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Editors

Chaos and Complex Systems

Proceedings of the 4th International Interdisciplinary Chaos Symposium

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SGS: To my beloved wife Olga and my son George Zoes (an expert in creating chaos)

SB: To my wife Mitali and kids Sumangal, Suvayan and Aheli, my perfectly organized chaotic life!

SHC and MO: To the memory of Prof. Dr. Dr. hc Onder Oztunali

Preface

Complexity science and chaos theory, as the study and analysis of nonlinear systems is known, is a very fascinating area of scientific research. Nonlinear systems, although deterministic, demonstrate a strange behavior, due to widely diverging outcomes, provoked by small variations in initial conditions or system parameters that lead to inobservance of long-term system-behavior prediction. This behavior is known as deterministic chaos (simply chaos). The interdisciplinary nature of complexity and chaos is a feature that provides scientists with a global theoretical tool. Complex systems give rise to emergent behaviors that lead to interesting phenomena in science, engineering, as well as social sciences.

The aim of all symposia on chaos and complex systems (CCS) is to convene scientists, engineers, economists, as well as social scientists and discuss the latest improvements in the area of corresponding nonlinear-system complex (chaotic) behavior.

Especially in the "4th International Interdisciplinary Chaos Symposium on Chaos and Complex Systems", which took place from April 29th to May 2nd, 2012 in Antalya, Turkey, the contents of the symposium have been enriched in an interdisciplinary-widening way, so as to allow work from circuits to econophysics and from nonlinear analysis to the history of chaos theory, to be presented. Thus, this symposium became an attractor (a strange for sure) for researchers. It should be mentioned that the organizers' expectations concerning the international resonance of the conference have been fulfilled.

Consequently, the "Conference proceedings: Chaos and Complex Systems—2012," published by Springer Team in the frame of the "Springer Complexity" series, aim to address emerging topics, but not strictly restricted to networks, circuits, systems, biology, evolution and ecology, nonlinear dynamics and pattern formation, as well as neural, psychological, psychosocial, socioeconomic, management complexity, and global systems. These proceedings also aspire to serve as a compact reference book on nonlinear systems, catering to research scholars, interested readers, and advanced learners from multidisciplinary areas.

On behalf of the organizing committee we would like to express our thanks to the Symposium's International Scientific Committee and the Advisory Committee, as well as all those who have contributed to this conference, for their support and advice. We are also grateful to Prof. Dr. Leon CHUA, for his support and the wonderful opening lecture. Our thanks are also due to all the invited lecturers: Prof. Dr. Fatihcan ATAY, Dr. Santo BANERJEE, Prof. Dr. Ernesto ESTRADA, Prof. Dr. Bahman KALANTARI, Prof. Dr. Bulent KARASOZEN, Prof. Dr. Arkady PIKOVSKY, and Prof. Dr. Michael ROSENBLUM. The organizers also thank Istanbul Kultur University and its Rector Prof. Dr. Dursun KOÇER, its Vice Rector Çetin BOLCAL, and Prof. Dr. Tamer KOÇEL for their support and incentive encouraging.

Finally, the editors of this tome are grateful to Springer for the quality of this edition of this volume.

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Contents

Two Element Chaotic and Hyperchaotic Circuits

Bharathwaj Muthuswamy, Andrew Przybylski, Chris Feilbach, and Joerg Mossbrucker

Abstract The goals of this work are twofold: one is to illustrate the use of Field Programmable Gate Arrays (FPGAs) for emulating circuit elements with memory (memristors, memcapacitors and meminductors). The second goal is to use the FPGA emulation to realize two element chaotic and hyperchaotic circuits. Such circuits utilize fully nonlinear models of memory devices in series-parallel configuration.

1 Introduction

The memristor was postulated as the fourth fundamental circuit element by Leon O. Chua in 1971 [1]. It thus took its place along side the rest of the more familiar circuit elements such as the resistor, capacitor, and inductor. The common thread that binds these elements together as the four basic elements of circuit theory is the fact that the characteristics of these elements relate the four fundamental circuit variables (voltage, current, flux-linkage and charge).

For over 30 years, the memristor was not significant in circuit theory. However in 2008, Strukov et al. [12] from Hewlett-Packard labs announced that they had fabricated a solid state implementation of the memristor. Ever since their announcement, a variety of circuit applications of memristors have been developed; refer to [6] for examples and further references.

