Springer Series on Atomic, Optical, and Plasma Physics 69

Andrey V. Korol
Andrey V. Solov'yov
Walter Greiner

Channeling and Radiation in Periodically Bent Crystals

Second Edition

Springer

Springer Series on Atomic, Optical, and Plasma Physics

Volume 69

For further volumes:
http://www.springer.com/series/411

The Springer Series on Atomic, Optical, and Plasma Physics covers in a comprehensive manner theory and experiment in the entire field of atoms and molecules and their interaction with electromagnetic radiation. Books in the series provide a rich source of new ideas and techniques with wide applications in fields such as chemistry, materials science, astrophysics, surface science, plasma technology, advanced optics, aeronomy, and engineering. Laser physics is a particular connecting theme that has provided much of the continuing impetus for new developments in the field, such as quantum computation and Bose-Einstein condensation. The purpose of the series is to cover the gap between standard undergraduate textbooks and the research literature with emphasis on the fundamental ideas, methods, techniques, and results in the field.

Andrey V. Korol · Andrey V. Solov'yov
Walter Greiner

Channeling and Radiation in Periodically Bent Crystals

Second Edition

Springer

Andrey V. Korol
Andrey V. Solov'yov
Physics Department
Goethe Universität,
Frankfurt amd Main
Frankfurt
Germany

Walter Greiner
Frankfurt Institute for Advanced Studies
Goethe Universität
Frankfurt
Germany

ISSN 1615-5653 ISSN 2197-6791 (electronic)
ISBN 978-3-642-54932-8 ISBN 978-3-642-54933-5 (eBook)
DOI 10.1007/978-3-642-54933-5
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2014937694

1st Edition: © Springer-Verlag Berlin Heidelberg 2013

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

Development of coherent radiation sources for a wavelength λ below 1 angstrom (i.e., in the hard X-ray and gamma ray ranges) is a challenging goal of modern physics. Sub-angstrom wavelength powerful spontaneous and, especially, coherent radiation will have many applications in basic science, technology and medicine. In particular, they may have a revolutionary impact on nuclear and solid-state physics as well as on life sciences.

The present state-of-the-art lasers are capable of emitting electromagnetic radiation from the infrared to ultraviolet range of the spectrum. Currently, there is one Free-Electron Laser (FEL) operating in the X-ray range ($\lambda \approx 1\,\text{Å}$) [99]. Several other FEL X-ray facilities are either under construction or undergoing advanced technical design work. Moving further, i.e., into the hard X-ray or/and gamma ray band, is not possible without new approaches and technologies.

In this book we present and discuss one of such novel approaches. The main phenomenon addressed is the radiation formed in a *Crystalline Undulator*. In this device, the electromagnetic radiation is generated by a bunch of ultra-relativistic particles channeling through a periodically bent crystalline structure. Such a system becomes a source of intensive spontaneous monochromatic radiation and, under certain conditions, also a source of the laser light. A laser based on the crystalline undulator could produce photons with $\lambda = 0.01$–$0.1\,\text{Å}$ (the corresponding photon energy range is from tens to hundreds of keV up to MeV region). Thus, its photon energy range starts where conventional FEL devices tail-off.

The feasibility of constructing a crystalline undulator is a very recent concept. The aim of this book is to represent the underlying fundamental physical ideas as well as the theoretical, experimental and technological advances made during the last one and a half decades in exploring the various features of crystalline undulators and the radiation formed in them. The book is addressed to a wide audience of researches and students since the phenomenon of crystalline undulator entangles the concepts from various research fields, such as material science, beam physics, physics of radiation, solid-state physics, acoustics, etc., whereas its investigation implies the use and further elaboration of a variety of theoretical and computational methods, experimental techniques, and technological and engineering approaches.

We are grateful to Andriy Kostyuk, Wolfram Krause and Mehdi Tabrizi together with whom many of the presented theoretical results were obtained. We express our gratitude to Erik Uggerhøj and Simon Connell for helpful discussions. We are indebted to Hartmut Backe, Werner Lauth and Ulrik Uggerhøj for numerous stimulating and clarifying discussions as well as for their support of our theoretical activity in the field and eagerness to carry out experimental investigations.

Financial support from Deutsche Forschungsgemeinschaft (DFG), Alexander von Humboldt Foundation and European Commission, granted to us at various stages of the research, is gratefully acknowledged.

Frankfurt, Germany

Andrey V. Korol
Andrey V. Solov'yov
Walter Greiner

Contents

Acronyms

AW	Acoustic Wave
BrS	Bremsstrahlung
BW	Band Width
ChR	Channeling Radiation
CU	Crystalline Undulator
CUL	Crystalline Undulator Laser
CUR	Crystalline Undulator Radiation
FEL	Free-Electron Laser
EM	Equations of Motion
MD	Molecular Dynamics
PBCh	Periodically Bent Channel
PBCr	Periodically Bent Crystal
SASE	Self-amplified Spontaneous Emission
UR	Undulator Radiation

Chapter 1
Introduction

Abstract The concept of a crystalline undulator as a source of high-energy electromagnetic radiation, both spontaneous and stimulated, is formulated. The distinguishing features of CU and of its radiation, the feasibility of CU and the methods of preparation of periodically bent crystalline structures are discussed in general terms.

1.1 Crystalline Undulator: Basic Ideas

The term Crystalline Undulator (CU)[1] stands for a system which consists of two essential parts. Firstly, it is a single crystal whose crystallographic planes are bent periodically. The second element of the system is the beam of ultra-relativistic charged particles undergoing channeling in the periodically bent crystal (PBCr). In such a system there appears, in addition to a well-known channeling radiation (ChR) [2], the radiation of the undulator type which is due to the periodic motion of channeling particles which follow the bending of crystallographic planes. The intensity and characteristic frequencies of the Crystalline Undulator Radiation (CUR) can be varied by changing the type of channeling particles, the beam energy, the crystal type and the parameters of periodic bending [3, 4].

The mechanism of the photon emission by means of CU is illustrated by Fig. 1.1. Short comments presented below aim at focusing on the principal features of the proposed scheme as well as on the list of relevant phenomena. At this point we do not elaborate all the important details, but do this further in the book.

The (yz)-plane in the figure is a cross section of a single crystal. The z-direction represents the cross section of a midplane of two neighbouring non-deformed crystallographic planes (not drawn in the figure) spaced by the interplanar distance d. Two sets of black circles denote the nuclei which belong to the periodically bent neighbouring planes which form a Periodically Bent Channel (PBCh). The amplitude

[1] This term was introduced but not clearly elaborated in [1].

