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Enrico Marcantoni

Collateralized Debt Obligations

*A Moment Matching Pricing
Technique based on Copula Functions*



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A Moment Matching Pricing Tech-
nique based on Copula Functions

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Foreword

The thesis deeply analyzes the existing approaches to price CDOs concerning the modeling of the individual default probability, the loss given default and particularly the default correlation between obligors. The most common approaches to recover the default correlation are represented by the reduced approach, where the dependencies of obligors are written in term of default intensities, and the structured approach based on the seminal paper by Merton (1974). Here the default correlations are expressed through the dependencies of the assets into the firm's portfolio on some common factors which allow to consider the granularity and the sectorial concentration and contagion risk. Obviously as the number of common factors increases, the valuation becomes more precise but the complexity of the model considerably increases. For this reason the reference paper of this thesis is Castagna et al. (2012) where, in a structured multi-factor model, they provide a closed-form for pricing CDO based on a Moment Matching technique.

However the original work of Castagna et al (2012) models the default correlation through a multivariate Gaussian copula function which does not allow to consider more general asymmetric dependency structures with tail dependencies; then the model has been rewritten here in an Archimedean framework such to propose a more general representation of the dependencies. Moreover the contribution of this thesis is not only theoretical because here for the first time the Moment Matching technique has been implemented to price a CDO. The data refer to a CDX composed by 125 names whose CDSs' quotes were collected on July 3rd 2007. As the default probabilities have been bootstrapped from the CDSs' prices with maturity 10 years, based on these marginal probabilities, a clustering approach allows to reduce the complexity in the numerical computation of the moments of the original distribution of the portfolio loss. The idea of the Moment Matching technique is to calibrate the moments of the original distribution on the moments of an approximated one, assumed to be the well known Vasicek's limiting model here, such to recover the parameters of the approximated distributions and then the tranches' price. It's very interesting to point out that the moment expansion that is very useful for pricing purposes, allows also to identify

the potential risk sources entering in the pricing process and to construct an hedging portfolio to cover them.

Bologna, October 17, 2013

Prof. Silvia Romagnoli

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