Handbook of Philosophical Logic 17

Dov M. Gabbay
Franz Guenthner *Editors*

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Second Edition

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SECOND EDITION

Volume 17

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Editors

Handbook of Philosophical Logic

Volume 17

Springer

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Preface to the Second Edition

It is with great pleasure that we are presenting to the community the second edition of this extraordinary Handbook. It has been over 15 years since the publication of the first edition and there have been great changes in the landscape of philosophical logic since then.

The first edition has proved invaluable to generations of students and researchers in formal philosophy and language, as well as to consumers of logic in many applied areas. The main logic article in the *Encyclopaedia Britannica 1999* has described the first edition as 'the best starting point for exploring any of the topics in logic'. We are confident that the second edition will prove to be just as good!

The first edition was the second Handbook published for the logic community. It followed the North Holland one-volume *Handbook of Mathematical Logic*, published in 1977, edited by the late Jon Barwise. The four-volume *Handbook of Philosophical Logic*, published in 1983–1989, came at a fortunate temporal junction at the evolution of logic. This was the time when logic was gaining ground in computer science and artificial intelligence circles.

These areas were under increasing commercial pressure to provide devices which help and/or replace the human in his daily activity. This pressure required the use of logic in the modelling of human activity and organisation on the one hand and to provide the theoretical basis for the computer program constructs on the other. The result was that the *Handbook of Philosophical Logic*, which covered most of the areas needed from logic for these active communities, became their bible.

The increased demand for philosophical logic from computer science and artificial intelligence and computational linguistics accelerated the development of the subject directly and indirectly. It directly pushed research forward, stimulated by the needs of applications. New logic areas became established and old areas were enriched and expanded. At the same time, it socially provided employment for generations of logicians residing in computer science, linguistics and electrical engineering departments which of course helped keep the logic community thriving. In addition to that, it so happens (perhaps not by accident) that many of the Handbook contributors became active in these application areas and took their place as time passed on, among the most famous leading figures of applied philosophical logic of our times. Today we have a Handbook with a most extraordinary collection of famous people as authors!

The table below will give our readers an idea of the landscape of logic and its relation to computer science and formal language and artificial intelligence. It shows that the first edition is very close to the mark of what was needed. Two topics were not included in the first edition, even though they were extensively discussed by all authors in a 3-day Handbook meeting. These are:

- A chapter on non-monotonic logic
- A chapter on combinatory logic and λ-calculus

We felt at the time (1979) that non-monotonic logic was not ready for a chapter yet and that combinatory logic and λ-calculus was too far removed.[1] Non-monotonic logic is now a very major area of philosophical logic, alongside default logics, labelled deductive systems, fibring logics, and multi-dimensional, multimodal and substructural logics. Intensive re-examinations of fragments of classical logic have produced fresh insights, including at times decision procedures and equivalence with non-classical systems.

Perhaps the most impressive achievement of philosophical logic as arising in the past decade has been the effective negotiation of research partnerships with fallacy theory, informal logic and argumentation theory, attested to by the Amsterdam Conference in Logic and Argumentation in 1995, and the two Bonn Conferences in Practical Reasoning in 1996 and 1997.

These subjects are becoming more and more useful in agent theory and intelligent and reactive databases.

Finally, 15 years after the start of the Handbook project, I would like to take this opportunity to put forward my current views about logic in computer science, computational linguistics and artificial intelligence. In the early 1980s, the perception of the role of logic in computer science was that of a specification and reasoning tool and that of a basis for possibly neat computer languages. The computer scientist was manipulating data structures and the use of logic was one of his options.

My own view at the time was that there was an opportunity for logic to play a key role in computer science and to exchange benefits with this rich and important application area and thus enhance its own evolution. The relationship between logic and computer science was perceived as very much like the relationship of applied mathematics to physics and engineering. Applied mathematics evolves through its use as an essential tool, and so we hoped for logic. Today my view has changed. As computer science and artificial intelligence deal more and more

[1] I am really sorry, in hindsight, about the omission of the non-monotonic logic chapter. I wonder how the subject would have developed, if the AI research community had had a theoretical model, in the form of a chapter, to look at. Perhaps the area would have developed in a more streamlined way!

with distributed and interactive systems, processes, concurrency, agents, causes, transitions, communication and control (to name a few), the researcher in this area is having more and more in common with the traditional philosopher who has been analysing such questions for centuries (unrestricted by the capabilities of any hardware).

The principles governing the interaction of several processes, for example, are abstract and similar to principles governing the cooperation of two large organisations. A detailed rule based effective but rigid bureaucracy is very much similar to a complex computer program handling and manipulating data. My guess is that the principles underlying one are very much the same as those underlying the other.

I believe the day is not far away in the future when the computer scientist will wake up one morning with the realisation that he is actually a kind of formal philosopher!

The projected number of volumes for this Handbook is about 18. The subject has evolved and its areas have become interrelated to such an extent that it no longer makes sense to dedicate volumes to topics. However, the volumes do follow some natural groupings of chapters.

I would like to thank our authors and readers for their contributions and their commitment in making this Handbook a success. Thanks also to our publication administrator Mrs J. Spurr for her usual dedication and excellence and to Kluwer Academic Publishers for their continuing support for the Handbook.

King's College London, and
Bar Ilan University, Israel, and
University of Luxembourg

Dov M. Gabbay

Logic	IT			
	Natural language processing	**Program control specification, verification, concurrency**	**Artificial intelligence**	**Logic programming**
Temporal logic	Expressive power of tense operators. Temporal indices. Separation of past from future	Expressive power for recurrent events. Specification of temporal control. Decision problems. Model checking	Planning. Time dependent data. Event calculus. Persistence through time—the Frame Problem. Temporal query language. Temporal transactions	Extension of Horn clause with time capability. Event calculus. Temporal logic programming
Modal logic. Multi-modal logics	Generalised quantifiers	Action logic	Belief revision. Inferential databases	Negation by failure and modality
Algorithmic proof	Discourse representation. Direct computation on linguistic input	New logics. Generic theorem provers	General theory of reasoning. Non-monotonic systems	Procedural approach to logic
Non-monotonic reasoning	Resolving ambiguities. Machine translation. Document classification. Relevance theory	Loop checking. Non-monotonic decisions about loops. Faults in systems	Intrinsic logical discipline for AI. Evolving and communicating databases	Negation by failure. Deductive databases
Probabilistic and fuzzy logic	Logical analysis of language	Real time systems	Expert systems. Machine learning	Semantics for logic programs
Intuitionistic logic	Quantifiers in logic	Constructive reasoning and proof theory about specification design	Intuitionistic logic is a better logical basis than classical logic	Horn clause logic is really intuitionistic. Extension of logic programming languages
Set theory, higher-order logic, λ-calculus, types	Montague semantics. Situation semantics	Non-well-founded sets	Hereditary finite predicates	λ-calculus extension to logic programs

