Amitava Chatterjee
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# Advances in Heuristic Signal Processing and Applications

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Editors

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# Preface

The development of sophisticated, state-of-the-art, signal processing algorithms and their suitable applications has evolved and remained a research activity of primary interest for more than 40 years now with special emphasis on the domains of communication, control theory, estimation, pattern recognition, design of electrical and electronic systems, mechanical systems, etc. Both one dimensional signal processing and multidimensional signal processing, namely image and video processing, have received tremendous research attention in recent years. Although the original focus was to develop traditional algorithms, perform in-depth analysis and then try to improve upon their performance from different perspectives, in recent times there has been a significant interest in applying heuristic based methods in solving signal processing problems.

This book makes a humble attempt in offering a collection of notable works that have recently contributed in the domain of heuristic signal processing, both in developing suitable general purpose algorithms and also in solving specialized application problems. Special emphasis was put on collecting several works that attempt to propose several heuristic, iterative optimization methods, essentially employing modern evolutionary and swarm intelligence based optimization techniques, specially employed for solving several relevant signal and image processing problems. Many of these problems under consideration originate from several important domains like the fields of communication engineering, estimation and tracking problems, the evergreen digital filter design problems, wireless sensor network problems, bioelectric signal classification problems, image denoising, image feature tracking problems, etc. We do hope that the sheer variety of the problems discussed in this book along with different techniques employed in solving them will arouse great interest among a large section of signal and image processing researchers all over the world.

Chapter 1 by Ling, Ho, and Teo shows how a two-channel linear phase FIR quadrature mirror filter bank minimax design problem can be formulated in terms of a nonconvex optimization problem. This optimization problem attempts to minimize a weighted sum of several performance indices like the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude, and the maximum stopband ripple magnitude of the prototype filter. The chapter discusses in great detail how a joint norm relaxed sequential quadratic programming and filled function method can be utilized for finding the global minimum of the nonconvex optimization problem. The authors show the effectiveness and utility of the proposed method using several suitable case studies.

Chapter 2 by de Lamare focuses on developing robust reduced-rank linearly constrained minimum variance (LCMV) beamforming algorithms, utilizing the notion of the joint iterative optimization of parameters. The author utilizes the concept of constrained robust joint iterative optimization of parameters based on the minimum variance criterion. He shows how this optimization procedure can be suitably utilized to adjust the parameters of a rank-reduction matrix, a reduced-rank beamformer, and the diagonal loading in an alternating manner. de Lamare also demonstrates how stochastic gradient and recursive least-squares adaptive algorithms can be devised for suitable implementation of this optimization technique based robust beamforming methodology.

In Chap. 3, Sen, Tang, and Nehorai propose a multi-objective optimization based methodology that can be used to design an orthogonal frequency division multiplexing (OFDM) radar signal, used for detection of a moving target in the presence of multipath reflections. They discuss in detail the development of a parametric OFDM measurement model for a particular range cell under test, and how it can be converted to an equivalent sparse-model by considering the target returns over all the possible signal paths and target velocities. They utilize the multi-objective optimization procedure for designing the spectral-parameters of the transmitting OFDM waveform by simultaneously optimizing three objective functions: (i) maximizing the Mahalanobis distance, (ii) minimizing the weighted trace of the Cramer–Rao bound matrix for the unknown parameters, and (iii) minimizing the upper bound on the sparse recovery error.

Chapter 4 by Kwolek demonstrates the utilization of the particle swarm optimization (PSO) algorithm for multi-target tracking problems. Here PSO is employed to track the local mode of the similarity measure and a suitable objective function is developed utilizing the region covariance matrix and multi-patch based object representation. In this process, the target locations and velocities are determined and thereafter they are employed in a PSO based procedure for further refinements of the extracted trajectories. In the last stage, the algorithm utilizes a conjugate method for the final optimization, and the suitability of the proposed algorithm is aptly demonstrated by evaluating performance on publicly available datasets.

Chapter 5 by Boussaïd, Chatterjee, Siarry, and Ahmed-Nacer studies the performance of a wireless sensor network in the context of binary detection of a deterministic signal. This work considers a decentralized organization of spatially distributed sensor nodes and considers the development of an optimal power allocation scheme that will minimize the total power spent by the whole sensor network under a desired detection error probability. The chapter considers two scenarios for the fusion of binary decisions, depending on whether the observations are independent and identically distributed or correlated. This work shows how this problem can be solved utilizing different variations of biogeography-based optimization algorithms and compares their performances vis-à-vis similar problems solved using GA and PSO.

In Chap. 6, Aslan and Saranlı focus on the problem of detection threshold optimization in a tracker-aware manner so that a feedback from the tracker to the detector is established, in a bid to maximize the overall system performance. This chapter puts high emphasis on the development of optimization schemes for the probabilistic data association filter utilizing the modified Riccati equation (MRE) and the hybrid conditional averaging (HYCA) algorithm. The chapter also proposes a closed-form solution for the MRE-based dynamic threshold optimization problem.

Chapter 7 by Luitel and Venayagamoorthy introduces the iterative design of finite impulse response filters using PSO with the quantum infusion (PSO-QI) algorithm. The design specification utilizes two methods for calculating performance indices: (i) minimizing the mean-squared error between the actual and the ideal filter response and (ii) minimizing the mean-squared error between the ripples in the passband and the stopband of the designed filter and the desired filter specification. The chapter shows performance comparisons vis-à-vis the constrained least-squares method of filter design.

Chapter 8 by Sengupta, Chakraborti, and Konar introduces Invasive Weed Optimization (IWO) based algorithms for solving two-dimensional IIR digital filter design problems. The authors develop an improved variant of IWO by introducing a constriction factor in the seed dispersal phase. The design algorithm is developed using temporal difference Q-Learning and it falls under the category of special types of adaptive Memetic Algorithms.

Chapter 9 by Castella, Moreau, and Zarzoso discusses a survey of kurtosis optimization schemes employed for MISO source separation and equalization problems. The chapter provides an in-depth review of some of the most widely employed iterative algorithms utilizing kurtosis for MISO source separation and equalization. These methods include gradient and Newton search based methods, algorithms with optimal step-size selection, and also algorithms based on reference signals. The authors show the efficacy of these algorithms by presenting the performance evaluation for case studies chosen from the fields of digital communications and biomedical signal processing.