A. V. Korol et al., *Channeling and Radiation in Periodically Bent Crystals*,
Springer Series on Atomic, Optical, and Plasma Physics 69,
DOI: 10.1007/978-3-642-54933-5_1, © Springer-Verlag Berlin Heidelberg 2014

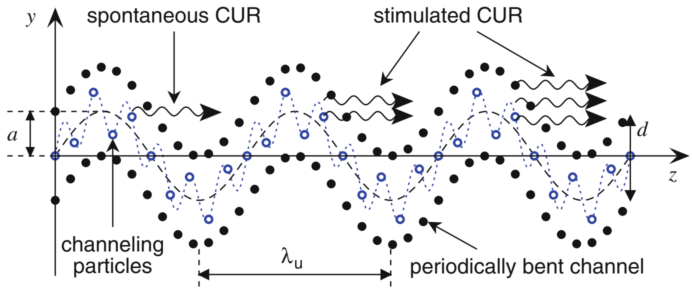

Fig. 1.1 Schematic representation of CU [4–6]. The *closed circles* mark the atoms belonging to two neighbouring crystallographic planes (separated by the interplanar distance d) which are periodically bent. The *centerline* of this channel (the *dashed line*) is described by a periodic shape function $y(z) = a \sin(2\pi z/\lambda_u)$. Its period λ_u and amplitude a satisfy the condition $\lambda_u \gg a \gg d$ (the y- and z-scales are incompatible in the figure!) Thin *dotted line* illustrates the trajectory of the particle (*open circles*), which propagates along the *centerline* (the undulator motion) and, simultaneously, undergoes channeling oscillations

of the bending, a, is defined as a maximum displacement of the deformed midplane (thick dashed curve) from the z-axis. The quantity λ_u stands for a spatial period of the bending. In principle, it is possible to consider various shapes, $y(z)$, of the periodically bent midplane. The harmonic (sinusoidal) form of this function, $y(z) = a \sin(2\pi z/\lambda_u)$, will be of a particular interest. We will call the CU, whose midplanes follow the sinusoidal profile as a *perfect* CU. For further referencing let us stress here that the main focus of this book is on the case when the quantities d, a and λ_u satisfy strong double inequality: $d \ll a \ll \lambda_u$. Typically, $d \sim 10^{-8}$ cm, $a \sim \left(10 - 10^2\right) d$, and $a \sim \left(10^{-5} - 10^{-4}\right) \lambda_u$.

Open circles in Fig. 1.1 denote the channeled ultra-relativistic particles. Initially, it was proposed to use positron beams in CU [3, 4]. Positively charged particles are repelled from the crystal nuclei and, therefore, they move between the crystal planes, where there are no atomic nuclei and the electron density is less than average. This reduces the probability of random collisions with the crystal constituents. Hence, the transverse momentum of the channeling particle increases slowly and the particle travels a longer distance in the channeling regime. In the cited papers as well as in a series of subsequent publications [6–21] the idea of this new type of radiation as well as the essential conditions and limitations which must be fulfilled to make possible the observation of the effect were formulated in a complete and adequate form for the first time. A number of corresponding novel numerical results were presented to illustrate the developed theory, including, in particular, the computation of spectral and angular distributions of CUR. (The detailed description of a positron-based CU and the CUR is given in Chaps. 4 and 5). The importance of ideas suggested and discussed in the cited papers has been also realized by other authors resulting in a significant increase of the number of publications on the properties of the positron-based CU within the last decade [22–42]. Theoretical activity in the field was accompanied by several experimental efforts for studying positron-based CU and its radiation. These were carried out in the course of the EU supported project PECU (Photon

Emission in Crystalline Undulator) [43] using the positron beam at CERN (the CERN collaboration [44]) and at DAΦNE Beam Test Facility at INFN/LNF (see review [45] on experimental aspects of CU experiments with positrons). Another experimental attempt to measure the undulator effect in positron-based CU was carried out at the Institute of High Energy Physics (Protvino, Russia). The description of the setup and of the measured data were reported in [22–24].

More recently, an electron based crystalline undulator was proposed [46–48]. On one hand, electrons are less preferable than positrons. Due to their negative charge, the electrons are attracted by the lattice ions and, therefore, are forced to oscillate around the crystal plane in the process of channeling. The probability of collisions with crystal constituents is enhanced. Thus, the dechanneling length is smaller by about two orders of magnitude in comparison to that of positrons at the same conditions. On the other hand, the electron beams are easier available and are usually of higher intensity and quality. Therefore, from the practical point of view, electron based crystalline undulator has its own advantages and deserves a thorough investigation. Experimental study of electron-based CU is currently on the way [49–52]. The overview of experimental studies of CU and CUR is given below in the book in Chap. 7.

In the case of a heavy projectile (a muon, a proton, an ion) the main restriction for the successful operation of CU is due to the photon attenuation [6, 48, 53], i.e., the decrease of the photon flux, which propagates in a crystal, due to the processes of absorption and scattering. Indeed for muon beam energies less than 100 GeV as well as for proton and ion beam energies below 1 TeV per nucleon the maximum emitted photon energy does not exceed several keV. This is exactly the energy range which characterized by a very strong absorption of the emitted photons via the atomic photoeffect. However, very recently the feasibility of a CU based on heavy particles channeling in PBCr was demonstrated for the first time [54]. It was shown that the emission within tens up to hundreds of keV range is achievable for muons of the energy starting with hundreds GeV and for proton beams within several TeV range, which are available in modern colliders [55]. The discussion of the properties of electron-based CU and that based on heavy particle channeling is presented in Chap. 6.