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Imperative vs. declarative languages	**Database theory**	**Complexity theory**	**Agent theory**	**Special comments: A look to the future**
Temporal logic as a declarative programming language. The changing past in databases. The imperative future	Temporal databases and temporal transactions	Complexity questions of decision procedures of the logics involved	An essential component	Temporal systems are becoming more and more sophisticated and extensively applied
Dynamic logic	Database updates and action logic	Ditto	Possible actions	Multimodal logics are on the rise. Quantification and context becoming very active
Types. Term rewrite systems. Abstract interpretation	Abduction, relevance	Ditto	Agent's implementation rely on proof theory	
	Inferential databases. Non-monotonic coding of databases	Ditto	Agent's reasoning is non-monotonic	A major area now. Important for formalising practical reasoning
	Fuzzy and probabilistic data	Ditto	Connection with decision theory	Major area now
Semantics for programming languages. Martin-Löf theories	Database transactions. Inductive learning	Ditto	Agent's constructive reasoning	Still a major central alternative to classical logic
Semantics for programming languages. Abstract interpretation. Domain recursion theory.		Ditto		More central than ever!

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Classical logic. Classical fragments	Basic background language	Program synthesis	A basic tool	
Labelled deductive systems	Extremely useful in modelling		A unifying frame-work. Context theory	Annotated logic programs
Resource and substructural logics	Lambek calculus		Truth maintenance systems	
Fibring and combining logics	Dynamic syntax	Modules. Combining languages	Logics of space and time	Combining features
Fallacy theory				
Logical dynamics	Widely applied here			
Argumentation theory games		Game semantics gaining ground		
Object level/ metalevel **Mechanisms: Abduction, default relevance**			Extensively used in AI Ditto	
Connection with neural nets				
Time-action-revision models			Ditto	
	Relational databases	Logical complexity classes	The workhorse of logic	The study of fragments is very active and promising.
	Labelling allows for context and control		Essential tool	The new unifying framework for logics
Linear logic			Agents have limited resources	

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	Linked databases. Reactive databases		Agents are built up of various fibred mechanisms	The notion of self-fibring allows for self-reference
				Fallacies are really valid modes of reasoning in the right context
			Potentially applicable	A dynamic view of logic
				On the rise in all areas of applied logic. Promises a great future
			Important feature of agents	Always central in all areas
			Very important for agents	Becoming part of the notion of a logic
				Of great importance to the future. Just starting
			A new theory of logical agent	A new kind of model

Contents

Chapter 1
Hybrid Logic

Torben Braüner

The starting point of this chapter[1] is the remarkable fact that proof procedures for wide classes of hybrid logics can be given in a uniform way, and moreover, this encompasses proof procedures like natural deduction and tableau systems which are suitable for actual[2] reasoning. A focus of the chapter is such proof procedures. Axiom systems, which are not meant for actual reasoning, are only mentioned in passing. We present a relatively small selection of procedures rather than trying to be encyclopedic. This allows us to give a reasonably detailed treatment of the selected procedures. Another focus of the chapter is the origin of hybrid logic in Arthur Prior's philosophical work.

In the first section of the chapter, Sect. 1.1, we give the basics of hybrid logic. In Sect. 1.2 we discuss the work of Arthur Prior and describe how hybrid logic has its origin in his work. In Sect. 1.3 we outline the development of hybrid logic since Prior. In Sect. 1.4 we introduce a natural deduction system for hybrid logic and in Sect. 1.5 we introduce tableau systems and tableau-based decision procedures for hybrid logic. In Sect. 1.6 we try to give an answer to the following question: Why does the proof-theory of hybrid logic behave so well compared to the proof-theory of ordinary modal logic?

---

[1]The chapter is composed of material adapted from the author's book (Braüner 2011). The author wishes to acknowledge the financial support received from The Danish Natural Science Research Council as funding for the projects HyLoMOL (2004–2008) and HYLOCORE (2009–2013).

[2]The word "actual" has here a broad meaning, not restricted to actual human reasoning. The logic does not care whether it is a human that carries out the reasoning, or the reasoning takes place in a computer, or in some other medium.

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1.1 The Basics of Hybrid Logic

In this section we give the basics of hybrid logic. We first give an informal motivation of hybrid logic. We then give the formal syntax and semantics and we give translations forwards and backwards between hybrid logic and first-order logic.

1.1.1 Informal Motivation

The term "hybrid logic" covers a number of logics obtained by adding further expressive power to ordinary modal logic.[3] The history of what now is known as hybrid logic goes back to Arthur Prior's work in the 1960s, which we shall come back to in Sect. 1.2. The term "hybrid logic" was coined in Patrick Blackburn and Jerry Seligman's paper published in 1995. The most basic hybrid logic is obtained by adding nominals, which are propositional symbols of a new sort interpreted in a restricted way that enables reference to individual points in a Kripke model. In what follows we shall give a more detailed explanation.

In the standard Kripke semantics for modal logic, the truth-value of a formula is relative to points in a set, that is, a formula is evaluated "locally" at a point. Usually, the points are taken to represent possible worlds, times, locations, epistemic states, states in a computer, or something else. Thus, in the Kripke semantics, a propositional symbol might have different truth-values at different points. This allows us to formalize natural language statements whose truth-values are relative to, for example times, like the statement

it is raining

which has clearly different truth-values at different times. Such statements can be formalized in ordinary modal logic using ordinary propositional symbols. Now, certain natural language statements are true at exactly one time, possible world, or something else. An example is the statement

it is 5 o'clock 10 May 2007

which is true at the time 5 o'clock 10 May 2007, but false at all other times. While the first kind of statement can be formalized in ordinary modal logic, the second kind of statement cannot, the reason being that there is only one sort of propositional symbol available, namely ordinary propositional symbols, which are not restricted to being true at exactly one point in the Kripke semantics.