Chapter 10 by Nobahari, Sharifi, and MohammadKarimi proposes a new class of filters, based on swarm intelligence, for the purpose of nonlinear systems state estimation. The authors show how such swarm filters can be formulated for a nonlinear system state estimation problem as a stochastic dynamic optimization problem. The chapter shows how successfully PSO and ant colony optimization can be utilized for this estimation purpose and how they perform vis-à-vis other popular nonlinear filters such as the unscented Kalman filter, etc.

In Chap. 11, Pan and Chang demonstrate how the design of multiplier-less digital filters can be carried out utilizing Canonic Signed Digit (CSD) code. The chapter solves the optimum design problem for digital filters utilizing GA. This chapter introduces the concept of CSD coded GA which can effectively reduce the time consumed in the process of the evolution and thus can accelerate the training speed. This chapter also introduces a new hybrid code for the filter coefficients that can be instrumental in improving the precision of the coefficients of a digital filter. The chapter examines the design of both Finite-Impulse Response and Infinite-Impulse Response Filters in this context.

Chapter 12 by Dutta, Chatterjee, and Munshi presents a thorough discussion on the development of robust algorithms for pathological classification of human gait signals. The chapter shows how cross-correlograms can be utilized for feature extraction and how both time and frequency domain based features can be extracted from cross-correlation procedures. These features are used as inputs for Elman's recurrent neural network (ERNN) based classifiers for automatic identification of healthy subjects and those with neurological disorder, and also the type of disorder, e.g., people suffering from Parkinson's disease (PD) or Huntington's disease (HD), or Amyotrophic Lateral Sclerosis (ALS). The chapter discusses how modular ERNNs can be utilized to develop composite classifiers to improve classification accuracy. The performances of such systems developed are compared with similar systems developed utilizing back propagation neural network (BPNN), learning vector quantization (LVQ), and least-squares support vector machine (LS-SVM) based classification algorithms.

In Chap. 13, Abdeldjalil Ouahabi presents a detailed review of image denoising techniques using wavelets which are specifically aimed at medical imaging applications. He specifically considers medical ultrasound and magnetic resonance images and discusses the denoising performances using the well known indices of SNR (or PSNR) and visual aspects of image quality. In the process, he highlights an important fact that image denoising using wavelet-based multi-resolution analysis requires employment of a judicious compromise between noise reduction and preserving significant image details. Hence, the author emphasizes the importance of employing heuristics to supplement theory and making it simpler for practical applications to involve less complexity.

Chapter 14 by Guo and Ruan presents a sparse representation method for single-channel signal separation with a priori knowledge. The key features of the proposed method include dictionary constructions and pursuit algorithms for finding sparse representations. The chapter also presents an overview of popular schemes that are commonly employed to achieve these two key features. The performance evaluation shows that the proposed method can efficiently separate the overlapping resonances and the baseline.

Chapter 15 by Raphael, Philippe, and Christine performs a detailed review of different approaches to the introduction of a color monogenic wavelet transform, that offer a geometric representation of grayscale images through an AM/FM model that facilitates invariance of coefficients to translations and rotations. The authors start from the grayscale monogenic wavelets together with a color extension of the monogenic signal based on geometric algebra and move on by giving a step-by-step description.

Chapter 16 by Pissaloux, Maybank, and Velázquez gives a vivid description of the state-of-the-art in image and feature matching, both in 2D and 3D, specifically aimed at the embedded or wearable real-time system implementations. The chapter first discusses relaxation, maximal clique, tree search, region growing, and dynamic programming based methods. This is followed by a discussion on the popular correlation-based methods, with a fixed size or adaptive sized window, pyramidal methods, the iterative closest point (ICP) algorithm, and probability (saliency)-based approaches.

In the end, we, the editors of this volume, would like to thank everyone who has contributed directly or indirectly in making this project happen. We would specially like to thank all chapter contributors who have made notable contributions in their own ways by writing their chapters and enriching this book. Their timely contributions and active cooperation helped the process to be smooth. Now we sincerely hope that the final product will satisfy our readers all over the world and will be useful, in a small way, in further enriching their subject knowledge and will help them to be better equipped in their future research endeavors.

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# Chapter 1
# Nonconvex Optimization via Joint Norm Relaxed SQP and Filled Function Method with Application to Minimax Two-Channel Linear Phase FIR QMF Bank Design

**Bingo Wing-Kuen Ling, Charlotte Yuk-Fan Ho, and Kok-Lay Teo**

**Abstract** In this chapter, a two-channel linear phase finite impulse response (FIR) quadrature mirror filter (QMF) bank minimax design problem is formulated as a nonconvex optimization problem so that a weighted sum of the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude, and the maximum stopband ripple magnitude of the prototype filter is minimized subject to specifications on these performances. A joint norm relaxed sequential quadratic programming and filled function method is proposed for finding the global minimum of the nonconvex optimization problem. Computer numerical simulations show that our proposed design method is efficient and effective.

## 1.1 Introduction

Filters are fundamental building blocks of many engineering systems, such as in multimedia [13] and communication [1] systems. Hence, developing efficient and effective filter design methods are essential. There are many filter design techniques such as the window design method. However, these design methods have some limitations. For example, it is necessary to find a closed form of the impulse responses of the filters. Also, the bandwidths of the transition bands and the ripples of different frequency bands are approximately the same. Because of these limitations, it is required to develop new methodologies for designing filters [8–11, 14, 28].

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On the other hand, there are many advantages of minimax two-channel linear phase FIR QMF banks. Since transition bandwidths of the filters in two-channel filter banks are usually larger than those in multi-channel filter banks, lengths of the filters in two-channel filter banks are usually shorter than those in multi-channel filter banks. Moreover, as only a single prototype filter is required for the design of a QMF bank and all other filters are derived from the prototype filter, the total number of filter coefficients required for the design of a QMF bank is usually smaller than those in general filter banks. Furthermore, as the linear phase property of the filters guarantees no phase distortion of the filter bank and the FIR property of the filters guarantees the bounded input bounded output (BIBO) stability of the filter bank, two-channel linear phase FIR QMF banks find many applications in image and video signal processing [22].

