Lecture Notes in Electrical Engineering 242

Jiuping Xu
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Asaf Hajiyev *Editors*

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Preface

Welcome to the proceedings of the Seventh International Conference on Management Science and Engineering Management (ICMSEM2013) held from November 7 to 9, 2013 at Drexel University, Philadelphia, Pennsylvania, USA.

International Conference on Management Science and Engineering Management is the annual conference organized by the International Society of Management Science and Engineering Management (ISMSEM). The goals of the Conference are to foster international research collaborations in Management Science and Engineering Management as well as to provide a forum to present current research results in the forms of technical sessions, round table discussions during the conference period in a relax and enjoyable atmosphere. 1420 papers from 35 countries were received and 130 papers from 12 countries were accepted for presentation or poster display at the conference after a serious review. These papers are from countries including USA, UK, Japan, Germany, Spain, Portugal, Turkey, China, Azerbaijan, Pakistan, Saudi Arabia and Australia. They are classified into 8 parts in the proceedings which are Computer and Networks, Information Technology, Decision Support System, Manufacturing, Supply Chain Management, Project Management, Ecological Engineering and Industrial Engineering. The key issues of the seventh ICMSEM cover various areas in MSEM, such as Decision Support System, Computational Mathematics, Information Systems, Logistics and Supply Chain Management, Relationship Management, Scheduling and Control, Data Warehousing and Data Mining, Electronic Commerce, Neural Networks, Stochastic models and Simulation, Heuristics Algorithms, Risk Control, and Carbon Credits. In order to further encourage the state-of-the-art research in the field of Management Science and Engineering Management, ISMSEM Advancement Prize for MSEM will be awarded at the conference for these researchers.

The conference also provides a suitable environment for discussions and exchanges of research ideas among participants during its well-organized post conference tours. Although we will present our research results in technical sessions, participate in round table discussions during the conference period, we will have extra and fruitful occasions to exchange research ideas with colleagues in this relaxed and enjoyable atmosphere of sightseeing.

We want to take this opportunity to thank all participants who have worked hard to make this conference a success. We appreciate the help from Drexel University and Sichuan University in conference organization. We also appreciate Springer-Verlag London for the wonderful publication of the proceedings. We are also grateful to all members of Organizing Committee, Local Arrangement Committee and Program Committee as well as all participants who have worked hard to make this conference a success. Finally we want to appreciate all authors for their excellent papers to this conference. Due to these excellent papers, ISMSEM Advancement Prize for MSEM will be awarded again at the conference for the papers which describe a practical application of Management Science and Engineering Management.

7-9 November 2013 *ICMSEM General and Program Chairs*
Philadelphia, Pennsylvania, USA

Contents

Organization

ICMSEM 2013 was organized by International Society of Management Science and Engineering Management, Sichuan University (Chengdu, China), Drexel University (Philadelphia, Pennsylvania, USA). It was held in cooperation with Lecture Notes in Electrical Engineering (LNEE) of Springer.

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Part V
Supply Chain Management

Chapter 66
On Effectivity of Delays in Queuing Systems

Asaf Hajiyev, Farah Ahmadzada and Irada Ibadova

Abstract For queuing systems with cyclic service for reducing of a customer average waiting time before service the delay of beginning service is introduced. The class of queuing systems for which it is advisable to introduce delay is described. The form of optimal function minimizing a customer average waiting time has found. Numerical examples demonstrating these results are given.

Keywords Customer average waiting time · Delay · Improvable service · Optimal function

66.1 Introduction

In this paper we consider queuing systems, for which $t_1, \cdots, t_n$ is the sequence of service starts instants. A stationary flow of customers with finite intensity arrives to service and this flow independent of t_i. At the instant t_i all customers that arrived over the interval $[t_{i-1}, t_i)$ are served instantaneously, i.e. it is assumed that server has infinite volume and service time equals zero. Such models are typical for applications and describe, for instance, behavior of transport systems. In the capacity of an efficiency index of a system we take a customer average time before service, which will be called w and σ is a variance of a customer waiting time before service. Our aim is by introducing of a control function to reduce a customer average waiting time before service. One of the simplest control policies is the introduction of service start delays that can be easily implemented in practice. The introduction of service start delays in systems with cyclic service has been studied in [1–3], where systems with service start delay preset as a constant were considered. In [4, 5], de-

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lays representing a certain class of control functions were studied in the case of systems with recurrent service.

66.2 Construction of Mathematical Models

Model 1. It is assumed that the sequence $t_1, t_2, \cdots, t_n$ is a simple renewal process. Define a customer waiting time before service as the time elapsed from the arrival of a customer into the system until the next service start. We will introduce control the service start instants; for this purpose we turn from the sequence $t_1, \cdots, t_n$ to the new sequence $t_1^*, \cdots, t_n^*$ using the following rule. Denote:

$$\eta_1 = t_1, \quad \eta_1 = t_i - t_{i-1}, \ i = 2, 3, \cdots,$$
$$\eta_1^* = t_1^*, \quad \eta_i^* = t_i^* - t_{i-1}^* = \eta_i + g(\eta_i), \ g \in G,$$

where G is a class of non-negative and measurable functions, i.e. $g(x) > 0$ and a random variable $\eta_i, \ i = 1, 2, \cdots$ has distribution function $F(x)$. As $t_1, \cdots, t_n$ is a renewal process, then $\eta_1, \eta_2, \cdots$ is a sequence of independent and identically distributed random variables with distribution function $F(x)$. Let $w(g)$ and $\sigma^2(g)$ be the expectation and the variance of the waiting time in a system with the control function $g \in G, c = E\eta^2/2E\eta$ and put $w(0) = 0$ and $\sigma^2(0) = \sigma^2$.

