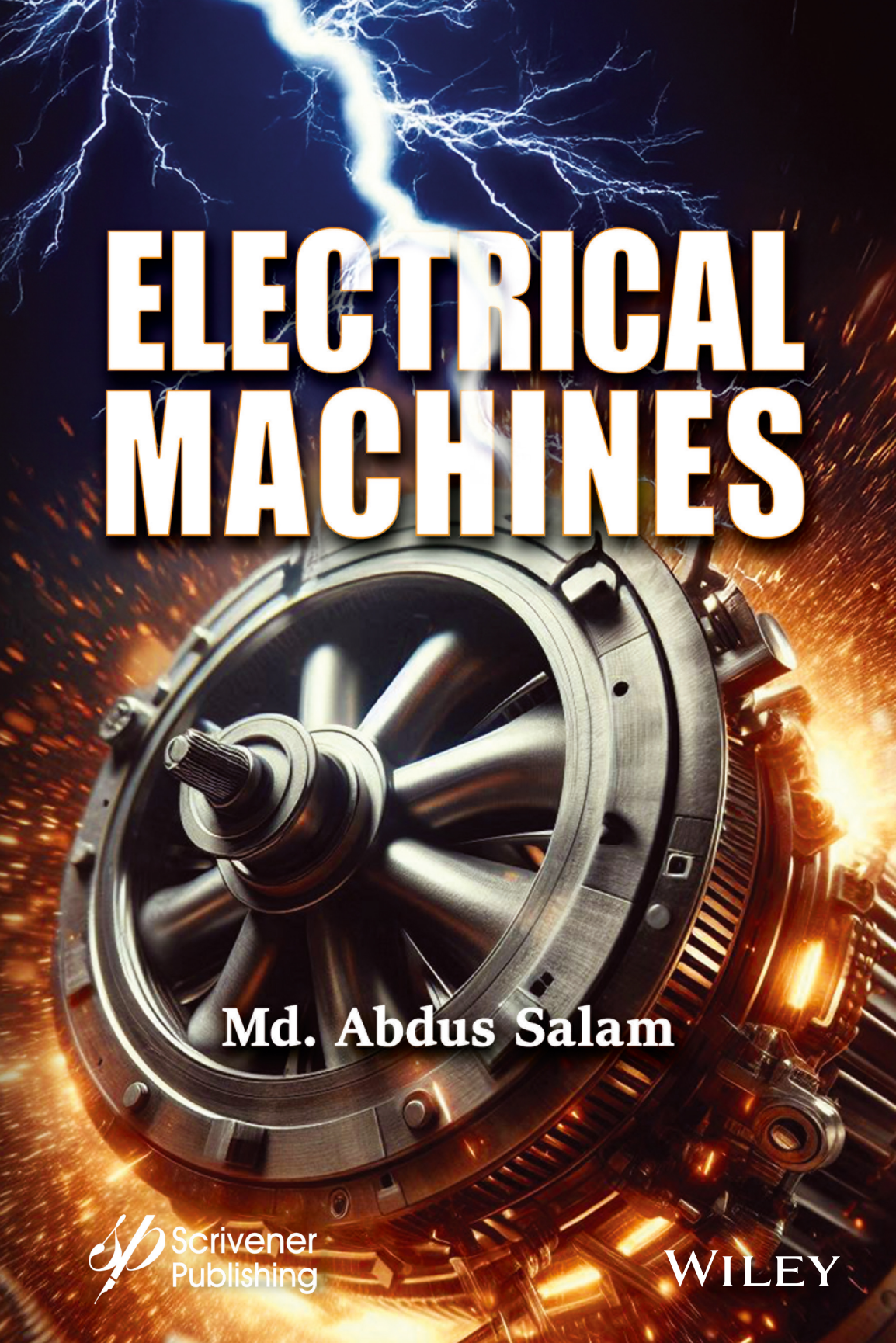




ELECTRICAL MACHINES

Md. Abdus Salam

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Electrical Machines

Scrivener Publishing

100 Cummings Center, Suite 541J
Beverly, MA 01915-6106

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Electrical Machines

Fundamentals and Analysis

Md. Abdus Salam



WILEY

This edition first published 2025 by John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, USA and Scrivener Publishing LLC, 100 Cummings Center, Suite 541J, Beverly, MA 01915, USA

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Library of Congress Cataloging-in-Publication Data

ISBN 978-1-394-23117-1

Front cover images supplied by Adobe Firefly

Cover design by Russell Richardson

Set in size of 11pt and Minion Pro by Manila Typesetting Company, Makati, Philippines

Printed in the USA

10 9 8 7 6 5 4 3 2 1

Dedication

To my parents, my wife (Asma Ara Bagum), my children (Syeed Hasan, Yusra Salam, and Sundus Salam), and my daughter-in-law (Naomi Hoque) for their kindness and endless support.