However, there are different considerations for two-channel linear phase FIR QMF bank designs compared to other filter bank designs [5–7, 12]. For example, unlike a multi-channel QMF bank [24, 26], a two-channel QMF bank cannot achieve the exact perfect reconstruction with the prototype filter having very good frequency selectivity [23]. Hence, it is useful to design a two-channel QMF bank so that a weighted sum of the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter is minimized subject to specifications on these performances. Nevertheless, this QMF bank minimax design problem is a nonconvex optimization problem subject to many nonlinear constraints. Since there are many nonlinear constraints, the corresponding dual problem consists of many Lagrange multipliers and the objective function of the dual problem is a nonlinear function of many variables. It is very difficult to find a locally optimal solution of the dual problem. Moreover, as nonconvex optimization problems usually consist of many local minima [32], it is usually stuck at these local minima. Hence, it is very difficult to find the global minima.

Gradient descent based methods are the most common approaches for finding locally optimal solutions of optimization problems. Due to the convergence issues, adaptive step sizes are used. However, when the step sizes are changed adaptively, in general it is not guaranteed to reach locally optimal solutions. There are mainly two different approaches for finding the global minima of nonconvex optimization problems. The first type of the approaches is nongradient based approaches, such as evolutionary algorithm based approaches [27, 29]. These approaches keep generating evaluation points randomly. Those evaluation points with better performances are kept, while those evaluation points with poor performances are ignored. However, these nongradient based approaches are not efficient. This is because most of the evaluation points are ignored and computational efforts are wasted. The second type of the approaches is filled function approaches [3, 4, 25, 30, 31, 33]. The working principles of the filled function methods are to find a sequence of locally optimal solutions and guarantee that their objective functional values are monotonic decreasing. For those optimization problems with a finite number of local minima, this method guarantees to reach globally optimal solutions. As the objective functional values of the locally optimal solutions are monotonic decreasing, better solutions are guaranteed for next iterations. Hence, this method is very efficient.

In this chapter, a joint norm relaxed sequential quadratic programming and filled function method is proposed for finding the global minimum of a two-channel linear phase FIR QMF bank minimax design problem. A locally optimal solution of the optimization problem in each iteration of the filled function method is found by the norm relaxed sequence quadratic programming method [2, 15–21, 34]. The globally optimal solution of the original optimization problem is found by the filled function method. The outline of this chapter is as follows. In Sect. 1.2, a two-channel linear phase FIR QMF bank minimax design problem is formulated as a nonconvex optimization problem. In Sect. 1.3, the globally optimal solution of the optimization problem is found by a joint norm relaxed sequential quadratic programming and filled function method. In Sect. 1.4, computer numerical simulations are presented. Finally, conclusions are drawn in Sect. 1.5.

## 1.2 Problem Formulation

Let us denote the transpose operator, the conjugate operator, and the conjugate transpose operator by the superscripts $T$, $*$ and $+$, respectively, and the modulus operator as $|\cdot|$. Let the transfer functions of the lowpass and the highpass analysis filters of a two-channel linear phase FIR QMF bank be $H_0(z)$ and $H_1(z)$, respectively, and those of the synthesis filters of the filter bank be $F_0(z)$ and $F_1(z)$, respectively. Here, $H_0(z)$ is the transfer function of the prototype filter. Let us denote the impulse response of the prototype filter as $h(n)$, the passband and the stopband of the prototype filter as $B_p$ and $B_s$, respectively, the length of the prototype filter as $N$, the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter as $\delta_p$ and $\delta_s$, respectively, the specifications on the acceptable bounds on the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter as $\varepsilon_p$ and $\varepsilon_s$, respectively, and the desired magnitude response of the prototype filter as $D(\omega)$. In this chapter, it is assumed that the prototype filter is of even length and symmetric. Let the polyphase components of $H_0(z)$ be $E_0(z^2)$ and $E_1(z^2)$, that is,

$$H_0(z) \equiv E_0\left(z^2\right) + z^{-1} E_1\left(z^2\right). \tag{1.1}$$

Let us denote the transfer function of the filter bank as $T(z)$, the maximum amplitude distortion of the filter bank as $\delta_a$, and the specification on the acceptable bound on the maximum amplitude distortion of the filter bank as $\varepsilon_a$. Let the vector containing these distortions and the even-time index filter coefficients be $\underline{\mathrm{x}}$, that is,

$$\underline{\mathrm{x}} \equiv \left[\delta_a, \delta_p, \delta_s, h(0), h(2), \ldots, h(N-2)\right]^T. \tag{1.2}$$

In order to achieve both the aliasing-free condition and the QMF pairs condition, the relationships among the analysis filters and the synthesis filters are governed by

$$H_1(z) = H_0(-z), \tag{1.3}$$

$$F_0(z) = 2H_0(z), \tag{1.4}$$

and

$$F_1(z) = -2H_0(-z). \tag{1.5}$$

As the prototype filter is of even length and symmetric, we have

$$H_0(z) = \sum_{n=0}^{\frac{N}{2}-1} h(2n)z^{-2n} + z^{-1} \sum_{n=0}^{\frac{N}{2}-1} h(2n)z^{-(N-2-2n)}, \tag{1.6}$$

$$E_0(z) = \sum_{n=0}^{\frac{N}{2}-1} h(2n)z^{-n}, \tag{1.7}$$

$$E_1(z) = \sum_{n=0}^{\frac{N}{2}-1} h(2n)z^{-(\frac{N}{2}-1-n)} = z^{-(\frac{N}{2}-1)} E_0\left(z^{-1}\right), \tag{1.8}$$

and

$$T(z) = 4z^{-1} E_0\left(z^2\right) E_1\left(z^2\right) = 4z^{-(N-1)} E_0\left(z^2\right) E_0\left(z^{-2}\right). \tag{1.9}$$

Let us denote

$$\eta(\omega) \equiv \left[0, 0, 0, 1, e^{-j\omega}, \ldots, e^{-j(\frac{N}{2}-1)\omega}\right]^T, \tag{1.10}$$

then

$$T(\omega) = 4e^{-j\omega(N-1)} \underline{\mathrm{x}}^T \left(\eta(2\omega)\right)^* \left(\eta(2\omega)\right)^T \underline{\mathrm{x}}. \tag{1.11}$$