A major motivation for hybrid logic is to add further expressive power to ordinary modal logic with the aim of being able to formalize the second kind of statement. This is obtained by adding to ordinary modal logic a second sort of propositional

[3] This should not be confused with the term "hybrid systems" which in the computer science community is used for systems that combine discrete and continuous features.

symbol called a nominal such that in the Kripke semantics each nominal is true at exactly one point. In other words, a nominal is interpreted with the restriction that the set of points at which it is true is a singleton set, not an arbitrary set. A natural language statement of the second kind (like the example statement with the time 5 o'clock 10 May 2007) is then formalized using a nominal, not an ordinary propositional symbol (which is used to formalize the example statement with rainy weather). The fact that a nominal is true at exactly one point implies that a nominal can be considered a term referring to a point, for example, if a is a nominal that stands for "it is 5 o'clock 10 May 2007", then the nominal a can be considered a term referring to the time 5 o'clock 10 May 2007.[4] Thus, in hybrid logic a term is a specific sort of propositional symbol whereas in first-order logic it is an argument to a predicate.

Most hybrid logics involve further additional machinery than nominals. There is a number of options for adding further machinery; here we shall consider a kind of operator called satisfaction operators. The motivation for adding satisfaction operators is to be able to formalize a statement being true at a particular time, possible world, or something else. For example, we want to be able to formalize that the statement "it is raining" is true at the time 5 o'clock 10 May 2007, that is, that

> at 5 o'clock 10 May 2007, it is raining.

---

[4]Considering a nominal as a symbol that refers to something is not the only way to view nominals. Two different views on nominals can be identified in the works of Arthur Prior, as is clear from the quotation below where Prior discusses the addition of nominals to a temporal version of modal logic called tense logic.

> We might . . . equate the instant a with a conjunction of all those propositions which would ordinarily be said to be true at that instant, or we might equate it with some proposition which would ordinarily be said to be true at that instant only, and so could serve as an index of it (Hasle et al. 2003, p. 124).

In the second half of the sentence, the nominal a is viewed as a proposition that can serve as an index of an instant, which is clearly in line with considering a nominal as a symbol that refers to an instant. On the other hand, in the first half of the sentence, the nominal a is viewed as a description of the content of an instant. The alternative view on nominals expressed in the first half of the sentence quoted above can also be found in a number of other places in Prior's works, for example the following.

> The essential trick is to treat the instant variables as a special sort of *propositional* variables, by identifying an 'instant' with the totality of what would ordinarily be said to be true *at* that instant, . . . (Hasle et al. 2003, p. 141).

See the discussion of Prior's work in Sect. 1.2 of the present handbook chapter, in particular Footnote 8 of that section. Moreover, see the discussion in Patrick Blackburn's paper (2006), the last paragraph of page 353, including Footnote 7, and the first complete paragraph of page 362, in particular Footnote 11. Incidentally, note that the description of the content of an instant as the conjunction of all propositions true at that instant is similar to a maximal consistent set of formulas.

This is formalized by the formula $@_a p$ where the nominal a stands for "it is 5 o'clock 10 May 2007" as above and where p is an ordinary propositional symbol that stands for "it is raining". It is the part $@_a$ of the formula $@_a p$ that is called a satisfaction operator. In general, if a is a nominal and ϕ is an arbitrary formula, then a new formula $@_a\phi$ can be built (in some literature the notation $a : \phi$ is used instead of $@_a\phi$). A formula of the form $@_a\phi$ is called a satisfaction statement. The satisfaction statement $@_a\phi$ expresses that the formula ϕ is true at one particular point, namely the point to which the nominal a refers.

To sum up, we have now added further expressive power to ordinary modal logic in the form of nominals and satisfaction operators. Informally, the nominal a has the truth-condition

a is true relative to a point w
if and only if
the reference of a is identical to w

and the satisfaction statement $@_a\phi$ has the truth-condition

$@_a\phi$ is true relative to a point w
if and only if
ϕ is true relative to the reference of a

Observe that actually the point w does not matter in the truth-condition for $@_a\phi$ since the satisfaction operator $@_a$ moves the point of evaluation to the reference of a whatever the identity of w. Note that the addition of nominals and satisfaction operators does not disturb the local character of the Kripke semantics: The truth-value of a formula is still relative to points in a set and the added machinery only involves reference to particular points, not all points in the set.

It is worth noting that nominals together with satisfaction operators allow us to express that two points are identical: If the nominals a and b refer to the points u and v, then the formula $@_a b$ expresses that u and v are identical. The following line of reasoning shows why.

$@_a b$ is true relative to a point w
if and only if
b is true relative to the reference of a
if and only if
b is true relative to u
if and only if
the reference of b is identical to u
if and only if
v is identical to u

The identity relation on a set has the well-known properties reflexivity, symmetry, and transitivity, which is reflected in the fact that the formulas

$@_a a$
$@_a b \rightarrow @_b a$
$(@_a b \wedge @_b c) \rightarrow @_a c$

are valid formulas of hybrid logic. To see that these hybrid-logical formulas correspond to the properties reflexivity, symmetry, and transitivity, read $@_a b$ as $a = b$ etc. Also the formula

$$(@_a b \wedge @_a \phi) \rightarrow @_b \phi$$

is valid. This hybrid-logical formula corresponds to the rule of replacement.

Beside nominals and satisfaction operators, in what follows we shall consider the binders $\forall$ and $\downarrow$, which allow us to build formulas $\forall a \phi$ and $\downarrow a \phi$. The binders bind nominals to points in two different ways: The $\forall$ binder quantifies over all points analogous to the standard first-order universal quantifier, that is, $\forall a \phi$ is true relative to w if and only if whatever point the nominal a refers to, ϕ is true relative to w. The $\downarrow$ binder binds a nominal to the point of evaluation, that is, $\downarrow a \phi$ is true relative to w if and only if ϕ is true relative to w when a refers to w. It turns out that the $\downarrow$ binder is definable in terms of $\forall$.