Additionally, I would like to thank my instructors and well-wishers who have supported my professional development over the years.

Md. Abdus Salam

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Preface

Electrical machines such as DC motors, DC generators, induction motors, synchronous machines, and special motors are widely used in different applications including the manufacturing industry. As technology continues to advance, the importance of understanding the principles and applications of electrical machines becomes increasingly critical. Electrical machines play a pivotal role in numerous aspects of our daily lives, from powering our homes and industries to driving innovations in renewable energy. Therefore, the basic principles of electrical machines along with their characteristics need to be known to proceed to higher semesters as well as to work in the practical field.

Welcome to a new textbook, *Electrical Machines: Fundamentals and Analysis*. This textbook has been thoroughly written to provide students, professors and engineers with a comprehensive, hands-on and accessible guide to the fascinating scope of electrical machines.

In addition, this textbook is designed to offer a balanced blend of theoretical concepts and practical applications, ensuring that readers not only grasp the underlying principles but also develop the skills necessary to apply them in real-world scenarios and applications, especially in the manufacturing industry.

Key Features

Fundamentals Concepts

The book begins with a solid foundation in the fundamentals of electrical circuits which will help readers gain a clear understanding of the essential theories that govern electrical machines' operation and characteristics.

Problem-Solving Approach

A step-by-step problem-solving approach is incorporated into each theory, encouraging active engagement and reinforcing knowledge. A variety of examples and exercises are included to enhance problem-solving skills.

Emerging Trends

Recognizing the dynamic and vibrant nature of the field, this textbook explores emerging trends and advancements in electrical machines, including discussions on the integration of smart technologies and software to start and control electrical machines.

Visual Learning

Rich and coloured illustrations, diagrams, and practical examples are employed to facilitate visual learning and enhance the clarity of complex concepts.

Practical Applications

Illustrations, diagrams, and practical examples are employed to facilitate visual learning. Real-world applications are included throughout the textbook, illustrating how electrical machines are used in different industries and inspiring engineers to see the direct impact of their knowledge.

Supplementary Resources

Based on adoption of this textbook, additional online resources, including tutorials, simulations, and supplementary materials will be made available to further support the learning process.

In the undergraduate course curriculum, universities normally offer two courses in electrical machines, each three credit hours. Most universities have already been evaluated or are in the process of evaluating their undergraduate programs in electrical and computer engineering to meet the Accreditation Board for Engineering and Technology (ABET) requirements.

This textbook strives to be your thorough companion, whether you are a student starting your journey into electrical engineering, a practicing engineer looking for a refresher, or just someone who is intrigued about the inner workings of the machines that power our modern world. Accept the fascinating realm of electrical machinery and start your educational journey now!

Aids for Instructors

Instructors who adopt this book as a text may obtain the solution manual as a supplement copy by contacting the publishers.

Acknowledgements

I would like to acknowledge the initial support of my former colleagues at the Electrical Engineering Department, College of Engineering, Cal Poly State University, San Luis Obispo, California. I also acknowledge with gratitude Danielle Wilken, President, Senior Vice Presidents, Vice Presidents, Dr. Manyul Im, Provost and Vice President for Academic Affairs, Administration, Professor Dr. Khaled Elleithy, Dean School of Engineering, Professor Dr. Navarun Gupta, Chair, Department of Electrical Engineering, Faculty members and staff for their kind inspiration during the time I was writing this textbook.

The improvement of the book is an ongoing process. Therefore, I would be grateful to academicians, students and professional engineers for their constructive comments on this edition of the book through my email: ma.abdus@gmail.com.

Finally, I would like to thank all staff of Scrivener Publishing House for their help in bringing this book project to a successful completion.