Obviously, the filter bank does not suffer from the phase distortion, and the amplitude distortion of the filter bank can be expressed as $|4\underline{\mathrm{x}}^T (\eta(2\omega))^* (\eta(2\omega))^T \underline{\mathrm{x}} - 1|$. Let us denote

$$\underline{\mathrm{Q}}(\omega) = 8\left(\eta(2\omega)\right)^* \left(\eta(2\omega)\right)^T, \tag{1.12}$$

then the amplitude distortion of the filter bank can be expressed as $|\frac{1}{2}\underline{\mathrm{x}}^T \underline{\mathrm{Q}}(\omega)\underline{\mathrm{x}} - 1|$. Let us denote

$$\iota_a \equiv [1, 0, \ldots, 0]^T, \tag{1.13}$$

then the constraint on the maximum amplitude distortion of the filter bank can be expressed as

$$\frac{1}{2}\underline{\mathrm{x}}^T \underline{\mathrm{Q}}(\omega)\underline{\mathrm{x}} - \iota_a^T \underline{\mathrm{x}} - 1 \le 0 \tag{1.14}$$

and

$$-\frac{1}{2}\underline{\mathrm{x}}^T \underline{\mathrm{Q}}(\omega)\underline{\mathrm{x}} - \iota_a^T \underline{\mathrm{x}} + 1 \le 0 \quad \forall\omega \in [-\pi, \pi]. \tag{1.15}$$

Let us denote

$$\kappa(\omega) \equiv 2\left[0, 0, 0, \cos\left(\frac{N-1}{2}\omega\right), \cos\left(\frac{N-5}{2}\omega\right), \dots, \cos\left(\frac{3-N}{2}\omega\right)\right]^T, \quad (1.16)$$

then

$$H_0(\omega) = \left(\eta(2\omega)\right)^T \underline{x} + e^{-j\omega(N-1)}\left(\eta(2\omega)\right)^+ \underline{x}, \quad (1.17)$$

therefore

$$\begin{aligned} H_0(\omega) = e^{-j\omega\frac{N-1}{2}} \times \Big(&\left[0, 0, 0, e^{j\omega\frac{N-1}{2}}, e^{j\omega\frac{N-5}{2}}, \dots, e^{-j\omega\frac{N-3}{2}}\right]\underline{x} \\ &+ \left[0, 0, 0, e^{-j\omega\frac{N-1}{2}}, e^{-j\omega\frac{N-5}{2}}, \dots, e^{j\omega\frac{N-3}{2}}\right]\underline{x}\Big). \end{aligned} \quad (1.18)$$

Finally,

$$H_0(\omega) = e^{-j\omega\frac{N-1}{2}} \left(\kappa(\omega)\right)^T \underline{x}, \quad (1.19)$$

and the passband ripple magnitude of the prototype filter can be expressed as $|(\kappa(\omega))^T \underline{x} - D(\omega)| \ \forall \omega \in B_p$. Let us define

$$\iota_p \equiv [0, 1, 0, \dots, 0]^T, \quad (1.20)$$

then the constraint on the maximum passband ripple magnitude of the prototype filter can be expressed as

$$\left|\left(\kappa(\omega)\right)^T \underline{x} - D(\omega)\right| \le \iota_p^T \underline{x} \quad \forall \omega \in B_p. \quad (1.21)$$

Let us define

$$\underline{A}_p(\omega) \equiv \left[\kappa(\omega) - \iota_p, -\kappa(\omega) - \iota_p\right]^T \quad (1.22)$$

and

$$\underline{c}_p(\omega) \equiv \left[D(\omega), -D(\omega)\right]^T, \quad (1.23)$$

then the constraint on the maximum passband ripple magnitude of the prototype filter can be further expressed as

$$\underline{A}_p(\omega)\underline{x} - \underline{c}_p(\omega) \le \underline{0} \quad \forall \omega \in B_p. \quad (1.24)$$

Similarly, let us define

$$\iota_s \equiv [0, 0, 1, 0, \dots, 0]^T, \quad (1.25)$$

$$\underline{A}_s(\omega) \equiv \left[\kappa(\omega) - \iota_s, -\kappa(\omega) - \iota_s\right]^T, \quad (1.26)$$

and

$$\underline{c}_s(\omega) \equiv \left[D(\omega), -D(\omega)\right]^T, \quad (1.27)$$

then the constraint on the maximum stopband ripple magnitude of the prototype filter can be expressed as

$$\underline{A}_s(\omega)\underline{x} - \underline{c}_s(\omega) \le \underline{0} \quad \forall \omega \in B_s. \tag{1.28}$$

Let us define

$$\underline{A}_b \equiv [\underline{I}, \underline{0}] \tag{1.29}$$

and

$$\underline{c}_b \equiv [\varepsilon_a, \varepsilon_p, \varepsilon_s]^T, \tag{1.30}$$

in which $\underline{I}$ is the $3 \times 3$ identity matrix, then the specifications on the acceptable bounds on the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude, and the maximum stopband ripple magnitude of the prototype filter can be expressed as

$$\underline{A}_b\underline{x} - \underline{c}_b \le \underline{0}. \tag{1.31}$$

In order to minimize a weighted sum of the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude, and the maximum stopband ripple magnitude of the prototype filter subject to the specifications on these performances, the filter bank design problem is formulated as the following optimization problem:

Problem ($\underline{P}$)

$$\min_{\underline{x}} f(\underline{x}) \equiv (\alpha \iota_a + \beta \iota_p + \gamma \iota_s)^T \underline{x}, \tag{1.32}$$

subject to

$$g_1(\underline{x}, \omega) \equiv \frac{1}{2}\underline{x}^T \underline{Q}(\omega)\underline{x} - \iota_a^T \underline{x} - 1 \le 0 \quad \forall \omega \in [-\pi, \pi], \tag{1.33}$$

$$g_2(\underline{x}, \omega) \equiv -\frac{1}{2}\underline{x}^T \underline{Q}(\omega)\underline{x} - \iota_a^T \underline{x} + 1 \le 0 \quad \forall \omega \in [-\pi, \pi], \tag{1.34}$$

$$g_3(\underline{x}, \omega) \equiv \underline{A}_p(\omega)\underline{x} - \underline{c}_p(\omega) \le \underline{0} \quad \forall \omega \in B_p, \tag{1.35}$$

$$g_4(\underline{x}, \omega) \equiv \underline{A}_s(\omega)\underline{x} - \underline{c}_s(\omega) \le \underline{0} \quad \forall \omega \in B_s, \tag{1.36}$$

and

$$g_5(\underline{x}) \equiv \underline{A}_b\underline{x} - \underline{c}_b \le \underline{0}, \tag{1.37}$$

where $\alpha$, $\beta$, and $\gamma$ are the weights of different criteria for formulating the objective function, $f(\underline{x})$ is the objective function, and $g_1(\underline{x}, \omega)$, $g_2(\underline{x}, \omega)$, $g_3(\underline{x}, \omega)$, $g_4(\underline{x}, \omega)$, and $g_5(\underline{x})$ are the constraint functions of the optimization problem.