Above we noted that nominals and satisfaction operators do not disturb the local character of the Kripke semantics. Also the $\downarrow$ binder leaves the local character of the semantics undisturbed since this binder just binds a nominal to the point of evaluation. Things are more complicated with the $\forall$ binder. This binder has a non-local character in the sense that it involves reference to all points in the Kripke semantics. Moreover, together with nominals and satisfaction operators, the $\forall$ binder gives rise to non-local expressivity in the form of full first-order expressive power (which we shall show in Sect. 1.1.3). However, the $\forall$ binder does not give rise to full first-order expressive power just together with nominals, that is, in the absence of satisfaction operators (or some similar machinery). Thus, it is really the interaction between the $\forall$ binder and satisfaction operators that gives rise to full first-order expressive power, and hence, non-local expressivity.[5]

To conclude, extending ordinary modal logic with hybrid-logical machinery (disregarding the extreme case involving both $\forall$ and satisfaction operators), gives us a more expressive logic without sacrificing the local character of the Kripke semantics.[6]

[5]In fact, the paper (Blackburn and Seligman 1995) gives a result (Proposition 4.5 on p. 264) indicating that the $\forall$ binder has a surprisingly local character when it is not accompanied by satisfaction operators or some similar machinery. Informally, this result says that the $\forall$ binder is then insensitive to the information at points outside the submodel generated by the point of evaluation, that is, it cannot detect the truth-values of formulas at such points.

[6]Further discussion of this point can be found in a number of places, notably the paper Blackburn (2006). This paper also discusses hybrid-logical versions of *bisimulations*, which give a mathematical way to illustrate the local character of the Kripke semantics. See also the paper Simons (2006) which discusses a number of logics of location involving what we here call satisfaction operators.

1.1.2 Formal Syntax and Semantics

In what follows we give the formal syntax and semantics of hybrid logic. In many cases we will adopt the terminology of Blackburn et al. (2001) and Areces et al. (2001a). The hybrid logic we consider is obtained by adding a second sort of propositional symbol, called *nominals*, to ordinary modal logic, that is, propositional logic extended with a modal operator $\Box$.[7] It is assumed that a set of ordinary propositional symbols and a countably infinite set of nominals are given. The sets are assumed to be disjoint. The metavariables p, q, r, ... range over ordinary propositional symbols and a, b, c, ... range over nominals. Besides nominals, an operator $@_a$ called a *satisfaction operator* is added for each nominal a. Sometimes the operator $@_a$ is called an *at operator*. Moreover, we shall consider the *binders* $\forall$ and $\downarrow$. The formulas of hybrid modal logic are defined by the grammar

$$S ::= p \mid a \mid S \wedge S \mid S \rightarrow S \mid \bot \mid \Box S \mid @_a S \mid \forall a S \mid \downarrow a S$$

where p ranges over ordinary propositional symbols and a ranges over nominals. In what follows, the metavariables ϕ, ψ, θ, ... range over formulas. Formulas of the form $@_a \phi$ are called *satisfaction statements* (cf. Blackburn 2000a). The notions of free and bound occurrences of nominals are defined as in first-order logic with the addition that the free nominal occurrences in $@_a \phi$ are the free nominal occurrences in ϕ together with the occurrence of a, and moreover, the free nominal occurrences in $\downarrow a\phi$ are the free nominal occurrences in ϕ except for occurrences of a. Also, if $\overline{a}$ is a list of pairwise distinct nominals and $\overline{c}$ is a list of nominals of the same length as $\overline{a}$, then $\psi[\overline{c}/\overline{a}]$ is the formula ψ where the nominals $\overline{c}$ have been simultaneously substituted for all free occurrences of the nominals $\overline{a}$. If a nominal a_i in $\overline{a}$ occurs free in ψ within the scope of $\forall c_i$ or $\downarrow c_i$, then the nominal c_i in ψ is renamed as appropriate (this can be done since there are infinitely many nominals). The connectives negation, nullary conjunction, disjunction, and bi-implication are defined by the conventions that $\neg\phi$ is an abbreviation for $\phi \rightarrow \bot$, $\top$ is an abbreviation for $\neg\bot$, $\phi \vee \psi$ is an abbreviation for $\neg(\neg\phi \wedge \neg\psi)$, and $\phi \leftrightarrow \psi$ is an abbreviation for $(\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$. Similarly, $\Diamond\phi$ is an abbreviation for $\neg\Box\neg\phi$ and $\exists a\phi$ is an abbreviation for $\neg\forall a\neg\phi$.

We now define models and frames.

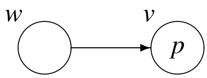
Definition 1.1. A *model* for hybrid logic is a tuple $(W, R, \{V_w\}_{w \in W})$ where

1. W is a non-empty set;
2. R is a binary relation on W; and
3. for each w, V_w is a function that to each ordinary propositional symbol assigns an element of $\{0, 1\}$.

[7] All results in the present handbook chapter can be generalized to cover an arbitrary, finite number of modal operators, but in the interest of simplicity, we shall stick to one modal operator unless otherwise is specified.

The pair (W,R) is called a *frame* and the model is said to be *based* on this frame. The elements of W are called *worlds* and the relation R is called the *accessibility relation*. A propositional symbol p is said to be *true* at w if $V_w(p) = 1$ and it is said to be *false* at w if $V_w(p) = 0$.

Note that a model for hybrid logic is the same as a model for ordinary modal logic. To give an extremely simple example of a model, we let $W = \{w, v\}$ and $R = \{(w,v)\}$, and moreover, we let $V_w(p) = 0$ and $V_v(p) = 1$. All other propositional symbols than p are ignored. This model can be depicted as



where circles represent worlds and an arrow indicates that two worlds are related by the accessibility relation. A propositional symbol in a circle means that the symbol is true and the absence of a propositional symbol means that it is false.

Given a model $\mathfrak{M} = (W, R, \{V_w\}_{w \in W})$, an *assignment* is a function g that to each nominal assigns an element of W. Given assignments g' and g, $g' \overset{a}{\sim} g$ means that g' agrees with g on all nominals save possibly a. The relation $\mathfrak{M}, g, w \models \phi$ is defined by induction, where g is an assignment, w is an element of W, and ϕ is a formula.

