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Takahiro Sagawa

Thermodynamics of Information Processing in Small Systems

Doctoral Thesis accepted by
The University of Tokyo, Japan

 Springer

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Chairman's Foreword

The origins of the Physics Department of the University of Tokyo may be found in “Bansho- Kurabesho” founded in the Edo era by the Tokugawa Shogunate in 1857. After the Meiji Restoration in 1868, this educational institution was taken over by the Meiji government, and was renamed “Kaisei Gakko”. It was in 1875 that a Physics Department opened at Tokyo Kaisei Gakko, which was then integrated into the first university in Japan, the University of Tokyo, founded in 1877. When the University of Tokyo issued its first bachelor degrees in 1879, 12 of the 55 recipients were from the Department of Physics.

Since those early days, the University of Tokyo has grown to acquire a reputation as one of the foremost institutions of tertiary education and scholarship in the world. And the Physics Department's successive generations of faculty and students who engaged in cutting-edge research and pioneering education, have played no small role in this transformation. Of the University's seven Nobel laureates, three are physicists who graduated from the Department, Leona Esaki (awarded 1973), Masatoshi Koshihara (awarded 2002), and Yoichiro Nambu (awarded 2008). The first Fields Medalist from Japan, Kunihiko Kodaira (awarded 1954) was also an alumnus of the Department. Today our staff continue this tradition, engaging in essential research in all aspects of physics, and working to shape the budding next generation of physicists.

It was a great pleasure to learn that our Department of Physics was invited, as one of the top universities in the world for physics, to participate in the Springer Theses initiative. Our first selections are three students among the 51 who wrote doctoral theses in English accepted by our Department in the Fiscal Year 2010. They are Dr. Takahiro Sagawa, Dr. Takayuki Yuasa, and Dr. Takuya Kanazawa. I am happy to express my heartfelt congratulations to these distinguished authors.

Prof. Ryugo Hayano
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Supervisor's Foreword

The second law of thermodynamics presupposes a clear-cut distinction between the controllable and uncontrollable degrees of freedom by means of macroscopic operations. The cutting-edge technologies in quantum information and nanoscience seem to require us to abandon such a working hypothesis in favor of the distinction between the accessible and inaccessible degrees of freedom. In his thesis, Takahiro Sagawa discusses the implications of this paradigm shift by addressing the question of how the second law of thermodynamics can be generalized in the presence of a feedback controller, and investigates the minimum work required for measurement and erasure of information.

Ever since the seminal discussion by James Clark Maxwell about an intelligent being presently known as Maxwell's demon, the relationship between thermodynamics and information has attracted much attention because it concerns the foundation of the second law of thermodynamics. In recent years, this relationship has attracted renewed interest because of the advances in nanotechnology which have enabled one to access atomic-scale objects in an controllable manner. Here, Maxwell's demon plays the role of a feedback controller that intervenes between a microscopic object and a macroscopic one. In modern terms, Maxwell's demon is formulated as an information processing device that performs measurement and feedback at the level of thermal fluctuations. By unifying information theory, measurement theory, and the recently developed theory of nonequilibrium statistical mechanics, Takahiro constructs theory of "information thermodynamics," in which information contents and thermodynamic variables are treated on an equal footing. Based on this theory, Takahiro successfully determines the maximum work that can be extracted by the demon and the minimum work that is needed for measurement and information erasure by the demon.

Because information processing is necessarily out of equilibrium, it is an interesting question to ask whether the Jarzynski equality can be generalized in the presence of feedback control and if so how it should be modified. Takahiro has

answered this question in the affirmative and generalized several nonequilibrium relations for classical stochastic systems. One of the generalized equalities has recently been verified experimentally by using sub-micron colloidal particles.

Thus, the work presented in this thesis establishes fundamental principles for information processing in small thermodynamic systems, and I expect that the obtained relations can be applied to nanomachines and nanodevices.

Tokyo, March 2012

Masahito Ueda

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