As the set of the filter coefficients satisfying the constraints (1.33) and (1.34) is nonconvex, the optimization problem is a nonconvex optimization problem. In general, it is difficult to find the global minimum of the nonconvex optimization problem.

## 1.3 Joint Norm Relaxed Sequential Quadratic Programming and Filled Function Method

A joint norm relaxed sequential quadratic programming and the filled function method is proposed for finding the globally optimal solution of Problem ($\underline{\text{P}}$). The details of the proposed method are discussed below.

### *1.3.1 Filled Function Method*

Some terminologies related to filled functions are discussed below. Notably, a basin of a function is defined as the subset of the domain of the optimization variables such that any points in this subset will yield the same local minimum of the function via conventional gradient based optimization methods. A hill of a function is defined as the subset of the domain of the optimization variables such that any points in this subset will yield the same local maximum of the function via conventional gradient based optimization methods. A higher basin of a function is a basin of the function with the objective functional value of the local minimum of the basin being higher than that of the current basin of the function. A lower basin of a function is a basin of the function with the objective functional value of the local minimum of the basin being lower than that of the current basin of the function.

A filled function is a function satisfying the following properties: (a) the current local minimum of the original objective function is the current local maximum of the filled function; (b) the whole current basin of the original objective function is a part of the current hill of the filled function; (c) the filled function has no stationary point in any higher basins of the original objective function; and (d) there exists a local minimum of the filled function which is in a lower basin of the original objective function.

The working principles of the filled function method are as follows. Due to property (a), by evaluating the filled function at a point slightly deviated from the current local minimum of the original objective function, a lower functional value will be obtained. Hence, the filled function could kick out from the current local minimum of the original objective function. Due to properties (b)–(d), the current local minimum of the filled function is neither in the current basin nor in any higher basins of the original objective function. Hence, the current local minimum of the filled function is in a lower basin of the original objective function. As a result, by finding the next local minimum of the original objective function via conventional gradient based methods with the initial point being the current local minimum of the filled function, a better local minimum of the original objective function can be obtained. Following these procedures, if the original objective function contains a finite number of local minima, then the global minimum of the original objective function will be eventually reached.

Based on the above working principles, the algorithm is summarized below.

**Algorithm 1**

Step 1: Initialize a minimum improvement factor $\varepsilon$, an acceptable error $\varepsilon'$, an initial search point $\tilde{\underline{x}}_1$, a positive definite matrix $\underline{R}$, and an iteration index $k = 1$.

Step 2: Find a local minimum of the following optimization Problem ($\underline{P}_f$) using the norm relaxed sequential quadratic programming method with the initial search point $\tilde{\underline{x}}_k$.

Problem ($\underline{P}_f$)

$$\min_{\underline{x}} f(\underline{x}) = (\alpha\iota_a + \beta\iota_p + \gamma\iota_s)^T \underline{x}, \tag{1.38}$$

subject to

$$g_1(\underline{x}, \omega) = \frac{1}{2}\underline{x}^T \underline{Q}(\omega)\underline{x} - \iota_a^T \underline{x} - 1 \le 0 \quad \forall \omega \in [-\pi, \pi], \tag{1.39}$$

$$g_2(\underline{x}, \omega) = -\frac{1}{2}\underline{x}^T \underline{Q}(\omega)\underline{x} - \iota_a^T \underline{x} + 1 \le 0 \quad \forall \omega \in [-\pi, \pi], \tag{1.40}$$

$$g_3(\underline{x}, \omega) = \underline{A}_p(\omega)\underline{x} - \underline{c}_p(\omega) \le \underline{0} \quad \forall \omega \in B_p, \tag{1.41}$$

$$g_4(\underline{x}, \omega) = \underline{A}_s(\omega)\underline{x} - \underline{c}_s(\omega) \le \underline{0} \quad \forall \omega \in B_s, \tag{1.42}$$

$$g_5(\underline{x}) = \underline{A}_b\underline{x} - \underline{c}_b \le \underline{0}, \tag{1.43}$$

$$g_6(\underline{x}) \equiv \iota_a^T \left(\underline{x} - (1-\varepsilon)\tilde{\underline{x}}_k\right) \le \underline{0}, \tag{1.44}$$

$$g_7(\underline{x}) \equiv \iota_p^T \left(\underline{x} - (1-\varepsilon)\tilde{\underline{x}}_k\right) \le \underline{0}, \tag{1.45}$$

and

$$g_8(\underline{x}) \equiv \iota_s^T \left(\underline{x} - (1-\varepsilon)\tilde{\underline{x}}_k\right) \le \underline{0}, \tag{1.46}$$

where $g_6(\underline{x})$, $g_7(\underline{x})$, and $g_8(\underline{x})$ are the constraint functions we imposed. Let us denote the obtained local minimum as $\underline{x}_k^*$.

Step 3: Find a local minimum of the following optimization Problem ( $\underline{P}_H$) using the norm relaxed sequential quadratic programming method with the initial search point $\underline{x}_k^*$.