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Analysis of Electrical Power

1.1 Introduction

AC generators, transformers, and other related high-voltage devices are required to generate, transmit and distribute alternating voltage and current. These types of equipment are rated by Mega Watt (MW) and Mega Voltage-Ampere (MVA). Electrical appliances, such as DVD players, televisions, microwaves, light bulbs, refrigerators, ceiling fans and many more are rated in a Watt (W). Careful attention is required in designing these kinds of equipment to address their power ratings, which in turn addresses the varying electricity bills. The electricity bills depend on the use of energy, and the unit of the energy is kWh. Power and time are also required to calculate the energy. The power is classified as electrical power, mechanical power and fluid power. Electrical power is usually generated and transferred over a long distance using an AC generator, transformer, transmission and distribution lines. Mechanical power is transferred to a short distance using the pulley, chain and conveyor belt, whereas fluid power is generated and distributed to a short distance using the pump, motor, valves and piping system. Therefore, knowledge power analysis plays an important role in designing and calculating the energy cost of a system. This chapter will discuss instantaneous power, average power, complex power, power factor, maximum power transfer theorem and power factor correction.

1.2 Power

The rate of receiving and delivering energy is known as power. Electrical power consists of DC power and AC power. DC power is calculated based on the supply voltage and draws current by the device from the source, whereas the calculation of AC power depends on the voltage, current and

2 ELECTRICAL MACHINES

power factor. Fluid power is also calculated from the fundamental definition of power. Mathematically, the power is defined as,

$$P = \frac{dW}{dt} = \frac{W}{t} \quad (1.1)$$

$$P = \frac{dW}{dq} \frac{dq}{dt} \quad (1.2)$$

The voltage is defined as the work done per unit charge and is expressed as,

$$V = \frac{dW}{dq} \quad (1.3)$$

The current is defined as the rate of change of charge and it is expressed as,

$$I = \frac{dq}{dt} \quad (1.4)$$

Substituting equations (1.3) and (1.4) into equation (1.2) yields,

$$P = VI \quad (1.5)$$

Again, a force F is applied to an object to move a distance d . The work done is expressed as,

$$W = F \times d \quad (1.6)$$

Substituting equation (1.6) into equation (1.1) yields,

$$P = \frac{F \times d}{t} = F \times \frac{d}{t} \quad (1.7)$$

$$P = \frac{F \times d}{t} = F \times v \text{ (lb-ft/s) or (N-m/s)} \quad (1.8)$$

But, 550 lb-ft/s is equal to one HP and 1000 N-m/s is equal to one kW. Then the fluid power in FPS and MKS (SI) units are expressed as,

$$P_{HP} = \frac{F \times v}{550} \quad (1.9)$$

$$P_{kW} = \frac{F \times v}{1000} \quad (1.10)$$

To derive the expression of power in an AC circuit, consider an AC voltage source $v(t)$ connected in series with an impedance as shown in Fig. 1.1. Let us consider that the expression of voltage is,

$$v(t) = V_m \sin \omega t \quad (1.11)$$

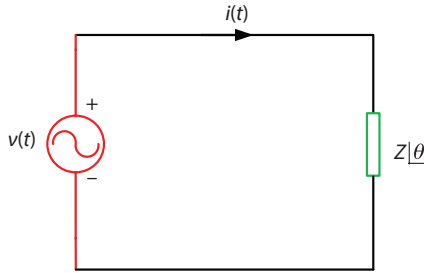


Fig. 1.1 A circuit with an impedance.

The series current $i(t)$ varies with time and the expression of the current is written as,

$$i(t) = \frac{v(t)}{Z|\theta} \quad (1.12)$$

Substituting equation (1.11) into equation (1.12) yields,

$$i(t) = \frac{V_m \sin \omega t}{Z \angle \theta} \quad (1.13)$$

$$i(t) = \frac{V_m \angle 0^\circ}{Z \angle \theta} = \frac{V_m}{Z} \angle -\theta \quad (1.14)$$

$$i(t) = I_m \angle -\theta = I_m \sin(\omega t - \theta) \quad (1.15)$$

The instantaneous power also varies with time, and it is defined as the product of the voltage and the current. The expression of instantaneous power is,

$$p(t) = v(t) \times i(t) \quad (1.16)$$

Substituting equations (1.11) and (1.15) into equation (1.16) yields,

$$p(t) = V_m \sin \omega t \times I_m \sin(\omega t - \theta) \quad (1.17)$$

$$p(t) = \frac{V_m I_m}{2} 2 \sin \omega t \times \sin(\omega t - \theta) \quad (1.18)$$

$$p(t) = \frac{V_m I_m}{2} [\cos(\omega t - \omega t + \theta) - \cos(\omega t + \omega t - \theta)] \quad (1.19)$$

$$p(t) = \frac{V_m I_m}{2} \cos \theta - \frac{V_m I_m}{2} \cos(2\omega t - \theta) \quad (1.20)$$

Equation (1.20) provides the expression for the instantaneous power for a series AC circuit.