The operational principle of a CU does not depend on the type of a projectile. Provided certain conditions are met, the particles, injected into the crystal, will undergo channeling in PBCh [3, 4]. The trajectory of a particle contains two elements, which are illustrated by Fig. 1.1 where the thin dotted line represents the trajectory of the particle. First, there are oscillations due to the action of the interplanar force,—the *channeling oscillations* [56], whose frequency $\Omega_{\rm ch} = c\sqrt{2U'_{\max}/d\varepsilon}$ (c is the speed of light) depends on the projectile energy ε and the parameters of the channel: the maximal gradient of the interplanar potential $U'_{\max}$ and the interplanar distance d. Second, there are oscillations due to the periodicity of the bendings, the *undulator oscillations*, whose frequency is $\Omega_{\rm u} \approx 2\pi c/\lambda_{\rm u}$. The spontaneous emission is associated with both of these oscillations. The typical frequency of the ChR is $\omega_{\rm ch} \approx 2\gamma^2\Omega_{\rm ch}$ and [2, 57], where $\gamma = \varepsilon/mc^2$ is the relativistic Lorentz factor of the projectile. The undulator oscillations give rise to photons with frequency

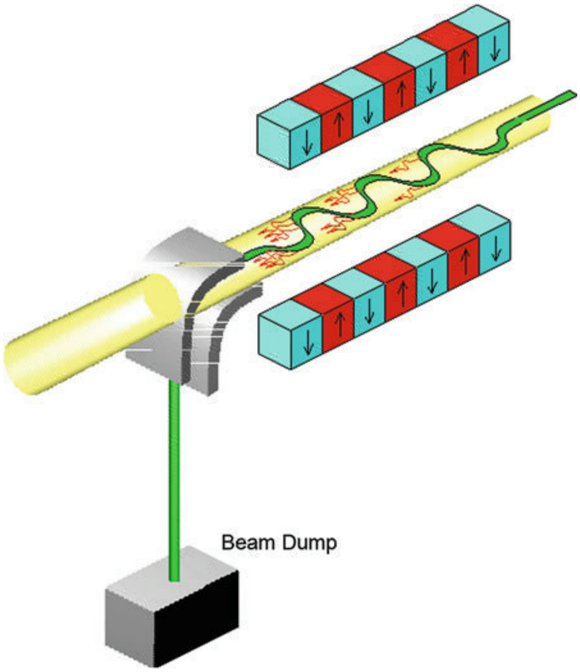

Fig. 1.2 Operational principle of a magnetic undulator (proposed by Ginzburg [58], experimental verification done by Motz et al. [63]). The beam of ultra-relativistic electrons propagates along the axis of a periodic lattice of alternating magnetic dipolar fields. The field forces the beam particles to move periodically in the transverse direction with a spatial period λ_u. As a result, the particle undulates, i.e., moves along periodic, sine-like trajectory. The periodicity of the motion gives rise to the electromagnetic radiation of a specific type, the undulator radiation (UR). Due to the interference effects the UR is emitted only at particular wavelengths, $\lambda_n = \lambda_1/n$ (where $n = 1, 2, 3 \dots$). The fundamental wavelength λ_1 is proportional to λ_u/γ, where γ is the relativistic Lorentz factor of the electron

$\omega_u \approx 4\gamma^2 \Omega_u/(2 + K^2)$, where $K = 2\pi\gamma a/\lambda_u$ is the so-called undulator parameter. If $\Omega_u \ll \Omega_{ch}$, then the frequencies of ChR and UR are well separated. In this case the characteristics of undulator radiation are practically independent on channeling oscillations [3, 4], and the operational principle of a crystalline undulator is the same as for a conventional one (see, e.g., [58–62]) in which the monochromaticity of radiation is the result of constructive interference of the photons emitted from similar parts of trajectory, see Fig. 1.2.

1.2 Why a Crystalline Undulator?

The motion of a projectile and the process of photon emission in the CU are very similar to that in an conventional undulator based on the action of periodic magnetic (or, electric) field. However, the interplanar electrostatic fields inside a crystal are so

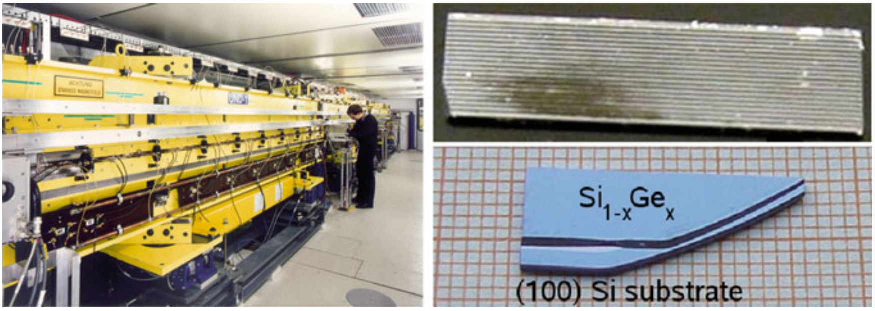

Fig. 1.3 *Left* Magnetic undulator for the X-ray laser XFEL [68]. The picture is taken from [69]. *Right top* laser-ablated diamond crystal. The crystal size is $4 \times 2 \times 0.3\,\mathrm{mm}^3$. The undulator period is $\lambda_u = 50\,\mu\mathrm{m}$ (The picture is taken from [70]). *Right bottom* a $\mathrm{Si}_{1-x}\mathrm{Ge}_x$ superlattice crystalline undulator with four periods. Periodically varied Ge content (from $x = 0$ to $x_{\max} = 0.5\,\%$) gives rise to the undulator period $\lambda_u = 50\,\mu\mathrm{m}$ (The picture is courtesy of J.L. Hansen, A. Nylandsted and U. Uggerhøj (University of Aarhus))

strong that they are able to steer the particles much more effectively than even the most advanced superconductive magnets. The electrostatic field strength is typically of the order of (10/e) GeV/cm = 10^{10} V/cm (here e stands for the elementary charge), which is equivalent to the magnetic field of approximately 3,000 T. The present state-of-the-art superconductive magnets produce the magnetic flux density of the order of 1–10 T [55] with 45 T being currently the highest value obtained by combining superconductive and resistive magnets [64]. Strong crystalline fields allow one to bring the period λ_u of bending down to the hundred or even ten micron range, which is two to five orders of magnitude smaller than the period of a conventional undulator [65–68]. As a result, the size of the undulator itself can be reduced by orders of magnitude. To illustrate this statement we present Fig. 1.3, which matches the magnetic undulator for the X-ray laser XFEL [68] with two CUs, manufactured in University of Aarhus by means of two different techniques (these will be described in Chap. 3) used in recent channeling experiments [50–52].

Apart from the 'geometrical' factor, the physical consequence of a very significant decrease in the magnitude of λ_u is that the UR, emitted in CU, has much shorter wavelength $\lambda \sim \lambda_u/2\gamma^2$, which can reach the (sub)picometer range, where conventional sources with comparable intensity are unavailable. Recently, it was demonstrated [17, 18] that the brilliance of radiation from a CU in the energy range from hundreds of keV up to tens of MeV is comparable to that of conventional light sources of the third generation [71] operating for much lower photon energies, as illustrated by Fig. 1.4 (see Sect. 5.4).