Definition 66.1. Service is improvable if there exists a function $g \in G$ such that $w(g) < w$, i.e. $\exists g(x) \in G$ such that $M_F(g) < 0$.

Statement 1. For service to be improvable it is necessary and sufficient holding of the following condition $x_0 < c$ such that $F(x_0) > 0$.

Definition 66.2. Function $g^*(x)$ is called an optimal if

$$\min_{g \in G} M_F(g) = M_F(g^*).$$

Statement 2. Under the conditions of the Statement 1 an optimal function has the following form:

$$g^*(x) = \max(0, c_1 - x) = (c_1 - x)^+ \text{ and } \sigma^2(g^*) \le \sigma^2(g) \le \sigma^2(0) = \sigma^2,$$

where $0 \le c_2 \le c_1$ and c_1 satisfies to the equation:

$$c_1^2 = \int_{c_1}^{\infty} (c_1 - x)^2 dF(x) \tag{66.1}$$

and moreover c_1 is a unique solution of the Equation (66.1).

Remark 66.1. In Fig. 66.1, the behavior of distribution function for initial system, and in Fig. 66.2, the distribution function for the optimal systems are shown.

$$F^*(x) = 0, \text{ if } x \le c_1 \text{ and } F^*(x) = F(x), \text{ if } x > c_1.$$

Remark 66.2. The Statement 2 reminds paradox. Let us consider the large time interval $[0,T)$, and assume that there is k services for initial (without control) system and m services for controlled system. (see Fig. 66.3). It is clear that $k < l$. But according to the Statement 2, a customer average waiting time before service in system with greater number of services is greater than the same efficiency index in the system. It is alike to paradox.

At the time interval $[0,T)$ there is k number of services in the initial system and l $(k > l)$ services in the controlled system. According to the Remark 66.2, $w < w^*$.

Example 66.1. Let us put $F(x) = 1 - e^{-x}$, $x \ge 0$. Numerical computations yield:

$$w = 1;\ \sigma^2 = 1;\ w^* = 0.90;\ \sigma^{2^*} = 0.67;\ g*(x) = (0.90 - x)^+ = \max(0, 0.90 - x).$$

Therefore, the gain in customer average waiting time equals 10%, and in variance 33%.

Model 2. Let $t_1, t_2, \cdots, t_n, \cdots$ be times when service starts in some system. We assume $t_i = id + x_i, i = 1, 2, \cdots$, where d a constant is and $x_1, x_2, \cdots, x_n$ is a sequence of independent and identically distributed random variables $(i.i.d.r.v.)$ with

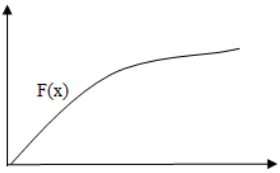


Fig. 66.1 The distribution function for initial system

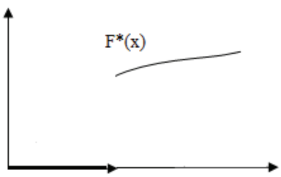


Fig. 66.2 The distribution function for the optimal systems

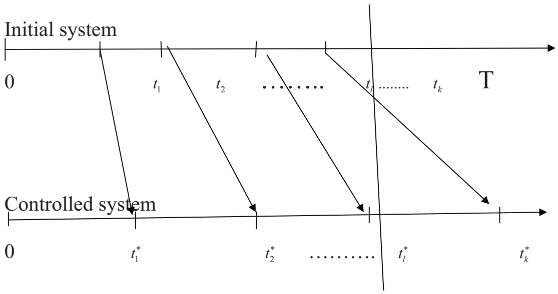


Fig. 66.3 k services for initial system and m services for controlled system

probability distribution function (d.f.) $G(x), |x| \leq d/2$ for all i. Such a scheme describes, for example, a behavior of public transport system where id $(i = 1,2,\cdots)$ is the ideal timetable for the arriving transport and $id + x_i$ $(i = 1,2,\cdots)$ is the real timetable. We denote $\eta_i = t_i - t_{i-1} = id + x_i - (i-1)d - x_{i-1} = d + x_i - x_{i-1}$. Then $\eta_1, \eta_2, \cdots, \eta_n, \cdots$ are identically distributed random variables with d.f.