$$
\begin{array}{rcl}
\mathfrak{M}, g, w \models p & \text{iff} & V_w(p) = 1 \\
\mathfrak{M}, g, w \models a & \text{iff} & w = g(a) \\
\mathfrak{M}, g, w \models \phi \wedge \psi & \text{iff} & \mathfrak{M}, g, w \models \phi \text{ and } \mathfrak{M}, g, w \models \psi \\
\mathfrak{M}, g, w \models \phi \to \psi & \text{iff} & \mathfrak{M}, g, w \models \phi \text{ implies } \mathfrak{M}, g, w \models \psi \\
\mathfrak{M}, g, w \models \bot & \text{iff} & \text{falsum} \\
\mathfrak{M}, g, w \models \Box\phi & \text{iff} & \text{for any } v \in W \text{ such that } wRv,\ \mathfrak{M}, g, v \models \phi \\
\mathfrak{M}, g, w \models @_a\phi & \text{iff} & \mathfrak{M}, g, g(a) \models \phi \\
\mathfrak{M}, g, w \models \forall a\phi & \text{iff} & \text{for any } g' \overset{a}{\sim} g,\ \mathfrak{M}, g', w \models \phi \\
\mathfrak{M}, g, w \models \downarrow a\phi & \text{iff} & \mathfrak{M}, g', w \models \phi \text{ where } g' \overset{a}{\sim} g \text{ and } g'(a) = w
\end{array}
$$

A formula ϕ is said to be *true* at w if $\mathfrak{M}, g, w \models \phi$; otherwise it is said to be *false* at w. By convention $\mathfrak{M}, g \models \phi$ means $\mathfrak{M}, g, w \models \phi$ for every element w of W and $\mathfrak{M} \models \phi$ means $\mathfrak{M}, g \models \phi$ for every assignment g. A formula ϕ is *valid* in a frame if and only if $\mathfrak{M} \models \phi$ for any model $\mathfrak{M}$ that is based on the frame. A formula ϕ is *valid* in a class of frames if and only if ϕ is valid in any frame in the class of frames in question. A formula ϕ is *valid* if and only if ϕ is valid in the class of all frames.

Now, let $\mathcal{O} \subseteq \{\downarrow, \forall\}$. In what follows $\mathcal{H}(\mathcal{O})$ denotes the fragment of hybrid logic in which the only binders are the binders in the set $\mathcal{O}$. If $\mathcal{O} = \emptyset$, then we simply write $\mathcal{H}$, and if $\mathcal{O} = \{\downarrow\}$, then we write $\mathcal{H}(\downarrow)$, etc. It is assumed that the set $\mathcal{O}$ of binders is fixed.

Note that $\downarrow$ is definable in terms of $\forall$ since the formula $\downarrow a\phi \leftrightarrow \forall a(a \to \phi)$ is valid. The fact that hybridizing ordinary modal logic actually does give more expressive power can for example be seen by considering the formula $\downarrow c\Box\neg c$. It is straightforward to check that this formula is valid in a frame if and only if the frame is irreflexive. Thus, irreflexivity can be expressed by a hybrid-logical formula, but it is well known that it cannot be expressed by any formula of ordinary modal logic. Irreflexivity can actually be expressed just by adding nominals to ordinary modal logic, namely by the formula $c \rightarrow \Box\neg c$. It is clear that if a frame is irreflexive, then $c \rightarrow \Box\neg c$ is valid in the frame. On the other hand, if $c \rightarrow \Box\neg c$ is valid in a frame, then the frame is irreflexive: Let (W,R) be a frame in which $c \rightarrow \Box\neg c$ is valid and let w be an element of W, then $\mathfrak{M}, g, w \models c \rightarrow \Box\neg c$ where $\mathfrak{M}$ is an arbitrarily chosen model based on (W,R) and g is an arbitrarily chosen assignment such that $g(c) = w$, and from this it follows that wRw is false. Hence, the formula $c \rightarrow \Box\neg c$ expresses irreflexivity. Other examples of properties expressible in hybrid logic, but not in ordinary modal logic, are asymmetry (expressed by $c \rightarrow \Box\neg\Diamond c$), antisymmetry (expressed by $c \rightarrow \Box(\Diamond c \rightarrow c)$), and universality (expressed by $\Diamond c$).

1.1.3 Translation into First-Order Logic

Hybrid logic can be translated into first-order logic with equality and (a fragment of) first-order logic with equality can be translated back into (a fragment of) hybrid logic. The translation from hybrid logic into first-order logic we consider in this subsection is an extension of the well-known *standard translation* from modal logic into first-order logic (see Areces et al. 2001a and van Benthem 1983).

The first-order language under consideration has a 1-place predicate symbol corresponding to each ordinary propositional symbol of modal logic, a 2-place predicate symbol corresponding to the modality, and a 2-place predicate symbol corresponding to equality. The language does not have constant or function symbols. It is assumed that a countably infinite set of first-order variables is given. The metavariables a, b, c, ... range over first-order variables. There are no function symbols or constants. So the formulas of the first-order language we consider are defined by the grammar

$$S ::= p^*(a) \mid R(a,b) \mid a = b \mid S \wedge S \mid S \rightarrow S \mid \bot \mid \forall a S$$

where p ranges over ordinary propositional symbols of hybrid logic, and a and b range over first-order variables. Note that according to the grammar above, for each ordinary propositional symbol p of the modal language there is a corresponding 1-place predicate symbol p^* in the first-order language. The predicate symbol p^* will be interpreted such that it relativises the interpretation of the corresponding modal propositional symbol p to worlds. In the grammar above, R is a designated predicate symbol which will be interpreted using the accessibility relation (with the same name). In what follows, we shall identify first-order variables with nominals of hybrid logic. Note in this connection that the set of metavariables ranging over first-order variables is identical to the set of metavariables ranging over nominals. Free and bound occurrences of variables are defined as usual for first-order logic.

Also, $\psi[c/a]$ is the formula ψ where the variable c has been substituted for all free occurrences of the variable a. As usual, if the variable a occurs free in ψ within the scope of $\forall c$, then the variable c in ψ is renamed as appropriate. It is assumed that a does not occur free in ψ within the scope of $\forall c$. The connectives $\neg$, $\top$, $\vee$, $\leftrightarrow$, and $\exists$ are defined in one of the usual ways.