The scheme presented in Fig. 1.1 leads also to the possibility of generating a stimulated emission of the FEL type. Thus, it is meaningful to discuss a novel source of electromagnetic radiation in hard X and gamma range,—a *Crystalline Undulator Laser* (CUL) [3, 4, 6, 19, 53, 74, 76–80] The emitted radiation can be very powerful if the probability density of the particles in the beam is

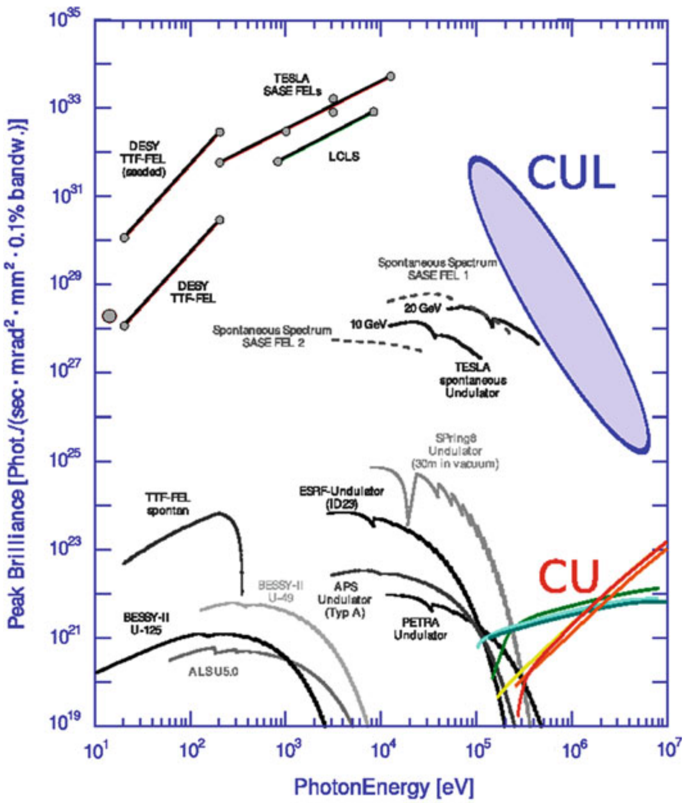

Fig. 1.4 Comparison of the peak brilliance for several modern undulators and FELs and for the CU based on different crystals (the *coloured curves* as indicated) (The data on undulators and FELs as well as general layout of the figure are taken from [72]). Peak brilliance of the spontaneous CUR was calculated [17, 18] for the KEKB positron beam and for the SLAC beam [73] (see also Sect. 5.4). The *dashed area* marks the estimation of the CUL brilliance [74] done with the parameters of the electron beam from the FLASH FEL [7]

modulated in the longitudinal direction with the period equal to the wavelength of the emitted radiation (see, e.g., [73]). In this case, the waves emitted by different particles have approximately the same phase, so that the intensity of radiation becomes proportional to the beam density squared. This increases the photon flux *by orders of magnitude* relative to the spontaneous emission from the crystalline undulator. Our estimates [74, 80] show that the brilliance of the CUL radiation can be as large as 10^{28}–10^{32} photons/s/mrad^2/mm^2/0.1 % BW in the photon energy range $\hbar\omega = 10^5$–10^6 eV (the corresponding wavelengths are 0.01–0.1 Å), see Fig. 1.4. These values of photon energies are inaccessible in conventional FELs [81]. Specific features of CUL as well as quantitative estimates of the parameters of stimulated emission are presented in Chap. 8.

1.3 Is This Realistic?

Despite the similarity of the operational principle, there are essential features which distinguish a seemingly simple scheme presented in Fig. 1.1 from a conventional undulator. In the latter the beam of particles and the photon flux move in vacuum whereas in the proposed scheme they propagate through a crystalline media.[2] The interaction of both beams with the crystal constituents makes the problem much more complicated from theoretical, experimental and technical viewpoints. Taking into consideration a number of side effects which accompany the beams dynamics, it is not at all evident a priori that the effect will not be smeared out. Therefore, to prove that the feasibility of CU as well as the radiation formed in it, it is necessary to analyze the influence, in most cases destructive, of various related phenomena. Therefore, prior to drawing a conclusion that the scheme illustrated by Fig. 1.1 is not of academic interest but can be made realistic and represent a new type of undulator, one has to understand to what extent general characteristics of UR (high intensity, high degree of monochromaticity of the spectral-angular distribution) are influenced by the presence of a crystalline media.

To fulfill this program and to establish the ranges of various parameters within which the operation of CU is feasible one has to analyze the following basic problems:

1. How to construct a periodically bent crystalline structure?
2. Which type of a projectile (positively or negatively charged, light or heavy) and which channeling regime (planar or axial) to be used, and what are the differences?
3. What are the conditions for stable channeling in a PBCr?
4. Which bending amplitudes (large or small compared to the interplanar/axial spacing) are most favourable?
5. To which extent the destructive role of dechanneling and photon attenuation influences the parameters of CU?
6. Are the energy losses of a channeling particle low enough to ensure the monochromaticity of CUR?
7. How strong is the influence of the structure imperfectness of a CU on the emission spectrum?

Most part of our book is devoted to step-by-step answering to the formulated questions. We demonstrate, that on the basis of such approach one can formulate the conditions which must be met and define the ranges of parameters of CU (the projectile energy ε, the amplitude a and the period λ_u of bending, the crystal length L, the number of undulator periods $N = L/\lambda_u$, the emitted photon energy $\hbar\omega$) within which all the criteria are fulfilled. In detail, this conditions are discussed in Chap. 4. Below we just formulate them and present a short description of the underlying physics.

---

[2] We will use the term *ideal* when addressing to an undulator in which the particles and photons propagate in vacuum.

1.3.1 Feasibility of CU

The necessary conditions, which must be met in order to treat a *CU as a feasible scheme* for devising a new source of electromagnetic radiation, are as follows (see [3–5, 7] and Sects. 4.1–4.4):

$$\begin{cases} C = 4\pi^2 \varepsilon a / U'_{\max} \lambda_u^2 < 1 & \text{– stable channeling,} \\ d < a \ll \lambda_u & \text{– large-amplitude regime,} \\ N = L/\lambda_u \gg 1 & \text{– large number of periods,} \\ L \sim \min\Big[L_d(C),\, L_a(\omega)\Big] & \text{– account for dechanneling and photon attenuation,} \\ \Delta\varepsilon/\varepsilon \ll 1 & \text{– low energy losses.} \end{cases} \tag{1.1}$$

The formulated conditions are of a general nature since they are applicable to any type of a projectile undergoing channeling in PBCr. Their application to the case of a specific projectile and/or a crystal channel allows one to analyze the feasibility of the CU by establishing the ranges of ε, a, λ_u, L, N and $\hbar\omega$ which can be achieved.