Problem ($\underline{P}_H$)

$$\min_{\underline{x}} H(\underline{x}) \equiv (\alpha\iota_a + \beta\iota_p + \gamma\iota_s)^T \underline{x} + \frac{1}{(\underline{x} - \underline{x}_k^*)^T \underline{R}(\underline{x} - \underline{x}_k^*)}, \tag{1.47}$$

subject to

$$g_1(\underline{x}, \omega) = \frac{1}{2}\underline{x}^T \underline{Q}(\omega)\underline{x} - \iota_a^T \underline{x} - 1 \le 0 \quad \forall \omega \in [-\pi, \pi], \tag{1.48}$$

$$g_2(\underline{x}, \omega) = -\frac{1}{2}\underline{x}^T \underline{Q}(\omega)\underline{x} - \iota_a^T \underline{x} + 1 \le 0 \quad \forall \omega \in [-\pi, \pi], \tag{1.49}$$

$$g_3(\underline{x}, \omega) = \underline{A}_p(\omega)\underline{x} - \underline{c}_p(\omega) \le \underline{0} \quad \forall \omega \in B_p, \tag{1.50}$$

$$g_4(\underline{x}, \omega) = \underline{A}_s(\omega)\underline{x} - \underline{c}_s(\omega) \le \underline{0} \quad \forall \omega \in B_s, \tag{1.51}$$

$$g_5(\underline{x}) = \underline{A}_b\underline{x} - \underline{c}_b \le \underline{0}, \tag{1.52}$$

$$g'_6(\underline{x}) \equiv \iota_a^T \left(\underline{x} - (1-\varepsilon)\underline{x}_k^*\right) \le \underline{0}, \tag{1.53}$$

$$g'_7(\underline{x}) \equiv \iota_p^T \left(\underline{x} - (1-\varepsilon)\underline{x}_k^*\right) \le \underline{0}, \tag{1.54}$$

and

$$g'_8(\underline{x}) \equiv \iota_s^T \left(\underline{x} - (1-\varepsilon)\underline{x}_k^*\right) \le \underline{0}, \tag{1.55}$$

where $H(\underline{x})$ is the filled function we defined, $g'_6(\underline{x})$, $g'_7(\underline{x})$, and $g'_8(\underline{x})$ are the constraint functions we imposed. Let us denote the obtained local minimum as $\tilde{\underline{x}}_{k+1}$. Increment the value of $k$.

Step 4: Iterate Step 2 and Step 3 until

$$\left\| (\alpha\iota_a + \beta\iota_p + \gamma\iota_s)^T \left(\underline{x}_k^* - \underline{x}_{k+1}^*\right) \right\| \le \varepsilon'. \tag{1.56}$$

Take the final vector of $\underline{x}_k^*$ as the global minimum of the original optimization problem.

Step 1 is an initialization of the proposed algorithm. In order not to terminate the algorithm when the convergence of the algorithm is slow and to have a high accuracy of the solution, both $\varepsilon$ and $\varepsilon'$ should be chosen as small values. Also, as $\tilde{\underline{x}}_1$ is an initial search point of the optimization algorithm, this initial search point should be in the feasible set. However, in general it is difficult to guarantee that $\tilde{\underline{x}}_1$ is in the feasible set, so it is chosen in such a way that most of the constraints are satisfied. Moreover, as $\underline{R}$ is a positive definite matrix, it controls the spread of the hill of $H(\underline{x})$ at $\underline{x}_k^*$. If $\underline{R}$ is a diagonal matrix with all diagonal elements being the same and positive, then small values of these diagonal elements will result to a wide spread of the hill of $H(\underline{x})$ at $\underline{x}_k^*$ and vice versa. Since the local minima of nonconvex optimization problems could be located far away from each other, the spread of the hill of $H(\underline{x})$ at $\underline{x}_k^*$ should be large and the diagonal elements of $\underline{R}$ should be chosen as small positive numbers. Step 2 is to find a local minimum of $f(\underline{x})$. As the constraints $g_6(\underline{x})$, $g_7(\underline{x})$, and $g_8(\underline{x})$ are imposed on the Problem ($\underline{P}_f$), the maximum amplitude distortion of the filter bank, the maximum ripple magnitude, and the maximum stopband ripple magnitude of the prototype filter corresponding to the new obtained local minimum are guaranteed to be lower than those corresponding to $\tilde{\underline{x}}_k$. Similarly, Step 3 is to find a local minimum of $H(\underline{x})$. As the constraints $g'_6(\underline{x})$, $g'_7(\underline{x})$, and $g'_8(\underline{x})$ are imposed on the Problem ($\underline{P}_H$), the maximum amplitude distortion of the filter bank, the maximum ripple magnitude, and the maximum stopband ripple magnitude of the prototype filter are guaranteed to be lower than those corresponding to $\underline{x}_k^*$. Step 4 is a termination test procedure. If the difference of the weighted performance between two consecutive iterations is smaller than a certain bound $\varepsilon'$, then the algorithm is terminated.

It has been discussed above that conventional filled function methods require that (a) the current local minimum of the original objective function be the current local maximum of the filled function; (b) the whole current basin of the original objective function be a part of the current hill of the filled function; (c) the filled function have no stationary point in any higher basins of the original objective function; and (d) there exist a local minimum of the filled function which is in a lower basin of the original objective function. As $\underline{R}$ is a positive definite matrix and $\underline{x}_k^*$ is in the denominator of $H(\underline{x})$, $H(\underline{x}) \to +\infty$ as $\underline{x} \to \underline{x}_k^*$. Hence, $\underline{x}_k^*$ is the global maximum of $H(\underline{x})$ and property (a) is guaranteed to be satisfied. As the constraints $g_6'(\underline{x})$, $g_7'(\underline{x})$, and $g_8'(\underline{x})$ are imposed on the Problem $(\underline{P}_H)$, when a new local minimum of $H(\underline{x})$ is found, this new local minimum of $H(\underline{x})$ will not be located at $\underline{x}_k^*$ and the original objective value evaluated at $\tilde{\underline{x}}_{k+1}$ will guarantee to be lower than that at $\underline{x}_k^*$. Hence, properties (b)–(d) are guaranteed to be satisfied. As a result, the proposed algorithm is guaranteed to reach the global minimum of the nonconvex optimization problem.

As the efficiency of general nonconvex optimization algorithms would depend on the initial search points, the total number of local minima of the optimization problems, and the stopping criteria of the optimization algorithms, there is always a tradeoff between the accuracy of the obtained solutions and the efficiency of the optimization algorithms. For nongradient based approaches, as most of the evaluation points are ignored, the effectiveness of these algorithms is low. On the other hand, our proposed method is guaranteed to obtain a local minimum in each iteration, the effectiveness of our proposed algorithm is high. Hence, for the same period of time, our proposed method will obtain a better solution than those of nongradient based approaches.