We first translate the hybrid logic $\mathcal{H}(\downarrow,\forall)$ into first-order logic with equality. It is assumed that two nominals a and b are given which do not occur in the formulas to be translated. The translations ST_a and ST_b are defined by mutual induction. We just give the translation ST_a.

$$\begin{aligned}
ST_a(p) &= p^*(a)\\
ST_a(c) &= a = c\\
ST_a(\phi \wedge \psi) &= ST_a(\phi) \wedge ST_a(\psi)\\
ST_a(\phi \rightarrow \psi) &= ST_a(\phi) \rightarrow ST_a(\psi)\\
ST_a(\bot) &= \bot\\
ST_a(\Box\phi) &= \forall b(R(a,b) \rightarrow ST_b(\phi))\\
ST_a(@_c\phi) &= ST_a(\phi)[c/a]\\
ST_a(\forall c\phi) &= \forall c ST_a(\phi)\\
ST_a(\downarrow c\phi) &= ST_a(\phi)[a/c]
\end{aligned}$$

The definition of ST_b is obtained by exchanging a and b. As an example, we demonstrate step by step how the hybrid-logical formula $\downarrow c\Box\neg c$ is translated into a first-order formula:

$$\begin{aligned}
ST_a(\downarrow c\Box\neg c) &= ST_a(\Box\neg c)[a/c]\\
&= \forall b(R(a,b) \rightarrow ST_b(\neg c))[a/c]\\
&= \forall b(R(a,b) \rightarrow \neg ST_b(c))[a/c]\\
&= \forall b(R(a,b) \rightarrow \neg b = c)[a/c]\\
&= \forall b(R(a,b) \rightarrow \neg b = a)
\end{aligned}$$

The resulting first-order formula is equivalent to $\neg R(a,a)$ which shows that $\downarrow c\Box\neg c$ indeed does correspond to the accessibility relation being irreflexive, cf. above. What has been done in the translation is that the semantics of hybrid logic has been formalised in terms of first-order logic; note how each clause in the translation formalizes a clause in the definition of the semantics, that is, the relation $\mathfrak{M}, g, w \models \phi$.

The translation ST_a is truth-preserving. To state this formally, we make use of the well-known observation that a model for hybrid logic can be considered as a model for first-order logic and vice versa.

Definition 1.2. Given a model $\mathfrak{M} = (W, R, \{V_w\}_{w\in W})$ for hybrid logic, a model $\mathfrak{M}^* = (W, V^*)$ for first-order logic is defined by letting

- $V^*(p^*) = \{w \mid V_w(p) = 1\}$ and
- $V^*(R) = R$.

It is straightforward to see that the map $(\cdot)^*$ which maps $\mathfrak{M}$ to $\mathfrak{M}^*$ is bijective. Moreover, an assignment in the sense of classical hybrid logic can be considered as an assignment in the sense of classical first-order logic and vice versa.

Given a model $\mathfrak{M}$ for first-order logic, the relation $\mathfrak{M}, g \models \phi$ is defined by induction in the standard way, where g is an assignment and ϕ is a first-order formula.

$$
\begin{array}{rll}
\mathfrak{M}, g \models p^*(a) & \text{iff} & g(a) \in V(p^*) \\
\mathfrak{M}, g \models R(a,b) & \text{iff} & g(a)V(R)g(b) \\
\mathfrak{M}, g \models a = b & \text{iff} & g(a) = g(b) \\
\mathfrak{M}, g \models \phi \wedge \psi & \text{iff} & \mathfrak{M}, g \models \phi \text{ and } \mathfrak{M}, g \models \psi \\
\mathfrak{M}, g \models \phi \rightarrow \psi & \text{iff} & \mathfrak{M}, g \models \phi \text{ implies } \mathfrak{M}, g \models \psi \\
\mathfrak{M}, g \models \bot & \text{iff} & \text{falsum} \\
\mathfrak{M}, g \models \forall a \phi & \text{iff} & \text{for any } g' \stackrel{a}{\sim} g, \mathfrak{M}, g' \models \phi
\end{array}
$$

The formula ϕ is said to be *true* if $\mathfrak{M}, g \models \phi$; otherwise it is said to be *false*. By convention $\mathfrak{M} \models \phi$ means $\mathfrak{M}, g \models \phi$ for every assignment g. We shall later make use of the first-order semantics in connection with the interpretation of geometric theories.

It can now be stated formally that the translation is truth-preserving.

Proposition 1.3. *Let $\mathfrak{M}$ be a model for hybrid logic and let ϕ be a hybrid-logical formula in which the nominals a and b do not occur. For any assignment g, it is the case that $\mathfrak{M}, g, g(a) \models \phi$ if and only if $\mathfrak{M}^*, g \models ST_a(\phi)$ (and the same for ST_b).*

Proof. Induction on the structure of ϕ. ■

Thus, hybrid logic, considered as a language for talking about models, has the same expressive power as the fragment of first-order logic obtained by taking the image of hybrid logic under the translation ST_a.

First-order logic with equality can be translated into the hybrid logic $\mathcal{H}(\forall)$ by the translation HT given below.

$$
\begin{aligned}
HT(p^*(a)) &= @_a p \\
HT(R(a,c)) &= @_a \Diamond c \\
HT(a = c) &= @_a c \\
HT(\phi \wedge \psi) &= HT(\phi) \wedge HT(\psi) \\
HT(\phi \rightarrow \psi) &= HT(\phi) \rightarrow HT(\psi) \\
HT(\bot) &= \bot \\
HT(\forall a \phi) &= \forall a HT(\phi)
\end{aligned}
$$

The translation HT is truth-preserving.

Proposition 1.4. *Let $\mathfrak{M}$ be a model for hybrid logic. For any first-order formula ϕ and any assignment g, it is the case that $\mathfrak{M}^*, g \models \phi$ if and only if $\mathfrak{M}, g \models HT(\phi)$.*

Proof. Induction on the structure of ϕ. ■

Thus, in the sense above the hybrid logic $\mathcal{H}(\forall)$ has the same expressive power as first-order logic with equality. It is implicit in the proposition above that the first-order formula ϕ is a formula of the first-order language defined by the grammar given earlier in the previous subsection. The history of the above observations goes back to the work of Arthur Prior, which we shall come back to in the next section.

In a way similar to the above translation, a fragment of first-order logic with equality which is called the *bounded fragment* can be translated into the hybrid logic $\mathcal{H}(\downarrow)$. This was pointed out in Areces et al. (2001a). The bounded fragment is obtained from the above grammar for first-order logic by replacing the clause $\forall aS$ by the new clause $\forall c(R(a,c) \rightarrow S$ where it is required that the variables a and c are distinct. In Areces et al. (2001a) a number of independent semantic characterizations of the bounded fragment are given. A translation from the bounded fragment to the hybrid logic $\mathcal{H}(\downarrow)$ can be obtained by replacing the last clause in the translation HT above by the following.

$$HT(\forall c(R(a,c) \rightarrow \phi)) = @_a \Box \downarrow c HT(\phi)$$

It is straightforward to check that Proposition 1.4 still holds, hence, the hybrid logic $\mathcal{H}(\downarrow)$ has the same expressive power as the bounded fragment of first-order logic (note that for any formula ϕ of $\mathcal{H}(\downarrow)$, the formula $ST_a(\phi)$ is in the bounded fragment).