$$F(x) = \int_{-d/2}^{d/2} G(x+y-d)dG(y),$$

but η_i, η_{i+1} are dependent. The flow of service customers is stationary with intensity $\mu < \infty$ and independent of $t_1, t_2, \cdots, t_n$. At the instant t_i, all customers, which arrived during the time interval $[t_{i-1}, t_i)$ are served immediately, in particular we assume that service time equals zero. If $t_1, t_2, \cdots, t_n, \cdots$ a stationary renewal process, $t_n = \eta_1 + \eta_2 + \cdots + \eta_n$ then according to Cox [6], we have:

$$w = \frac{E\eta^2}{2E\eta}, \; \sigma^2 = \frac{E\eta^3}{3E\eta} - \left(\frac{E\eta^2}{2E\eta}\right)^2, \tag{66.2}$$

where η is a random variable with d.f. $F(x)$. Next we shall show Equation (66.2) is true also or our model. Generally formula (66.2) is true for any identically distributed random (even dependent) variables η_i for which $\eta_i \geq 0, E\eta_i^2 < \infty$ for all i. (Proof see, Hajiyev [5]). Now we shall take delays into account, i.e. from the random variables $\eta_1, \eta_2, \cdots, \eta_n, \cdots$. we pass to random variables $\eta_1^*, \eta_2^*, \cdots, \eta_n^*, \cdots$ where $\eta_i^* = \eta_i + g(\eta_i), i = 1,2,\cdots, g \in G$. Here G is a class of measurable and non-negative functions. Let W^*, σ^{2*} be the expectations and variance of the delayed waiting times until service. Our interest concerns the following problem. For which systems can the introduction of delays diminish W. (i.e. give better service). Similarly Equation (66.2), we have:

$$w^* = \frac{E\eta^{2*}}{2E\eta^*}, \; \sigma^{2*} = \frac{E\eta^{3*}}{3E\eta^*} - \left(\frac{E\eta^{2*}}{2E\eta^*}\right)^2.$$

Statement 3. Service in the model 2 can be improved by delays if and only if there exists an $x_0 < c$ such that $F(x_0) > 0$, where $c = \frac{E\eta^2}{2E\eta}$.

Definition 66.3. We call $\tilde{g}(x)$ an optimal function if

$$\min_{g \in G} M_F(g) = M_F(\tilde{g}).$$

Statement 4. Under the conditions of the Statement 3 the optimal function has the form:

$$\tilde{g}(x) = (c_1 - x)^+ = \max(0, \; (c_1 - x)),$$

where c_1 is the unique solution of the equation:

$$c_1^2 = \int_{c_1}^{2d} (x - c_1)^2 dF(x).$$

Example 66.2. Let $d = 1$ and x_i has a uniform distribution on $\left[-\frac{1}{2}, \frac{1}{2}\right]$. Simple calculations yield $W = 0.583, \sigma^2 = 0.160$ and $W_* = 0.577, \sigma_*^2 = 0.156$ and $\tilde{g}(x) = (0.577 - x)^+$.

66.3 Queues with Complicated Structure

Model 3. Consider a model in which delay of one service start affects the next service, i.e., the sequence of controlled service start instants has the form:

$$\begin{aligned}
&\xi_1^* = t_1^* = \xi_1^* + g_1(\xi_1^*),\\
&\xi_2^* = t_2^* - t_1^* = (\xi_1 + \xi_2 - \xi_1^*)^+ + g_2(\xi_1 + \xi_2 - \xi_1^*)^+\\
&\vdots\\
&\xi_n^* = t_n^* - t_{n-1}^* = (\xi_1 + \xi_2 + \cdots + \xi_n - \xi_1^* - \xi_2^* - \cdots - \xi_{n-1}^*)^+ +\\
&\qquad g_n(\xi_1 + \xi_2 - \xi_1^* - \xi_2^* - \cdots - \xi_{n-1}^*)^+,
\end{aligned}$$

where $g_i(x) \in G$ is the class of non-negative measurable functions. Such models are typical for applications, because increasing an interval between two neighbor services we reduce next interval between services. Denote:

$$\begin{aligned}
\eta_j &= (\xi_1 + \xi_2 + \cdots + \xi_j - \xi_1^* - \xi_2^* - \cdots - \xi_{j-1}^*)^+\\
&= \left(\sum_{i=1}^{j} \xi_i - \sum_{i=1}^{j-1} \xi_i^*\right)^+, \ \eta_j \geq 0, i = 1, 2, \cdots.
\end{aligned}$$

Assume $\xi_1, \xi_2, \cdots, \xi_n$ are independent and identically distributed random variables with $E\xi_1 = d$ and $Var(\xi_i) = \sigma^2$.
Statement 5. For model 3 is true

$$w = \lim_{n\to\infty} \left\{ \frac{(1/N)\sum_i^n E\eta_i}{(1/n)\sum_{i=`1}^n 2E\eta_i} \right\}, \quad w^* = \lim_{n\to\infty} \left\{ \frac{(1/N)\sum_i^n E\eta_i^*}{(1/n)\sum_{i=`1}^n 2E\eta_i^*} \right\}.$$

Statement 6. If for the model 3 an optimal function exists then it has the following form

$$\tilde{g}(x) = (d - x)^+ = \max(0,\, d - x), \ \ d = \mathrm{E}\xi.$$

Corollary 66.1.

$$\tilde{w} = E\xi_1/2.$$

Statement 7. Service is improvable if and only if Var $\xi_i > 0$.

Corollary 66.2. *For the optimal function $\tilde{g}$, the following expression is true:*

$$W(\tilde{g}) = \mathrm{E}\xi/2, \quad \sigma^2(\tilde{g}) = \mathrm{E}\xi/12.$$

Example 66.3. Let $F(x) = 1 - e^{-x}$, $x \geq 0$, then $\tilde{g}(x) = (1-x)^+$, $W(\tilde{g}) = 1/2$, $\sigma^2(\tilde{g}) = 0.083$, i.e., the gain in mean waiting time is 50%.