Example 1.1

The excitation voltage and the impedance of a series circuit are given by $v(t) = 15 \sin \omega t$ V and $Z = 5 \angle 10^\circ \Omega$, respectively. Calculate the instantaneous power.

Solution

The value of the current is calculated as,

$$i(t) = \frac{15 \angle 0^\circ}{5 \angle 10^\circ} = 3 \angle -10^\circ \text{ A} \quad (1.21)$$

The instantaneous power is calculated as,

$$p(t) = \frac{15 \times 3}{2} 2 \sin \omega t \times \sin(\omega t - 10^\circ) \quad (1.22)$$

$$p(t) = \frac{45}{2} \cos 10^\circ - \frac{45}{2} \cos(2\omega t - 10^\circ) \quad (1.23)$$

$$p(t) = 22.16 - 22.5 \cos(2\omega t - 10^\circ) \text{ W} \quad (1.24)$$

Practice Problem 1.1

The current and impedance of a series circuit are given by $i(t) = 10 \sin(\omega t + 15^\circ)$ A and $Z = 2 \angle 20^\circ \Omega$, respectively. Calculate the instantaneous voltage and power.

1.3 Average Power and Reactive Power

The average power is related to the maximum value of the sinusoidal voltage and current, which are shown in equations (1.11) and (1.15), respectively.

The average power for a periodic waveform over one cycle can be derived as,

$$P = \frac{1}{T} \int_0^T p(t) dt \quad (1.25)$$

Substituting equation (1.20) into the equation (1.25) yields,

$$P = \frac{V_m I_m}{2T} \int_0^T \cos \theta dt - \frac{V_m I_m}{2T} \int_0^T \cos(2\omega t - \theta) dt \quad (1.26)$$

$$P = \frac{V_m I_m}{2T} \cos \theta [T] - \frac{V_m I_m}{2T} \int_0^T \cos(2\omega t - \theta) dt \quad (1.27)$$

The second term of equation (1.27) is a cosine waveform. The average value of any cosine waveform over one cycle is zero. Therefore, from equation (1.27), the final expression of the average power can be represented as,

$$P = \frac{V_m I_m}{2} \cos \theta \quad (1.28)$$

Similarly, the expression of the average reactive power can be written as,

$$Q = \frac{V_m I_m}{2} \sin \theta \quad (1.29)$$

The term $\cos \theta$ in equation (1.28) is the power factor of the circuit, and it is determined by the phase angle θ of the circuit impedance, where θ is the phase difference between the voltage and current phases i. e., $\theta = \theta_v - \theta_i$. The average power is often known as the true power or real power. The units of average power and reactive power are watts (W) and volt-ampere reactive (Var), respectively. The average power from equation (1.28) can be represented in terms of rms values of the voltage and current as,

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \theta = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \theta \quad (1.30)$$

Substituting $V_{rms} = \frac{V_m}{\sqrt{2}}$ and $I_{rms} = \frac{I_m}{\sqrt{2}}$ into equation (1.30) yields,

$$P = V_{rms} I_{rms} \cos \theta \quad (1.31)$$

Similarly, from equation (1.29), the reactive power can be represented as,

$$Q = V_{rms} I_{rms} \sin \theta \quad (1.32)$$

The current flows back and forth between the source and the network due to a specific magnitude of the reactive power. The reactive power does not dissipate any energy in the load. However, in practice, it produces energy losses in the line. In addition to that, it is required to create a magnetic field to operate AC and DC machines. Therefore, extra attention needs to be taken in designing the power system networks.