- A *stable channeling* of a projectile in a periodically bent channel occurs if the maximum centrifugal force F_{cf} is less than the maximal interplanar force $U'_{\max}$, i.e. $C = F_{cf}/U'_{\max} < 1$. Expressing F_{cf} through the energy ε of the projectile, the period and amplitude of the bending one formulates this condition as it is written in (1.1).
- The operation of a CU should be considered in the *large-amplitude regime*. The limit $a/d > 1$ accompanied by the condition $C \ll 1$ is mostly advantageous, since in this case the characteristic frequencies of UR and ChR are well separated: $\omega_u^2/\omega_{ch}^2 \sim Cd/a \ll 1$. As a result, the channeling motion does not affect the parameters the UR, the intensity of which can be comparable or higher than that of ChR. A strong inequality $a \ll \lambda_u$ ensured elastic deformation of the crystal.
- The term "undulator" implies that the *number of periods, N, is large*. Only then the emitted radiation bears the features of an UR (narrow, well-separated peaks in spectral-angular distribution, see Sect. 2.2). This is stressed by the third condition.
- A CU essentially differs from a conventional undulator, in which the beams of particles and photons move in vacuum, In CU the both beams propagate in crystalline medium and, thus, are affected by *the dechanneling and the photon attenuation*. The dechanneling effect stands for a gradual increase in the transverse energy of a channeled particle due to inelastic collisions with the crystal constituents [56]. At some point the particle gains a transverse energy higher than the planar potential barrier and leaves the channel. The average interval for a particle to penetrate into a crystal until it dechannels is called the dechanneling length, L_d. In a straight channel this quantity depends on the crystal, on the energy and the type of a projectile. In a periodically bent channel there appears an additional dependence on the parameter C. The intensity of the photon flux, which propagates through a crystal, decreases due to the processes of absorption and scattering. The interval within which the intensity decreases by a factor of e is called the attenuation length,

$L_a(\omega)$. This quantity is tabulated for a number of elements and for a wide range of photon frequencies (see, e.g., [55]).
The fourth condition in (1.1) takes into account severe limitation of the allowed values of the length L of a CU due to the dechanneling and the attenuation.

- Finally, let us comment on the last condition, which is of most importance for light projectiles, positrons and electrons. For sufficiently large photon energies ($\hbar\omega \gtrsim 10^1 - 10^2$ keV depending on the type of the crystal atom) the restriction due to the attenuation becomes less severe than due to the dechanneling effect. Then, the value of $L_{\rm d}(C)$ effectively introduces an upper limit on the length of a CU. Since for an ultra-relativistic particle $L_{\rm d} \propto \varepsilon$ (see, e.g., [82]), it seems natural that to increase the effective length one can consider higher energies. However, at this point another limitation manifests itself [7]. The coherence of UR is only possible when the energy loss $\Delta\varepsilon$ of the particle during its passage through the undulator is small, $\Delta\varepsilon \ll \varepsilon$. This statement, together with the fact, that for ultra-relativistic electrons and positrons $\Delta\varepsilon$ is mainly due to the photon emission, leads to the conclusion that L must be much smaller than the radiation length $L_{\rm r}$, the distance over which a particle converts its energy into radiation.

For a positron-based CU a thorough analysis of the system (1.1) was carried out for the first time in [3–7, 17]. Later on, the feasibility of the CU utilizing the planar channeling of electrons was demonstrated [46, 47]. Recently, similar analysis was carried out for heavy ultra-relativistic projectiles (muon, proton, ion) [54].

1.3.2 Methods of Preparation of CU

Two important issues, which are mentioned in the list of questions formulated in the beginning of this section but not answered directly by the conditions (1.1), refer to the feasibility of creating periodically bent crystalline structures and to the quality of the periodic bending. These topics will be addressed in detail in Chap. 3, Sects. 3.4 and 5.5. At this stage, we give just a general overview.

Up to now, several methods have been proposed to construct a PBCr suitable for generation of CUR.

Historically, the first proposed approach concerns the propagation of a transverse ultrasonic wave along a particular crystallographic direction [1, 3, 4, 83–87]. One of the possibilities for achieving this is in placing a piezo sample atop the crystal and generating radio frequencies to excite the oscillations. The advantage of this method is its flexibility: the bending amplitude and period can be changed by varying the intensity and frequency of the ultrasonic wave. Although the applicability of this method has not yet been checked experimentally, it does not seem unrealistic keeping in mind, that a number of experiments has been carried out on the stimulation of channeling radiation by longitudinal and transverse ultrasonic waves excited in piezoelectric crystals (see [88] and references therein).

Periodic bending of crystal channels can be achieved by using the technologies of growing $Si_{1-x}Ge_x$ mixtures [89]. For a non-varying germanium content x, which results in a constant curvature bending, the possibility of a low-energy proton beam bending by means of the $Si_{1-x}Ge_x$ was demonstrated in [89, 90]. To create a PBCh the germanium content must be varied periodically [15, 91]. A similar effect is expected to be achieved by using doped diamond super lattices [70]. The CU based on strained-layer $Si_{1-x}Ge_x$ superlattices, produced in the MBE laboratory of Aarhus (an example of such a CU is presented in Fig. 1.3, right bottom), have been used in the ongoing channeling experiments at the Mainz Microtron [50, 52].

Periodically bent crystallographic structure can be obtained by making regularly spaced trenches on the crystal surface. This can be done either mechanically by a diamond blade [23, 25, 92] or by means of laser ablation [70] (see Fig. 1.3 right top for the laser-ablated diamond crystal). The latter method is by far superior due to its reproducibility, homogeneity and accuracy. Additionally, the trench spacing can be controlled with a few-micron accuracy, which is an order of magnitude better than that achieved by the diamond-blade scratching technique. So far, the laser ablation technique was applied to sufficiently thick crystals (1 mm and beyond).

The surface stress can be created by a deposition of tensile Si_3N_4 strips onto a surface of a Si crystal [92–94]. It was demonstrated in the cited papers that this is a tractable method to construct a CU. The resulting periodic deformation is present in the bulk of the Si crystal with an essentially uniform amplitude, making the entire volume of the crystal available for channeling and in turn for emission of UR.