1.2 The Origin of Hybrid Logic in Prior's Work

In this section we discuss the work of Arthur Prior, and we describe how hybrid logic has its origin in his work. The precise origin of hybrid logic is Prior's hybrid tense logic, which is a hybridized version of ordinary tense logic. Arthur Prior (1914–1969) is usually considered the founding father of modern temporal logic, his main contribution being the formal logic of tenses. In his memorial paper on Prior (Kenny 1970), A.J.P. Kenny summed up Prior's life and work as follows.

> Prior's greatest scholarly achievement was undoubtedly the creation and development of tense-logic. But his research and reflection on this topic led him to elaborate, piece by piece, a whole metaphysical system of an individual and characteristic stamp. He had many different interests at different periods of his life, but from different angles he constantly returned to the same central and unchanging themes. Throughout his life, for instance, he worked away at the knot of problems surrounding determinism: first as a predestinarian theologian, then as a moral philosopher, finally as a metaphysician and logician (Kenny 1970, p. 348).

Prior's reflections on determinism and other issues related to the philosophy of time were a major motivation for his formulation of tense logic. With the aim of discussing tense logic and hybrid tense logic further, we shall give a formal definition of hybrid tense logic: The language of hybrid tense logic is simply the language of hybrid logic defined above except that there are two modal operators, namely G and H, instead of the single modal operator $\Box$. The two new modal operators are called *tense operators*. The semantics of hybrid tense logic is the semantics of hybrid logic, cf. earlier, with the clause for $\Box$ replaced by clauses for the tense operators G and H.

$$
\begin{array}{lll}
\mathfrak{M}, g, w \models G\phi & \text{iff} & \text{for any } v \in W \text{ such that } wRv,\ \mathfrak{M}, g, v \models \phi \\
\mathfrak{M}, g, w \models H\phi & \text{iff} & \text{for any } v \in W \text{ such that } vRw,\ \mathfrak{M}, g, v \models \phi
\end{array}
$$

Thus, there are now two modal operators, namely one that "looks forwards" along the accessibility relation R and one that "looks backwards". In tense logic the elements of the set W are called *moments* or *instants* and the accessibility relation R is now also called the *earlier-later relation*.

It is straightforward to modify the translations ST_a and HT in the previous section such that translations are obtained between a tense-logical version of $\mathcal{H}(\forall)$ and first-order logic with equality. The first-order logic under consideration is what Prior called *first-order earlier-later logic*. Given the translations, it follows that Prior's first-order earlier-later logic has the same expressive power as the tense-logical version of $\mathcal{H}(\forall)$, that is, hybrid tense-logic.

Now, Prior introduced hybrid tense logic in connection with what he called four grades of tense-logical involvement. The four grades were presented in the book Prior (1968), Chapter XI (also Chapter XI in the new edition (Hasle et al. 2003)). Moreover, see the book Prior (1967), Chapter V.6 and Appendix B.3–4. For a more general discussion of the four grades, see the posthumously published book Fine and Prior (1977). The stages progress from pure first-order earlier-later logic to what can be regarded as a pure tense logic, where the second grade is a "neutral" logic encompassing first-order earlier-later logic and tense logic on the same footing. The motivation for Prior's four grades of tense-logical involvement was philosophical. Prior considered instants to be "artificial" entities which due to their abstractness should not be taken as primitive concepts.

> …my desire to sweep 'instants' under the metaphysical table is not prompted by any worries about their punctual or dimensionless character but purely by their abstractness. … 'instants' as literal objects, or as cross-sections of a literal object, go along with a picture of 'time' as a literal object, a sort of snake which either eats its tail or doesn't, either has ends or doesn't, either is made of separate segments or isn't; and this picture I think we must drop (Prior 1967, p. 189).

Given the explicit reference to instants in first-order earlier-later logic, Prior found that first-order earlier-later logic gives rise to undesired ontological import. Instead of first-order earlier-later logic, he preferred tense logic.

> Some of us at least would prefer to see 'instants', and the 'time-series' which they are supposed to constitute, as mere logical constructions out of tensed facts (Hasle et al. 2003, p. 120).

This is why Prior's goal was to extend tense logic such that it could be considered as encompassing first-order earlier-later logic. Technically, the goal was to extend tense logic such that first-order earlier-later logic could be translated into it. It was with this goal in mind Prior introduced what he called *instant-propositions*.

> What I shall call the third grade of tense-logical involvement consists in treating the instant-variables a, b, c, etc. as also representing propositions (Hasle et al. 2003, p. 124).

In the context of modal logic, Prior called such propositions *possible-world-propositions*. Of course, this is what we here call nominals. Prior also introduced the binder $\forall$ and what we here call satisfaction operators (he used the notation $T(a,\phi)$ instead of $@_a\phi$ for satisfaction operators). The extended tense-logic thus obtained is the logic he called third grade tense logic, hence, the third grade tense logic is identical to the tense-logical version of $\mathcal{H}(\forall)$, hybrid tense logic, which has the same expressive power as first-order earlier-later logic, as remarked above.

Prior gave an alternative, but equivalent, formulation of the third grade tense logic in which the satisfaction operator is replaced by a modal operator A called the *universal* modality (some authors call it the *global* modality). The universal modality has a fixed interpretation: The truth-condition is that a formula $A\phi$ is true (at any world) if and only if the formula ϕ is true at all worlds. Thus, the universal modality is interpreted using the universal binary relation. Formally, the clause for the satisfaction operator in the semantics is replaced by a clause for the modal operator A.

$$\mathfrak{M}, g, w \models A\phi \quad \text{iff} \quad \text{for any } v \in W,\ \mathfrak{M}, g, v \models \phi.$$

Thus, besides the tense operators G and H, the language under consideration here also contains the modal operator A. The two formulations of the third degree are equivalent since the satisfaction operator and the universal modality are interdefinable in the presence of nominals and the $\forall$ binder, this being the case as the formulas $A\phi \leftrightarrow \forall a(@_a\phi)$ and $@_a\phi \leftrightarrow A(a \rightarrow \phi)$ are valid.