### *1.3.2 Norm Relaxed Sequential Quadratic Programming*

To find locally optimal solutions of both Problem $(\underline{P}_f)$ and Problem $(\underline{P}_H)$, the norm relaxed sequential quadratic programming method is employed. This method is based on the assumptions that the initial point is in the feasible set of the optimization problem and both the objective function and the constraint functions are smooth. The working principles of the norm relaxed sequential quadratic programming method are to find directions of descents via solving quadratic programming problems and to construct a set of new points based on the obtained directions of descents, where the new points are in the feasible set of the original optimization problem and the objective functional values of these new points are monotonic decreasing.

Based on the above working principles, the algorithm is summarized below.

**Algorithm 2**

Step 1: Denote the initial searching vector for the norm relaxed sequential quadratic programming method obtained from Algorithm 1 as $\overline{\underline{x}}_0$, initialize constants $\delta_1 > 0$, $\delta_2 > 0$, $\beta_{-1} > 0$, $\sigma \in (0, 1)$, $\alpha \in (0, 1)$, and $\overline{P} > 0$, as well as define a symmetric positive definite matrix $\underline{B}_0$. Set $k = 0$. By using the integration approach, the functional inequality constraints can be converted to conventional inequality constraints. Let us define

$$\overline{f}(\underline{x}) \equiv f(\underline{x}) \quad \texttt{for Problem} \, (\underline{P}_f), \tag{1.57}$$

$$\overline{f}(\underline{x}) \equiv H(\underline{x}) \quad \texttt{for Problem} \, (\underline{P}_H), \tag{1.58}$$

$$\overline{g}_j(\underline{x}) \equiv \int \left(\max\left(g_j(\underline{x}, \omega), 0\right)\right)^2 d\omega \quad \texttt{for } j = 1, \ldots, 4, \tag{1.59}$$

$$\overline{g}_5(\underline{x}) \equiv g_5(\underline{x}), \tag{1.60}$$

$$\overline{g}_j(\underline{x}) \equiv g_j(\underline{x}) \quad \texttt{for } j = 6, 7, 8, \texttt{ and for Problem} \, (\underline{P}_f), \tag{1.61}$$

$$\overline{g}_j(\underline{x}) \equiv g'_j(\underline{x}) \quad \texttt{for } j = 6, 7, 8, \texttt{ and for Problem} \, (\underline{P}_H), \tag{1.62}$$

$$P(\underline{x}) \equiv \max\left(0, \overline{g}_1(\underline{x}), \ldots, \overline{g}_8(\underline{x})\right), \tag{1.63}$$

and

$$I_0(\underline{x}) \equiv \left\{ j \in \{1, \ldots, 8\} \quad \texttt{such that} \quad \overline{g}_j(\underline{x}) = P(\underline{x}) \right\}. \tag{1.64}$$

Step 2: Solve the following quadratic programming problem:
Problem $(\underline{QP}_k)$

$$\min_{(\underline{d}, z)} z + \frac{1}{2} \underline{d}^T \underline{B}_k \underline{d}, \tag{1.65}$$

subject to

$$\nabla \overline{f}(\underline{x}_k)^T \underline{d} \leq z \tag{1.66}$$

and

$$\overline{g}_j(\underline{x}_k) + \nabla \overline{g}_j(\underline{x}_k)^T \underline{d} \leq z, \quad \texttt{for } j = 1, \ldots, 8. \tag{1.67}$$

Let us denote the obtained solution as $(\underline{d}_k, z_k)$. If $\underline{d}_k = \underline{0}$, then the algorithm terminates.

Step 3: Let us define

$$\triangle_k \equiv \frac{2P(\underline{x}_k) - \underline{d}_k^T \underline{B}_k \underline{d}_k}{\underline{d}_k^T \underline{B}_k \underline{d}_k} + \delta_1, \tag{1.68}$$

$$\beta_k = \beta_{k-1} \quad \texttt{for } \beta_{k-1} \geq \triangle_k, \tag{1.69}$$

$$\beta_k = \triangle_k + \delta_2 \quad \texttt{for } \beta_{k-1} < \triangle_k, \tag{1.70}$$

$$\psi_{\beta_k}(\underline{x}) \equiv \overline{f}(\underline{x}) + \beta_k P(\underline{x}), \tag{1.71}$$

$$P'(\underline{x}_k, \underline{d}_k) \equiv \max\left(\nabla \overline{g}_j(\underline{x}_k)^T \underline{d}_k \quad \texttt{for } j \in I_0(\underline{x}_k), 0\right) \quad \texttt{for } P(\underline{x}_k) = 0, \tag{1.72}$$

$$P'(\underline{x}_k, \underline{d}_k) \equiv \max\left(\nabla \overline{g}_j(\underline{x}_k)^T \underline{d}_k \quad \texttt{for } j \in I_0(\underline{x}_k)\right) \quad \texttt{for } P(\underline{x}_k) > 0, \tag{1.73}$$

and

$$\psi'_{\beta_k}(\underline{\overline{x}}_k, \underline{d}_k) \equiv \nabla \overline{f}(\underline{\overline{x}}_k)^T \underline{d}_k + \beta_k P'(\underline{\overline{x}}_k, \underline{d}_k). \tag{1.74}$$

Find the step size $t_k$ which is defined as the first value in the sequence $\{1, \sigma, \sigma^2, \ldots\}$ such that:

if $P(\underline{\overline{x}}_k) \leq \overline{P}$, then

$$\psi_{\beta_k}(\underline{\overline{x}}_k + t_k \underline{d}_k) \leq \psi_{\beta_k}(\underline{\overline{x}}_k) + \alpha t_k \psi'_{\beta_k}(\underline{\overline{x}}_k, \underline{d}_k), \tag{1.75}$$

if $P(\underline{\overline{x}}_k) > \overline{P}$, then

$$\psi_{\beta_k}(\underline{\overline{x}}_k + t_k \underline{d}_k) \leq \psi_{\beta_k}(\underline{\overline{x}}_k) + \alpha t_k \psi'_{\beta_k}(\underline{\overline{x}}_k, \underline{d}_k), \tag{1.76}$$

and

$$P(\underline{\overline{x}}_k + t_k \underline{d}_k) \leq P(\underline{\overline{x}}_k). \tag{1.77}$$

Step 4: Find a new symmetric positive definite matrix $\underline{B}_{k+1}$ using existing algorithms. Set

$$\underline{\overline{x}}_{k+1} = \underline{\overline{x}}_k + t_k \underline{d}_k. \tag{1.78}$$

Increment the value of $k$ and go back to Step 2.