66.4 Queues with Cyclic Services

Consider N terminals and one server, which serves customers at the each terminal (from i-th terminal to $(i+1)$-th terminal). At the each terminal Poisson flow of customers with finite intensity λ_i, $i = 1,2,\cdots,n$ arrives to service. Denote $t_1^i, t_2^i, \cdots, t_n^i$ the instants when service starts at the i-th terminal. At the t_j^i instant all customers, which arrived at the time interval $[t_{j-1}^i, t_j^i)$ immediately will get service, i.e. we consider server with infinite volume and assume that service time equals zero (see Fig. 66.4). Denote $X_j^i = t_{j+1}^i - t_j^i$ $(i = 1,2,\cdots,n;\ j = 1,2,\cdots)$, i.e. the time interval between $(j+1)$-th and j-th service at the i-th terminal. In the capacity of efficiency index (w) we take a customer average waiting time before service. Similarly as above, we will introduce delay of beginning service to reduce an average waiting time before service.

Assume that for any fixed i the random variables X_j^i $(j = 1,2,\cdots)$ independent and identically distributed random variables $(E|X_j^i| < \infty, E(X_j^i)^2 < \infty)$ and at the i-th terminal each customer pays penalty c_i $(i = 1,2,\cdots,n)$ for any unit time of waiting. Suppose $t_1^1 = 0$. Then for first terminal we have:

$$\mathrm{t}_1^1 = 0, \mathrm{t}_2^1 = \mathrm{t}_1^1 + X_1^1 + X_1^2 + \cdots + X_1^n, \cdots, \mathrm{t}_{\mathrm{m}+1}^1 = \mathrm{t}_{\mathrm{m}}^1 + X_m^1 + X_m^2 + \cdots + X_m^n.$$

For second $\mathrm{t}_1^2 = \mathrm{t}_1^1 + X_1^1, \mathrm{t}_2^2 = \mathrm{t}_2^1 + X_2^1, \cdots, \mathrm{t}_{\mathrm{m}+1}^2 = \mathrm{t}_{\mathrm{m}+1}^1 + X_{m+1}^1$ and so on. For n-th $\mathrm{t}_1^{\mathrm{n}} = \mathrm{t}_1^{\mathrm{n}-1} + X_1^{n-1}, \mathrm{t}_2^{\mathrm{n}} = \mathrm{t}_2^{\mathrm{n}-1} + X_2^{n-1}, \cdots, \mathrm{t}_{\mathrm{m}+1}^{\mathrm{n}} = \mathrm{t}_{\mathrm{m}+1}^{\mathrm{n}-1} + X_{m+1}^{n-1}$.

Consider the random variables $\eta_j^1 = \mathrm{t}_{\mathrm{j}+1}^1 - \mathrm{t}_{\mathrm{j}}^1 = X_j^1 + X_j^2 + \cdots + X_j^n$, $\eta_j^2 = \mathrm{t}_{\mathrm{j}+1}^2 - \mathrm{t}_{\mathrm{j}}^2 = X_{j+1}^1 + X_j^2 + \cdots + X_j^n, \cdots, \eta_j^n = \mathrm{t}_{\mathrm{j}+1}^{\mathrm{n}} - \mathrm{t}_{\mathrm{j}}^{\mathrm{n}} = X_{j+1}^1 + \cdots + X_{j+1}^{n-1} + X_j^n$. It is clear

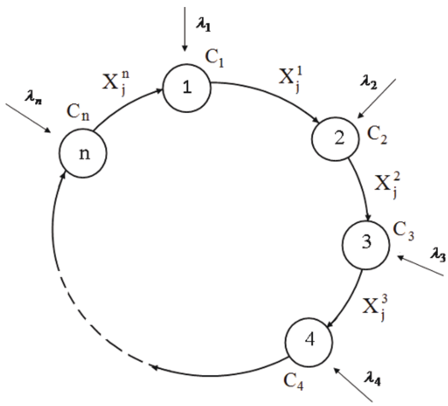


Fig. 66.4 N terminals and one server queues with cyclic services

that for fixed i the random variables η_j^i $(j=1,2,\cdots)$ are independent and identically distributed. Then according to Hajiyev [5], we have $W_i = \frac{E(\eta_j^i)^2}{2E\eta_j^j}$, $i=1,2,\cdots,n$. It follows:

$$W_i = \frac{E(\sum_{k=1}^n X_1^k)^2}{2E(\sum_{k=1}^n X_1^k)},\ i=1,2,\cdots,n$$

and for any customer its average penalty is:

$$C_0 = \frac{\sum_{k=1}^n \lambda_k c_k W_k}{\sum_{k=1}^n \lambda_k} = \frac{\sum_{k=1}^n \lambda_k c_k}{\sum_{k=1}^n \lambda_k} W_1$$

and $W_1 = W_2 = \cdots = W_n$.