Usually, when discussing the properties of a CU and the radiation from it, one considers the case of a *perfect* CU. This term designates the crystal whose planes are bent periodically following a perfect harmonic shape, $y(z) = a \sin(2\pi z/\lambda_u)$, see Fig. 1.1. In this case, for each value of the emission angle the spectrum of radiation consists of a set of narrow, well-separated and powerful peaks corresponding to different harmonics of radiation. The CUs, which were used some of recent experiments [22–24, 43–45, 49, 50] were prepared by making regularly spaced grooves on the crystal surface by means the methods described above. Regular surface deformation results in the periodic bending of crystallographic planes in the bulk. The question which appears in connection with these methods concerns the quality of the periodical bending. Indeed, for a crystal of a finite thickness the surface deformations, regularly spaced with the period λ_u, result in the volume deformations of the same period but of a varied bending amplitude, $a \neq const$. The latter has the maximum value in the surface layer but decreases with the penetration distance. Therefore, it is important to carry out a quantitative analysis (a) of the structure of this *imperfect* periodic bending in the bulk, and (b) of its influence on the spectrum of CUR.

The influence of imperfect structure of a CU on the emission spectrum was analyzed recently in [21]. It was demonstrated that variation of the bending amplitude over the the crystal thickness h and the presence of harmonics with smaller bending periods both lead to a loss of monochromaticity of CUR. Typical scale, within which the parameters vary noticeably, is equal to the period of the surface deformations. One can choose either of the following two strategies to partly restore the monochromaticity. First, one can use thin crystals, $h < \lambda_u$. In this case, neither variation of the amplitude nor higher harmonics induce dramatic changes in the radiation spectrum. However, this limit corresponds to very thin crystals, if one takes into account that the period of surface deformations lies within the range $10^1 - 10^3$ μm [25, 70, 92]. The second approach prescribes the use of a thick crystal but in combination with a narrow beam of particles injected into the central part of the crystal.

Chapter 2
Related Phenomena

Abstract Brief review of the phenomena closely related to the main subject,—channeling and radiation in PBCr, is given. These include: general features of radiation by relativistic charges, specific types of electromagnetic radiation in external fields (undulator radiation, incoherent and coherent BrS), channeling in straight and bent crystals, channeling radiation.

Prior to analyzing the feasibility of CU and various aspects of the electromagnetic radiation emitted a beam of ultra-relativistic charged particles channeling in a PBCr, below in this chapter we review the phenomena closely related to the main subject. These include: general features of radiation by relativistic charges, specific types of electromagnetic radiation in external fields (undulator radiation, incoherent and coherent BrS), channeling in straight and bent crystals, channeling radiation.

We do not pretend to cover the whole range of problems concerning the mentioned phenomena but present a brief description of the effects. More detailed information on each of the discussed topics one finds in the review chapters and books cited in each section.

2.1 Radiation from Relativistic Charges: Classical, Quantum and Quasiclassical Approaches

An important issue in the study of radiation formed in a CU concerns the choice of the formalism used to describe the phenomenon. This point could have been seen as merely a technical one but it is not so. Contrary to the case of conventional undulators, based on the action of magnetic fields, the physics of CUs is, essentially, a newly arisen research field. Therefore, any theoretical study of the effect, which pretends to go a bit farther than purely academic research, must contain a great part of numerical analysis and numerical data on the basis of which real experimental investigations can be planned. In turn, to obtain the reliable data it is necessary to

A. V. Korol et al., *Channeling and Radiation in Periodically Bent Crystals*,
Springer Series on Atomic, Optical, and Plasma Physics 69,
DOI: 10.1007/978-3-642-54933-5_2,

choose a theoretical tool which allows one, on the one hand, to treat adequately all principal physical phenomena involved into the problem, and, on the other hand, to effectively carry out the corresponding numerical analysis.

In the CU problem there are three basic phenomena which must be accurately described: (a) the motion of an ultra-relativistic particle in a strong external field (the electrostatic crystalline field), (b) the process of photon emission by the particle, (c) the problem of the radiative recoil, which results in the radiative energy losses of the projectile.

2.1.1 Classical Description

In many cases, the motion of an ultra-relativistic particle, moving in an external field, can be treated, within the framework classical mechanics (see, e.g., [136, 196]). General criterion of the applicability of the classical description is in the condition that the variation of the de Bloglie wavelength $\lambda_{\mathrm{B}} = h/p$ of the projectile must be negligible over the distances of the order of λ_{B}. This condition can be written in the form (see, e.g., [198]) $m\hbar U'_{\max}/p^3 \ll 1$, where m and $p \approx \varepsilon/c$ are projectile's mass and momentum, and $U'_{\max}$ stands for the maximum gradient of the external field (i.e., the maximum force). Taking into account that $U'_{\max} \sim 10^1$–$10^2\,\mathrm{GeV}\big/\mathrm{cm}$ for an planar crystalline potential and by approximately an order of magnitude higher for an axial potential (see, e.g., [37]), one demonstrate that the condition is well-fulfilled for projectile positrons and electrons with $\varepsilon \sim 10^2\,\mathrm{MeV}$ and higher (this energy range is of prime interest in the CU problem, as it will be demonstrated below in the book).[1]

The process of photon emission can be treated classically provided the photon energy is small compared to that of a projectile: $\hbar\omega/\varepsilon \ll 1$.

Hence, if both of the mentioned conditions are met, one can calculate the spectral-angular distribution the radiated energy E using the following formula of classical electrodynamics (see, e.g., [136]):

$$\frac{\mathrm{d}^3 E}{\mathrm{d}\omega \mathrm{d}\Omega} = \frac{e^2}{c}\,\frac{q^2\omega^2}{8\pi^2}\int_0^{\tau}\mathrm{d}t_1\int_0^{\tau}\mathrm{d}t_2\,\mathrm{e}^{\mathrm{i}\omega\left(\varphi(t_1)-\varphi(t_2)\right)}\left(\frac{\mathbf{v}_1\cdot\mathbf{v}_2}{c^2}-1\right). \tag{2.1}$$

Here $\mathrm{d}\Omega$ is the solid angle in the direction $\mathbf{n}$ of the emission, q is the projectile charge in units of the elementary charge e, τ is the time of flight through a spatial domain within which the external field acts on the projectile. The quantities $\mathbf{v}_{1,2} \equiv \mathbf{v}(t_{1,2})$ stand for the projectile velocities at the instants t_1 and t_2. It is assumed that for an ultra-relativistic particle $v_{1,2} \approx c$. The function $\varphi(t)$ is defined as follows

[1] More accurately, the condition of the applicability of the classical description of the channeling motion is formulated in Sect. 2.3.4.