Step 1 is an initialization of the norm relaxed sequential quadratic programming method. Step 2 is to find the directions of descents via solving quadratic programming problems. The constraints imposed in Problem $(\underline{QP}_k)$ guarantee that the objective functional values of the obtained solutions are monotonic decreasing and within the feasible set of the original optimization problem. As a result, the converged solution of the quadratic programming problems is guaranteed to be a locally optimal solution of the original optimization problem.

As the quadratic programming problems are only subject to linear constraints, the corresponding dual problems only involve simple functions of these Lagrange multipliers. Hence, the quadratic programming problems can be solved via efficient algorithms such as interior point methods. As a result, the proposed method is very efficient and effective for finding locally optimal solutions of the optimization problems in each iteration of the filled function method.

## 1.4 Computer Numerical Simulation Results

In order to have a fair comparison, the performance of the QMF banks designed via our proposed method is compared to that designed via existing minimax approaches [23]. We choose the same passband, stopband, filter length, maximum passband ripple magnitude, maximum stopband ripple magnitude, and desirable magnitude response of the prototype filter as that in [23], that is,

$$B_p = [-0.4\pi, 0.4\pi], \tag{1.79}$$

$$B_s = [0.6\pi, \pi] \cup [-\pi, -0.6\pi], \tag{1.80}$$

$$N = 36, \tag{1.81}$$

$$\varepsilon_p = -50 \text{ dB}, \tag{1.82}$$

$$\varepsilon_s = -50 \text{ dB}, \tag{1.83}$$

$$D(\omega) = 1 \quad \texttt{for } \omega \in B_p, \tag{1.84}$$

and

$$D(\omega) = 0 \quad \texttt{for } \omega \in B_s. \tag{1.85}$$

In order to guarantee that the performance of the QMF bank designed via our proposed method is better than that in [23], the specification on the maximum amplitude distortion of the filter bank is chosen as $\varepsilon_a = -58$ dB, which is better than that in [23] ($\varepsilon_a = 0.003 = -50.4576$ dB). In order not to have any bias among the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude, and the maximum stopband ripple magnitude of the prototype filter, all the weights in the objective function are chosen to be the same, that is, $\alpha = \beta = \gamma = 1$. In this chapter, $\varepsilon = \varepsilon' = 10^{-6}$ are chosen, which is small enough for most applications. $\underline{\tilde{x}}_1$ is chosen as the filter coefficients obtained via the Remez exchange algorithm, which is guaranteed to satisfy the specifications on the maximum passband ripple magnitude and the maximum stopband ripple magnitude of the prototype filter. $\underline{R}$ is chosen as the diagonal matrix with all diagonal elements equal to $10^{-3}$, which is small enough for most applications.

To compare the efficiency of the designed method, our proposed method only takes three iterations to converge and the total time required for the computer numerical simulations is 0.8 seconds. On the other hand, the method discussed in [23] takes 68 iterations to converge and the total time required for the computer numerical simulations is 80 seconds. Hence, it can be concluded that the method discussed in [23] requires more computational efforts than our proposed method and our proposed method is more efficient than that discussed in [23]. The magnitude responses of the filter banks as well as the magnitude responses of the prototype filters in both the passband and the stopband designed via our proposed method are shown in Figs. 1.1, 1.2, 1.3. It can be seen from these figures that the prototype filter designed by our proposed method can achieve $\delta_p = -64.2416$ dB and $\delta_s = -50.3625$ dB, and the QMF bank could achieve $\delta_a = -58.1557$ dB. It can be checked easily that the QMF bank designed via our proposed method achieves better performance with respect to the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude, and the maximum stopband ripple magnitude ripple of the prototype filter than that designed by the method discussed in [23]. This is because the QMF bank designed by the method discussed in [23] is not the global minimum, while that designed by our proposed method is the global minimum.

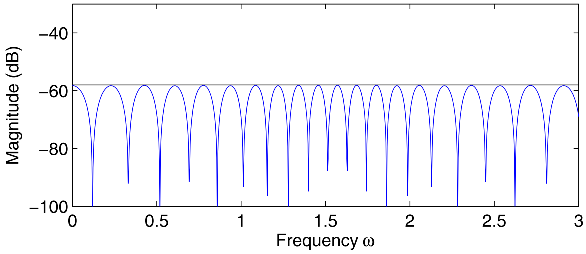


**Fig. 1.1** Magnitude response of the filter bank

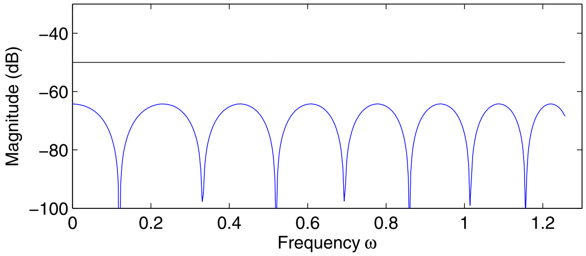


**Fig. 1.2** Magnitude response of the prototype filter in the passband

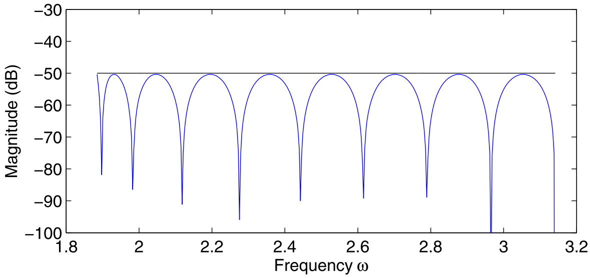


**Fig. 1.3** Magnitude response of the prototype filter in the stopband

## 1.5 Conclusions

This chapter proposes a joint norm relaxed sequential quadratic programming and filled function method for the design of a two-channel linear phase FIR QMF bank so that a weighted sum of the maximum amplitude distortion of the filter bank, the maximum passband ripple magnitude, and the maximum stopband ripple magnitude of the prototype filter is minimized. In particular, a locally optimal solution of the optimization problem in each iteration of the filled function method is found by the norm relaxed sequence quadratic programming method. The globally optimal solution of the original optimization problem is found by the filled function method. Computer numerical simulation results show that the proposed method can find the global minimum of the nonconvex optimization problem efficiently.

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