Prior's fourth grade tense logic is obtained from the third grade tense logic by replacing the satisfaction operator (or the universal modality in the alternative formulation of the third grade) by a defined modal operator L such that

$$\mathfrak{M}, g, w \models L\phi \quad \text{iff} \quad \text{for any } v \in W \text{ such that } wR^*v,\ \mathfrak{M}, g, v \models \phi$$

where the binary relation R^* is the reflective, symmetric, and transitive closure of the earlier-later relation R. Prior considered two ways to define the operator L in what he took to be purely tense-logical terms. In the first case he allowed what amounts to infinite conjunctions of formulas. If infinite conjunctions are allowed, the operator L can be defined by the conventions that

$$L\phi = L^0\phi \wedge L^1\phi \wedge \ldots$$

and

$$\begin{aligned} L^0\phi &= \phi \\ L^{n+1}\phi &= GL^n\phi \wedge HL^n\phi \end{aligned}$$

Note that for any given natural number k, $L^k\phi$ is a formula in the object language (which does not involve natural numbers). For example, if $k = 1$ and $\phi = p$, then $L^1\phi = Gp \wedge Hp$. In the second case Prior assumed time to have a structure making $L\phi$ equivalent to

$$L^0\phi \wedge L^1\phi \wedge \ldots \wedge L^k\phi$$

for some fixed natural number k whereby infinite conjunctions are avoided. If for example time is linear, that is, transitive, backwards linear, and forwards linear, then $k = 1$ will do. If time is branching, that is, transitive and backwards linear, then $k = 2$ will do. In whichever way the operator L is defined, the fourth grade tense logic has the same expressive power as first-order earlier-later logic if it is assumed that the time-series is unique, that is, if it is assumed that any two instants are connected by some number of steps in either direction along the earlier-later relation R. For Prior it was natural to assume that the time-series is unique, as is witnessed by the following quotation.

> For is not the question as to whether 'our' time-series (whatever its structure) is unique, a genuine one? I would urge the following consideration against saying that it is, or at all events against saying it too hurriedly: It is only if we have a more-or-less 'Platonistic' conception of what a time-series is, that we can raise this question. If, as I would contend, it is only by tensed statements that we can give the cash-value of assertions which purport to be about 'time', the question as to whether there are or could be unconnected time-series is a senseless one. We think we can give it a sense because it is as easy to draw unconnected lines and networks as it is to draw connected ones; but these diagrams cannot represent *time*, as they cannot be translated into the basic non-figurative temporal language (Prior 1967, pp. 198–199).

The reason why the fourth grade tense logic has first-order expressive power when the time-series is unique, is that the fourth-grade modality L then has the same effect as the universal modality A which is used in (the alternative formulation of) the third-grade logic, and the third-grade logic has first-order expressive power, as we argued above. This is discussed in more detail in the paper Braüner (2002b).

To sum up, Prior obtained tense logics having the same expressive power as first-order earlier-later logic, namely the third and fourth grade tense logics, by adding to ordinary tense logic further expressive power in the form of hybrid-logical machinery (and in the case of the fourth grade tense logic by making appropriate assumptions about the structure of time, including an assumption that the time-series is unique). So Prior clearly reached his technical goal. Prior also found that he reached his philosophical goal, namely that of avoiding an ontology including instants.

> The 'entities' which we 'countenance' in our 'ontology' ... depend on what variables we take seriously as individual variables in a first-order theory, i.e. as subjects of predicates rather than as *assertibilia* which may be qualified by modalities. If we prefer to handle instant-variables, for example, or person-variables, as subjects of predicates, then we may be taken to believe in the existence of instants, or of persons. If, on the other hand, we prefer to treat either of these as *propositional* variables, i.e. as arguments of truth-functions and of modal functions, then we may be taken as *not* believing in the existence of instants, etc. (they don't exist; rather, they are or are not the case) (Hasle et al. 2003, p. 220).

However, it has been debated whether or not Prior managed to avoid an instant ontology. We shall return to this later in Sect. 1.2.1 (where we also return to the person-variables mentioned in the quotation above).

The discussion on Prior's third grade tense logic and first-order earlier-later logic is closely related to the discussion on two different conceptions of time, namely the *A-series* and *B-series* conceptions, a terminology introduced in 1908 by the philosopher McTaggart (cf. McTaggart 1908). According to the A-series conception, also called the *dynamic* view, the past, present, and future tenses are primitive concepts from which other temporal concepts, in particular instants and the earlier-later relation, are to be derived. On the other hand, according to the B-series conception, also called the *static* view, instants and the earlier-later relation are primitive. The A-series conception embodies the local way in which human beings experience the flow of time whereas the B-series conception embodies a Gods-eye-view of time, where time is a sequence of objectively and tenselessly existing instants. It is notable that representations of both the A-series and B-series conceptions can be found in natural language (the A-series conception of course in the form of tense inflection of verbs and the B-series conception in particular in the form of nominal constructions like "5 o'clock 10 May 2007"). Of course, first-order earlier-later logic is associated with the B-series conception and Prior's third grade tense logic is associated with the A-series conception, which was Prior's own view, as succinctly expressed in the following quotation.

> So far, then, as I have anything that you could call a philosophical creed, its first article is this: I believe in the reality of the distinction between past, present, and future. I believe that what we see as a progress of events *is* a progress of events, a *coming to pass* of one thing after another, and not just a timeless tapestry with everything stuck there for good and all (Prior 1996, p. 47).

The discussion of A-series and B-series is reflected in discussions of time in Artificial Intelligence, see the paper Galton (2006). The paper by Patrick Blackburn (2006) discusses all the above issues as well as a number of other issues in hybrid logic and their origin in Prior's work. The above issues are also discussed in many papers of the collection (Copeland 1996), in particular in Richard Sylvan's paper (1996). See the paper Øhrstrøm and Hasle (1993), the book Øhrstrøm and Hasle (1995), and the handbook chapters Øhrstrøm and Hasle (2005b) and Øhrstrøm and Hasle (2005a) for general accounts of Prior's work. See also the encyclopedia article Copeland (2007). A recent assessment of Prior's philosophical and logical views can be found in Müller (2007).

1.2.1 *Did Prior Reach His Philosophical Goal?*

It has been debated whether Prior reached his philosophical goal with the third and fourth grade logics, namely that of avoiding an ontology including instants.