66.4.1 Delay at the One Terminal

Let us introduce delays at the first terminal. Introduce the delay function g$(X^1, X^2, \cdots, X^n)$, which depends on previous intervals between services, where $g(\cdot)$ is measurable and non-negative function. Then

$$\xi_j^1 = \eta_j^1 + g(X_j), \xi_j^2 = \eta_j^2 + g(X_j), \cdots, \xi_j^n = \eta_j^n + g(X_j),$$

where $X_j = (X_j^1, X_j^2, \cdots, X_j^n),\ j=1,2,\cdots$ and we have:

$$W_1(g) = \frac{E(\xi_1^1)^2}{2E\xi_1^1} = \frac{E(X_1^1+X_1^2+\cdots+X_1^n+g(X_1))^2}{2E(X_1^1+X_1^2+\cdots+X_1^n+g(X_1))} = \frac{E(\sum\limits_{i=1}^n X_1^i + g(X_1))^2}{2E(\sum\limits_{i=1}^n X_1^i + g(X_1))},$$

$$W_2(g) = \frac{E(\xi_1^2)^2}{2E\xi_1^2} = \frac{E(X_2^1+X_1^2+\cdots+X_1^n+g(X_1))^2}{2E(X_2^1+X_1^2+\cdots+X_1^n+g(X_1))} = \frac{E(X_2^1 + \sum\limits_{i=2}^n X_1^i + g(X_1))^2}{2E(X_2^1 + \sum\limits_{i=2}^n X_1^i + g(X_1))},$$

$$\vdots$$

$$W_n(g) = \frac{E(\xi_1^n)^2}{2E\xi_1^n} = \frac{E(X_2^1+X_2^2+\cdots+X_2^{n-1}+X_1^n+g(X_1))^2}{2E(X_2^1+X_2^2+\cdots+X_2^{n-1}+X_1^n+g(X_1))}$$

$$= \frac{E(\sum\limits_{i=1}^{n-1} X_2^i + X_1^n + g(X_1))^2}{2E(\sum\limits_{i=1}^{n-1} X_2^i + X_1^n + g(X_1))}.$$

For penalties we have:

$$C(g) = \frac{\lambda_1 c_1 W_1(g) + \lambda_2 c_2 W_2(g) + \cdots + \lambda_n c_n W_n(g)}{\lambda_1 + \lambda_2 + \cdots + \lambda_n} G$$

is the class of all measurable and non-negative functions which are defined on:

$$R_+^n = \{\bar{x} : \bar{x} = (x_1, x_2, \cdots, x_n), x_1 \geq 0, x_2 \geq 0, \cdots, x_n \geq 0\}.$$

Our aim to minimize the value of penalties on the class G.

$$\mathrm{C(g)} \to \min, \mathrm{g} \in \mathrm{G}.$$

Definition 66.4. Function $g^*(\cdot)$ is called an optimal if:

$$\min C(g) = C(g^*), \quad g \in G.$$

Introduce the following notations:

$$a = \frac{E(X_1^1 + X_1^2 + \cdots + X_1^n)^2}{2E(X_1^1 + X_1^2 + \cdots + X_1^n)} = \frac{E\left(\sum\limits_{i=1}^n X_1^i\right)^2}{2E\left(\sum\limits_{i=1}^n X_1^i\right)},$$

$$g_b(\bar{x}) = \left(b - \frac{\sum\limits_{i=1}^n \left(\sum\limits_{k=1}^{n-i} \lambda_k c_k E X_1^k + \sum\limits_{k=n-i+1}^n \lambda_k c_k x_k \right)}{\sum\limits_{i=1}^n \lambda_i c_i} \right),$$

where $\bar{x} = (x_1, x_2, \cdots, x_n)$. It is clear that,

$$C_0 = C(0) = \frac{\sum\limits_{k=1}^n \lambda_k c_k}{\sum\limits_{k=1}^n \lambda_k} a,$$

where c_0 means payment in the system without control. Denote $F(\bar{x})$ the joint probability distribution function of the random variables $X_1^1, X_1^2, \cdots, X_1^n$. Then

$$F(\bar{x}) = F_1(x_1) F_2(x_2) \cdots F_n(x_n),$$

where $F_1(x_1), F_2(x_2), \cdots, F_n(x_n)$ corresponding probability distribution function of the random variables $X_1^1, X_1^2, \cdots, X_1^n$. As for fixed i $(i = 1, 2, \cdots, n)$ the random variables X_j^i $(j = 1, 2, \cdots)$ have identically distribution and X_j^i $(i = 1, 2, 3;\ j = 1, 2, \cdots)$ are independent then:

$$\mathrm{C(g)} = \frac{\sum\limits_{k=1}^{n} \lambda_k c_k}{\sum\limits_{k=1}^{n} \lambda_k} \times \frac{\int\limits_{R_+^n} g(\bar{x})(\frac{g(\bar{x})}{2} - g_a(\bar{x}))dF(\bar{x})}{\sum\limits_{k=1}^{n} EX_1^k + \int\limits_{R_+^n} g(\bar{x})dF(\bar{x})} + \mathrm{C_0}.$$

Statement 8. Service is improvable if

$$\mathrm{g_a}(\bar{x}_0) > 0,\ F(\bar{x}_0) > 0.$$

Statement 9. Under the conditions of the Statement 5 the following expression is true:

$$\tilde{g}(\bar{x}) = \max(g_d(\bar{x}), 0).$$

Remark 66.3. We can show that d satisfies to the following condition:

$$2dE\left(\sum_{i=1}^{n} X_1^i\right) + E(\max(0, g_d(X_1)))^2 = E\left(\sum_{i=1}^{n} X_1^i\right)^2. \tag{66.3}$$

At the interval $(0, a)$, Equation (66.2) has a unique solution.