For a purely resistive circuit, the voltage and the current are in phase. This means that the phase angle between them is zero,

$$\theta = 0^\circ \quad (1.33)$$

Substituting equation (1.33) into equation (1.28) yields,

$$P_R = \frac{V_m I_m}{2} \cos 0^\circ \quad (1.34)$$

$$P_R = \frac{V_m I_m}{2} \quad (1.35)$$

Equation (1.35) can be rearranged by substituting $V_m = I_m R$ or $I_m = V_m / R$ as,

$$P_R = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R} \quad (1.36)$$

The phase difference between the voltage and current for the inductance and capacitance is,

$$\theta = \pm 90^\circ \quad (1.37)$$

Substituting equation (1.37) into equation (1.28) yields the average power for either an inductance or a capacitance,

$$P_L = P_C = \frac{V_m I_m}{2} \cos 90^\circ = 0 \quad (1.38)$$

The reactive power is usually stored in a circuit and it can be expressed for the inductor and capacitor as,

$$Q_L = I_L^2 X_L = \frac{V_L^2}{X_L} \quad (1.39)$$

$$Q_C = I_C^2 X_C = \frac{V_C^2}{X_C} \quad (1.40)$$

From equations (1.28) and (1.38), it can be concluded that the resistive load absorbs power whereas inductive or capacitive loads do not absorb any power.

Example 1.2

Fig. 1.2 shows an electrical series circuit with resistance, inductive and capacitive reactance. Find the average power supplied by the source and the power absorbed by the resistor.

Solution

The net impedance is calculated as,

$$Z_t = 2 + j5 - j9 = 4.47 \angle -63.43^\circ \Omega \quad (1.41)$$

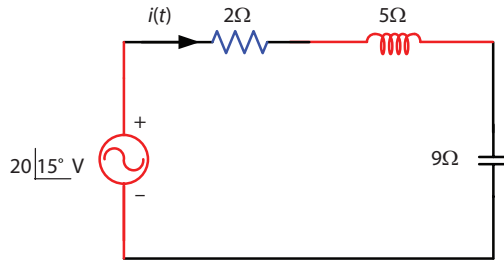


Fig. 1.2 Circuit for example 1.2.

The source current in phasor form is calculated as,

$$I = \frac{20\angle 15^\circ}{4.47\angle -63.43^\circ} = 4.47\angle 78.43^\circ \text{ A} \quad (1.42)$$

The average power supplied by the source is calculated as,

$$P_s = \frac{20 \times 4.47}{2} \cos(15^\circ - 78.43^\circ) = 20 \text{ W} \quad (1.43)$$

The average power absorbed by the resistor is calculated as,

$$P_R = \frac{4.47^2 \times 2}{2} = 20 \text{ W} \quad (1.44)$$

Practice Problem 1.2

Fig. 1.3 shows a series-parallel circuit with resistance, inductive and capacitive reactance. Calculate the average power supplied by the source and the power absorbed by the resistors.

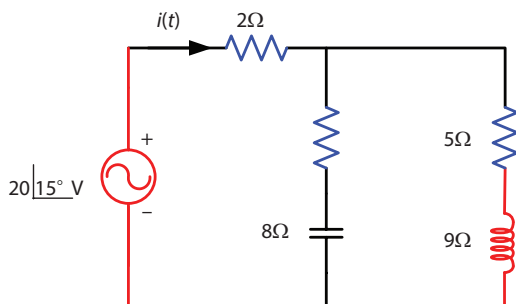


Fig. 1.3 Circuit for practice problem 1.2.

1.4 Apparent Power

The apparent power is the product of the voltage and the current. It is also defined as the vector sum of real power and reactive power. Real power is used to do useful work and can transmit over a long distance. Reactive power is the power that flows back into the source from the energy storage elements inductor and capacitor. Reactive power helps to regulate voltages on the transmission system to flow the real power. In addition to that, reactive power flow wastes the energy and capability of the machines and causes voltage drops. A leading reactive power (current leads the voltage) is supplied to correct the lagging power flow which tends to bring the current in phase with the voltage. In this condition (current in phase with the voltage), system losses decrease, increase the system capability and rise the voltage drops. The units of the reactive power are Var, kVar and Mvar. Therefore, the apparent power is related to the sinusoidal voltage and current. Let us consider that the expressions for sinusoidal voltage and current are,

$$v(t) = V_m \sin(\omega t + \theta_v) \quad (1.45)$$

$$i(t) = I_m \sin(\omega t + \theta_i) \quad (1.46)$$

The phasor forms of these voltage and current components are,

$$V = V_m \angle \theta_v \quad (1.47)$$