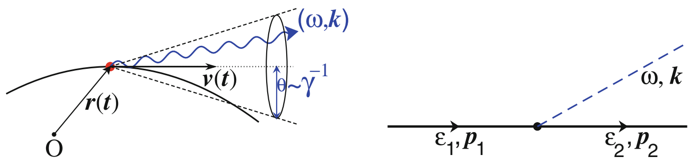

Fig. 2.1 Classical (*left*) and quantum (*right*) approaches to the radiation process. Classical ultra-relativistic charged projectile (*red dot* on *left panel*), being accelerated (decelerated) by external field, moves along a well-defined trajectory $\mathbf{r} = \mathbf{r}(t)$. The electromagnetic radiation of frequency ω and wave-vector $\mathbf{k}$ is essentially emitted within the cone $\theta \sim \gamma^{-1}$ along the vector of the instant velocity. Within the quantum picture (the *right panel* represents the Feynman diagram) the radiative transition from the initial state of the projectile (initial energy and the asymptotic momentum are ε_1 and $\mathbf{p}_1$) to the final state with ε_2, $\mathbf{p}_2$ is accompanied by the photon emission (*dashed line*). The *circle* denotes the vertex of the particle–photon interaction

$$\varphi(t) = t - \frac{\mathbf{n} \cdot \mathbf{r}(t)}{c}\,. \tag{2.2}$$

The dependence of the position vector on time, $\mathbf{r} = \mathbf{r}(t)$, is found from the classical equations of motions.

The classical description of the radiative process is illustrated by Fig. 2.1 (left), where the solid curve represents the trajectory of the charged particle. The radiation formed in a segment of the trajectory is emitted predominantly within the cone $\theta \sim 1/\gamma$ ($\gamma = \varepsilon/mc^2$ is the Lorentz relativistic factor) along the vector of the instant velocity $\mathbf{v}(t)$.

The classical approach based on (2.1) is commonly used to describe various types of electromagnetic radiation: BrS, synchrotron radiation, undulator radiation and channeling radiation. For more specific and retails information see [8, 14, 36, 37, 46, 53, 54, 136, 142, 193, 196, 269].

The main drawback of the classical framework is that it does not allow a self-consistent description of the decrease of the projectile energy due to the radiation emission. Hence, this scheme implies that the particle moves along the trajectory having the constant value of the total energy, $\varepsilon = \text{const}$.

2.1.2 Quantum Description

The most rigorous approach to the radiation process is based on the formalism of quantum electrodynamics (see, e.g., [61]), where the amplitude of the process is described in terms of a single free-free matrix element of the photon emission taken between the initial and final states of an ultra-relativistic particle in the interplanar field. The Feynman diagram of the process is presented in Fig. 2.1 (right), where the solid line denotes the projectile in the initial (the subscript 1) and the final (the subscript 2) states, the dashed line stands for the emitted photon and the dots marks the vertex of the particle—photon interaction. The energy conservation law implies

$\varepsilon_1 - \varepsilon_2 = \hbar\omega$. The corresponding analytical expression for the amplitude M_{21} is given by

$$M_{21} = qe \int \mathrm{d}\mathbf{r}\, \Psi^{\dagger}_{\varepsilon_2\, \mathbf{p}_2\, \nu_2}(\mathbf{r})\, (\mathbf{e}\cdot\boldsymbol{\alpha})\, \exp\left(-\mathrm{i}\mathbf{k}\cdot\mathbf{r}\right) \Psi_{\varepsilon_1\, \mathbf{p}_1\, \nu_1}(\mathbf{r}). \tag{2.3}$$

Here the bi-spinor wavefunction $\Psi_{\varepsilon\, \mathbf{p}\, \nu}(\mathbf{r})$ is the solution of the Dirac equation with the external potential U (a so-called Furry approximation, see, e.g., [61]) corresponding to the total energy ε, the asymptotic momentum $\mathbf{p}$. Other quantum numbers which characterize the particle, including its polarization, are incorporated in the subscript ν. The symbol $\dagger$ denotes the hermitian conjugation, $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$ where γ^0 and $\boldsymbol{\gamma}$ are the Dirac matrices. The vectors $\mathbf{k}$ and $\mathbf{e}$ denote the photon wave vector and polarization.

The power of radiation P (the energy per unit time) emitted within the frequency interval $\mathrm{d}\omega$ and within the cone $\mathrm{d}\Omega$ can be expressed in terms of the differential cross section $\mathrm{d}^3\sigma/\mathrm{d}\omega\,\mathrm{d}\Omega$ of the process:

$$\frac{\mathrm{d}^3 P}{\hbar \mathrm{d}\omega\, \mathrm{d}\Omega} = j\,\omega\, \frac{\mathrm{d}^3\sigma}{\mathrm{d}\omega\, \mathrm{d}\Omega} = \frac{\omega^3\, p_2\, \varepsilon_2}{(2\pi)^5 c^5} \sum_{\mathrm{pol}} \int_{(4\pi)} \mathrm{d}\Omega_{\mathbf{p}_2}\, |M_{21}|^2\,. \tag{2.4}$$

Here j stands for the flux of the incoming particles, the summation is carried out over the particle polarizations in the initial and final states as well as over the photon polarizations, the integration is carried out over the scattering angles.

Equations (2.3) and (2.4) are applicable in the whole range of the emitted photon energies, starting from the soft photons, $\hbar\omega \ll \varepsilon_1$ so that $\varepsilon_1 \approx \varepsilon_2$, up to the tip end of the spectrum, when nearly all the initial (kinetic) energy $\varepsilon_1 - mc^2$ is radiated.

In the ultra-relativistic domain the quantum-electrodynamic approach has been used for theoretical and numerical studies of various radiative processes. These include BrS in electron–atom (or/and ion), electron–electron etc collisions (see, e.g., [6, 61, 110, 136] and references therein), coherent BrS (e.g., [7]), synchrotron radiation [136, 147], and channeling radiation (e.g., [14, 194, 237]).