Remark 66.4. An average expenses for optimal function equals:

$$C(g^*) = \frac{\lambda_1 c_1 + \lambda_2 c_2 + \cdots + \lambda_n c_n}{\lambda_1 + \lambda_2 + \cdots + \lambda_n} d,$$

where d is defined in Equation (66.3).

Remark 66.5. If $\lambda_1 c_1 = \lambda_2 c_2 = \cdots = \lambda_n c_n$, then:

$$\begin{aligned} g^*(\bar{x}) &= \left(d - \frac{\sum\limits_{i=1}^{n} \left((n-i)EX_1^i + ix_i\right)}{n}\right)^+ \\ &= \left(d - \left(\frac{\left((n-1)EX_1^1 + x_1\right)}{n} + \frac{\left((n-2)EX_1^2 + 2x_2\right)}{n} + \cdots + x_n\right)\right)^+. \end{aligned}$$

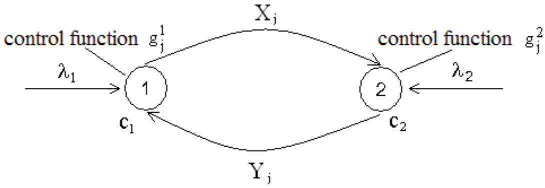
If $n = 3$ then:

$$g^*(\bar{x}) = \left(d - \left(\frac{2EX_1^1 + x_1}{3} + \frac{EX_1^2 + 2x_2}{3} + x_3\right)\right)^+.$$

66.4.2 Two Terminals Delays at the Both Terminals

Consider two terminals system (Fig. 66.5). Introduce the following function:

Fig. 66.5 Two terminals system



$$g^1 = (g_1^1, g_2^1, \cdots) = (g_j^1)_{j=1,\infty}, \ g^2 = (g_1^2, g_2^2, \cdots) = (g_j^2)_{j=1,\infty}.$$

g_j^i is delay function at the i-th terminal on j-th service, $X_j, Y_j (j = 1, \cdots, n)$.

$$X_1 \to g_1^2 \to Y_1 \to g_1^1 \to X_2 \to \cdots \to X_j \to g_j^2 \to Y_j \to g_j^1 \to X_{j+1} \to g_{j+1}^2 \to \cdots.$$

Then we have for first terminal:

$$\mathrm{t}_1^1 = 0, \mathrm{t}_2^1 = \mathrm{t}_1^1 + X_1 + \mathrm{g}_1^2 + Y_1 + \mathrm{g}_1^1, \cdots, \mathrm{t}_{\mathrm{m}+1}^1 = \mathrm{t}_{\mathrm{m}}^1 + X_m + \mathrm{g}_{\mathrm{m}}^2 + Y_{\mathrm{m}} + \mathrm{g}_{\mathrm{m}}^1,$$

for second:

$$\mathrm{t}_1^2 = X_1 + g_1^2, \mathrm{t}_2^2 = \mathrm{t}_1^2 + Y_1 + \mathrm{g}_1^1 + X_2 + \mathrm{g}_2^2, \cdots, \mathrm{t}_{\mathrm{m}+1}^2 = \mathrm{t}_{\mathrm{m}}^2 + Y_{\mathrm{m}} + \mathrm{g}_{\mathrm{m}}^1 + X_{m+1} + \mathrm{g}_{\mathrm{m}+1}^2$$

and

$$\xi_j = \mathrm{t}_{\mathrm{j}}^1 - \mathrm{t}_{\mathrm{j}}^1 = X_j + \mathrm{g}_{\mathrm{j}}^2 + Y_j + \mathrm{g}_{\mathrm{j}}^1,$$
$$\eta_{\mathrm{j}} = \mathrm{t}_{\mathrm{j}}^2 - \mathrm{t}_{\mathrm{j}}^2 = Y_j + \mathrm{g}_{\mathrm{j}}^1 + X_{j+1} + \mathrm{g}_{\mathrm{j}+1}^2.$$

According to Hajiyev [5], we have:

$$W_1(g^1, g^2) = \frac{\lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_{j=1}^{N} E{\xi_j}^2}{2 \lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_{j=1}^{N} E\xi_j} = \frac{\lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_{j=1}^{N} E(X_j + \mathrm{g}_{\mathrm{j}}^2 + Y_j + \mathrm{g}_{\mathrm{j}}^1)^2}{2 \lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_{j=1}^{N} E(X_j + \mathrm{g}_{\mathrm{j}}^2 + Y_j + \mathrm{g}_{\mathrm{j}}^1)},$$

$$W_2(g^1, g^2) = \frac{\lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_{j=1}^{N} E{\eta_j}^2}{2 \lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_{j=1}^{N} E\eta_j} = \frac{\lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_{j=1}^{N} E(Y_j + \mathrm{g}_{\mathrm{j}}^1 + X_{j+1} + \mathrm{g}_{\mathrm{j}+1}^2)^2}{2 \lim\limits_{N\to\infty} \frac{1}{N} \sum\limits_{j=1}^{N} E(Y_j + \mathrm{g}_{\mathrm{j}}^1 + X_{j+1} + \mathrm{g}_{\mathrm{j}+1}^2)},$$

$$C(g^1, g^2) = \frac{\lambda_1 c_1 W_1(g^1, g^2) + \lambda_2 c_2 W_2(g^1, g^2)}{\lambda_1 + \lambda_2}.$$