In application to the channeling motion and ChR, as well as to the CUR, the main (technical) limitation of the quantum approach is due to the fact, that in the ultra-relativistic limit, when $\gamma \gg 1$, the number of the energy levels of the transverse motion in the effective interplanar (or, axial) potential increases significantly. Consequently, an accurate numerical calculations of the particle dynamics becomes a formidable task [146]. It is exactly this sort of difficulties which resulted in the absence of any numerical analysis and the data for the emission spectra in [50, 87, 133], where CUR was treated in terms of quantum electrodynamics.

2.1.3 Quasi-Classical Description of Radiation Emission

An adequate approach to the radiation emission by ultra-relativistic projectiles was developed by Baier and Katkov in the late 1960th [34] and was called by the authors the "operator quasi-classical method". The details of the formalism, as well as its application to a variety of radiative processes, can be found in [36, 37, 61] and will not be reproduced here.

A remarkable feature of this method is that it allows one to combine the classical description of the particle motion in an external field and the quantum effect of radiative recoil.

The classical description of the motion is valid provided the characteristic energy of the projectile in an external field, $\hbar\tilde{\omega}_0$, is much less than its total energy, $\varepsilon = m\gamma c^2$. The relation $\hbar\tilde{\omega}_0/\varepsilon \propto \gamma^{-1} \ll 1$ is fully applicable in the case of an ultra-relativistic projectile.

The role of radiative recoil, i.e., the change of the projectile energy due to the photon emission, is governed by the ratio $\hbar\omega/\varepsilon$. In the limit $\hbar\omega/\varepsilon \ll 1$ a purely classical description (2.1) of the radiative process can be used. For $\hbar\omega/\varepsilon \leq 1$ quantum corrections must be accounted for.

The quasi-classical approach neglects the terms $\sim\hbar\tilde{\omega}_0/\varepsilon$ but explicitly takes into account the quantum corrections due to the radiative recoil. The method is applicable in the whole range of the emitted photon energies, except for the extreme high-energy tail of the spectrum $(1 - \hbar\omega)/\varepsilon \ll 1$.

Within the framework of quasi-classical approach one derives the following expression for the distribution of the energy radiated in given direction **n** by an ultra-relativistic particle (see [37, 61]):

$$\frac{\mathrm{d}^3 E}{\hbar \mathrm{d}\omega\, \mathrm{d}\Omega} = \alpha\, \frac{q^2\omega^2}{4\pi^2} \int\limits_0^{\mathcal{T}} \mathrm{d}t_1 \int\limits_0^{\mathcal{T}} \mathrm{d}t_2\, \mathrm{e}^{\mathrm{i}\omega'\left(\varphi(t_1)-\varphi(t_2)\right)}\, f(t_1,t_2). \tag{2.5}$$

All notations, except for ω' and $f(t_1, t_2)$, are the same as in the classical formula (2.1). The function $f(t_1, t_2)$ is defined as follows

$$f(t_1,t_2) = \frac{1}{2}\left[\left(1 + (1+u)^2\right)\left(\frac{\mathbf{v}_1\cdot\mathbf{v}_2}{c^2} - 1\right) + \frac{u^2}{\gamma^2}\right] \tag{2.6}$$

The key point of the quasi-classical method,—the radiative recoil, i.e. the account for the terms $\hbar\omega/\varepsilon$, is contained in the parameters ω' and u, which are defined as follows:

$$u = \frac{\hbar\omega}{\varepsilon - \hbar\omega}, \qquad \omega' = (1+u)\,\omega. \tag{2.7}$$

In the classical limit $u \approx \hbar\omega/\varepsilon \to 0$ and $\omega' \to \omega$, so that (2.5) and (2.6) reproduce (2.1).

Application of the general quasi-classical formula (2.5) to a variety of radiative processes in ultra-relativistic collisions in linear crystals is discussed in the monographs [36, 37]. It was also applied to the problem of synchrotron-type radiation emitted by an ultra-relativistic projectile channeling in a non-periodically bent crystal [21, 22, 255].

2.2 UR from an Ideal Planar Undulator

For the sake of completeness and for further referencing, in this section we present a collection of formulae describing the characteristics of radiation (spectral-angular and spectral distributions, position and width of the peaks of emitted harmonics, etc) by an ultra-relativistic charged particle moving in vacuum with a constant velocity v in the (y, z) plane along the trajectory

$$y(z) = a \sin k_{\mathrm{u}} z, \qquad \text{where} \quad k_{\mathrm{u}} = \frac{2\pi z}{\lambda_{\mathrm{u}}}, \tag{2.8}$$

consisting of N segments (periods) each of the length λ_{u}, which is called an undulator period. We will term a device in which an ultrarelativistic projectile *moves in vacuum along the sinusoidal line* as an "*ideal undulator*".

Such a motion can be realized in a planar magnetic undulator, in which bending of the particle trajectory is achieved by applying a periodic magnetic field directed perpendicular to the (y, z) plane: $\mathbf{B} = (B_x, 0, 0)$ with $B_x = B_0 \sin(k_{\mathrm{u}} z)$ (see, e.g., [8, 46, 109, 236]).

2.2.1 General Formalism

In an undulator the particle moves quasi-periodically, i.e. during the time interval T it completes a full oscillation along the y direction and simultaneously advances by the interval λ_{u} along the z direction, which is called the undulator axis, see Fig. 2.2. Hence, the position vector and the velocity of the particle satisfy the conditions

$$\mathbf{r}(t+T) = \mathbf{r}(t) + \langle \mathbf{v}_0 \rangle T, \qquad \mathbf{v}(t+T) = \mathbf{v}(t), \tag{2.9}$$

where $\langle \mathbf{v}_0 \rangle = T^{-1} \int_0^T \mathbf{v}(t) \mathrm{d}t$ is the mean velocity which is directed along the undulator axis and $\langle v_0 \rangle \approx c$.

Assuming that the Lorentz relativistic factor satisfies a strong inequality $\gamma \gg 1$, one expands the functions $\varphi(t)$ and $f(t_1, t_2)$ in powers of γ^{-1}. Then, retaining the dominant non-vanishing terms, one represents the right-hand side of (2.5) as follows:

$$\frac{\mathrm{d}^3 E}{\hbar \mathrm{d}\omega \mathrm{d}\Omega} = \alpha q^2 \frac{\omega^2 (1+u)(1+w)}{4\pi^2} \left[\frac{w |I_0|^2}{\gamma^2 (1+w)} + |\theta I_0 - \cos\phi I_1|^2 + \sin^2\phi |I_1|^2 \right] \tag{2.10}$$

Here $w = u^2/2(1+u)$ and (θ, ϕ) are the emission angles with respect to the undulator axis. The notations I_0 and I_1 stand for the integrals