Denote:

$$d = \frac{\lambda_1 + \lambda_2}{\lambda_1 c_1 + \lambda_2 c_2} \inf_{(g_1, g_2) \in G} C(g_1, g_2)$$

and consider the functional:

$$I(g^1,g^2)= \alpha_1 \lim_{N\to\infty}\frac{1}{N}\sum_{j=1}^{N}E(X_j+\mathrm{g}_\mathrm{j}^2+Y_j+\mathrm{g}_\mathrm{j}^1-d)^2 + \alpha_2 \lim_{N\to\infty}\frac{1}{N}\sum_{j=1}^{N}E(Y_j+\mathrm{g}_\mathrm{j}^1+X_{j+1}+\mathrm{g}_\mathrm{j+1}^2-d)^2-d^2,$$

where

$$\alpha_1=\frac{\lambda_1 c_1}{\lambda_1 c_1+\lambda_2 c_2},\ \ \alpha_2=\frac{\lambda_2 c_2}{\lambda_1 c_1+\lambda_2 c_2},\alpha_1+\alpha_2=1.$$

Introduce:

$$\mathrm{g}_\mathrm{j,d}^1=d-(\alpha_1 X_j+\alpha_2 EX+Y_j),\, j=1,2,\cdots,$$
$$\mathrm{g}_\mathrm{j+1,d}^2=d-(X_{j+1}+\alpha_2 Y_j+\alpha_1 EY).$$

It follows that $(\bar{g}^1,\bar{g}^2)$ is an optimal function for $(g^1,g^2)\in G_1$ if the following expression:

$$\lim_{N\to\infty}\frac{1}{N}\sum_{j=1}^{N}\Big(E(\bar{\mathrm{g}}_\mathrm{j}^1+\alpha_1\bar{\mathrm{g}}_\mathrm{j}^2+\alpha_2\bar{\mathrm{g}}_\mathrm{j+1}^2-\mathrm{g}_\mathrm{j,d}^1)(g_j^1-\bar{g}_j^1) + E(\bar{\mathrm{g}}_\mathrm{j+1}^2+\alpha_1\bar{\mathrm{g}}_\mathrm{j+1}^1+\alpha_2\bar{\mathrm{g}}_\mathrm{j}^1-\mathrm{g}_\mathrm{j+1,d}^2)(g_{j+1}^2-\bar{g}_{j+1}^2)\Big)\geq 0$$

is true. Hence,

$$\hat{g}_{m_k}^2=\left(g_{m_k,d}^2-\alpha_2\hat{g}_{m_k-1}^1\right)^+,$$
$$\hat{g}_{m_k-1}^1=\left(g_{m_k-1,d}^1-\alpha_1\hat{g}_{m_k-1}^2-\alpha_2 E(\hat{g}_{m_k}^2|K)\right)^+,$$
$$\hat{g}_{m_k-1}^2=\left(g_{m_k-1,d}^2-\alpha_1(\hat{g}_{m_k-1}^1|K)-\alpha_2\hat{g}_{m_k-2}^1\right)^+, \tag{66.4}$$
$$\hat{g}_j^1=\left(g_{j,d}^1-\alpha_1\hat{g}_j^2-\alpha_2 E(\hat{g}_{j+1}^2|K)\right)^+. \tag{66.5}$$

From the Equations (66.4) and (66.5), we can find $\hat{g}_{m_k-1}^1$:

$$\hat{g}_{m_k-1}^1=f_1\left(X_{m_k-1},Y_{m_k-1},\hat{g}_{m_k-1}^2\right) \tag{66.6}$$

and from the Equations (66.5) and (66.6), we can find $\hat{g}_{m_k-1}^2$:

$$\hat{g}_{m_k-1}^2=f_2\left(Y_{m_k-2},X_{m_k-1},\hat{g}_{m_k-2}^1\right). \tag{66.7}$$

Finally we can find $\hat{g}^i_j$. Thus, starting from the control function $\hat{g}^1_{m_k} = 0$ moving back we sequatially can find $\hat{g}^i_j$. Then we can find a minimum of expenses function $C_{\min} = C(g^1, g^2)$.

Remark 66.6. d is a solution of the non-linear equation:

$$2dE(X+Y) + \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \left(\alpha_1 E(\mathrm{g}^2_\mathrm{j} + \mathrm{g}^1_\mathrm{j})^2 + \alpha_2 E(\mathrm{g}^1_\mathrm{j} + \mathrm{g}^2_\mathrm{j+1})^2 \right) = E(X+Y)^2.$$

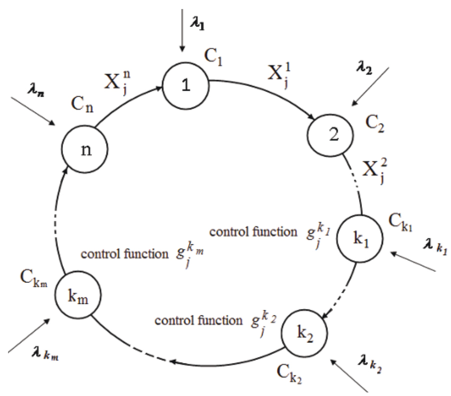


Fig. 66.6 Queueing system with n terminals

66.4.3 Queues with n Terminals. Control at the Several Terminals

Consider queueing system with n terminals. Control function is introduced at the m terminals $(m \le n)$, see Fig. 66.6. It is supposed that control function is introduced at the terminals with numbers $k_1, k_2, \cdots, k_m$, where $1 \le k_1 < k_2 < \cdots < k_m \le n$. Below is giving order of realizations $\cdots \to X^1_j \to \cdots \to X^{k_1-1}_j \to g^{k_1}_j \to X^{k_1}_j \to \cdots \to X^{k_m-1}_j \to g^{k_m}_j \to X^{k_m}_j \to \cdots \to X^n_j \to X^1_{j+1} \to \cdots$. We get the following integral equations:

$$\begin{aligned} \bar{\mathrm{g}}^{\mathrm{k_h}}_{\mathrm{j+1}} = & \left(\mathrm{g}^{\mathrm{k_h}}_{\mathrm{j+1,d}} - \sum_{i=1}^{h-1} \left[\left(1 - \sum_{l=k_i}^{k_h-1} \alpha_l \right) \bar{\mathrm{g}}^{\mathrm{k_i}}_{\mathrm{j+1}} + \sum_{l=k_i}^{k_h-1} \alpha_l E\left(\bar{\mathrm{g}}^{\mathrm{k_i}}_{\mathrm{j+2}} | K\right) \right] \right. \\ & \left. - \sum_{i=h+1}^{m} \left[\sum_{l=k_h}^{k_i-1} \alpha_l \bar{\mathrm{g}}^{\mathrm{k_i}}_{\mathrm{j}} + \left(1 - \sum_{l=k_h}^{k_i-1} \alpha_l \right) E\left(\bar{\mathrm{g}}^{\mathrm{k_i}}_{\mathrm{j+1}} | K\right) \right] \right)^+, \quad h = 1, 2, \cdots, m, \end{aligned}$$

where

$$g^k_{j+1,d} = \left(d - \sum_{i=1}^{k-1}\left(\sum_{l=i+1}^{k-1}\alpha_l EX_1^i + \left(1 - \sum_{l=i+1}^{k-1}\alpha_l\right)X_{j+1}^i\right)\right.$$
$$\left. - \sum_{i=k}^{n}\left(\left(1 - \sum_{l=k}^{i}\alpha_l\right)EX_1^i + \sum_{l=k}^{i}\alpha_l X_j^i\right)\right).$$

66.4.4 Numerical Examples

It was prepared a special program on a computer for numerical calculation. Moving time between terminals corresponds to the exponential distribution with parameters α_i $(i = 1, 2, \cdots, n)$.

Table 66.1 System with 2 terminals, $n = 2$

N	λ_1	λ_2	c_1	c_2	α_1	α_2	d	C_0	$C_{\min} = C(\tilde{g})$	Gain (%)
1.	2	3	1	2	1/5	1/6	6.82	13.24	10.9	17.6
2.	4	3	2	3	1/6	1/4	6.48	18.46	15.75	14.7
3.	3	5	4	2	1/3	2/7	4.40	13.43	12.11	9.9

Table 66.2 System with 3 terminals, $n = 3$

N	λ_1	λ_2	λ_3	c_1	c_2	c_3	α_1	α_2	α_3	d	C_0	$C_{\min}$	Gain (%)
1.	2	3	4	3	5	2	0.3	0.25	1/6	8.27	29.11	26.64	8.5
2.	3	1	2	2	5	4	0.4	0.6	0.7	3.22	11.99	10.18	15.1
3.	5	4	3	3	2	6	0.8	0.7	0.9	2.3	8.65	7.85	9.3

Table 66.3 System with 2 terminals, $n = 2$

N	λ_1	λ_2	c_1	c_2	I_1	I_2	d	C_0	$C_{\min} = C(\tilde{g})$	Gain (%)
1.	2	3	2	1	[2,3]	[4,5]	3.51	4.92	4.92	0
2.	4	3	2	3	[7,11]	[5,7]	6.86	18.35	16.66	9.2
3.	3	5	4	2	[8,12]	[6,8]	8.34	23.51	22.94	2.4

Table 66.4 System with 5 terminals, $n = 5$, (1)

N	λ_1	λ_2	λ_3	λ_4	λ_5	c_1	c_2	c_3	c_4	c_5
1.	2	4	5	3	2	3	2	1	3	5
2.	4	6	5	4	3	3	2	3	4	5
3.	6	4	2	5	4	4	5	5	3	7

Table 66.5 System with 5 terminals, $n = 5$, (2)

N	α_1	α_2	α_3	α_4	α_5	C_0	d	$C_{\min}$	Gain (%)
1.	0.9	0.75	0.9	0.95	0.8	133.7	2.06	78.4	41
2.	0.8	0.85	1.2	1	0.95	223.8	1.92	135.1	39
3.	1.5	1.7	1.1	1.3	0.9	233.6	1.64	159.5	31