Chua and Kang [2] first extended the notion of the memristor to a general class of memristive systems. DiVentra et al. [4] incorporated capacitors and inductors into the notion of memory devices, as shown in Fig. 1. This "memory element"

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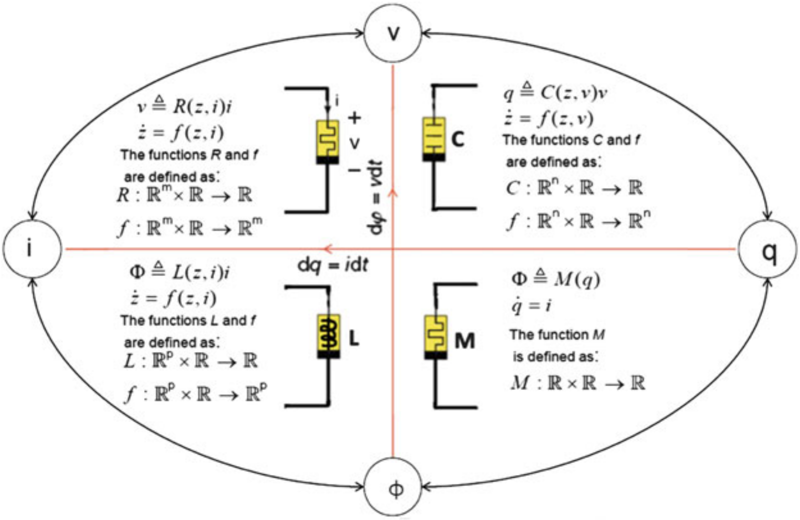

Fig. 1 We can generalize the four basic circuit elements to elements with memory. A resistor is a device that establishes a relation between current (i) and voltage (v), a memristor is a device whose resistance depends on a state variable (z). Similarly memcapacitors establish a memory relationship between charge (q) and voltage; meminductors establish a relationship between flux-linkage (ϕ) and current. Note that an ideal memristor is a special case of a general memristive system—the ideal memristor's internal state is simply the charge flowing through or the flux-linkage across the device

abstraction enables us to model nanoscale systems [7] where the dynamical properties of electrons and ions strongly depend on the history of the system.

In this work, we are concerned with utilizing memory elements to design two element chaotic and hyperchaotic circuits. We picked two elements because such circuits have been shown to correspond to models of physical systems [10].

Since memory elements are commercially unavailable as of this writing, we also show how one can use FPGAs to emulate such elements. Although microcontrollers have been used to emulate memory elements [8], FPGAs are an inherently parallel architecture and hence enable us to emulate memory devices at much higher frequencies than a sequential microcontroller. For instance, our FPGA emulator functions up to frequencies of 6 KHz. The micro controller emulation functions at a maximum frequency of 50 Hz [8]. Although an FPGA has been used to implement memristive chaotic circuits [9], our work is the first to emulate circuits with memcapacitors and meminductors.

This work is organized as follows: in Sect. 2 we review the principal ideas behind memory elements. In Sect. 3 we give an example of a two element chaotic circuit and in Sect. 4 we give an example of the two element hyperchaotic circuit. Section 5 discuss the FPGA physical emulation platform. Finally, we conclude with suggestions for future work.

In lieu of page constraints, we have focused on the main ideas in this work. For details regarding physical realization of memory elements using FPGAs (including the design source), please email the first author.

2 An Overview of Circuit Elements with Memory

Based on Fig. 1, we can generally define $z(t)$ to be a set of state variables describing the internal state of a system [2]. Let $x(t)$ and $y(t)$ be any two complementary constitutive variables (i.e., current, charge, voltage, or flux-linkage) denoting input and output of the system and g be a generalized response [4]. We can now define a general class of nth-order x-controlled memory devices as those described by Eqs. (1) and (2).

$$y(t) = g(z, x, t)x(t) \tag{1}$$

$$\dot{z} = f(z, x, t) \tag{2}$$

In Fig. 1, g is either $R(z, i)$ (memristance), $C(z, v)$ (memcapacitance), $L(z, i)$ (memductance) or $M(q)(q \equiv z$, memristance).

Practically speaking, different memory effects (namely, memristive, memcapacitive and/or meminductive features) could coexist in physical devices [10]. Hence we will utilize the fully nonlinear models of the memristor (Definition (1)), memcapacitor (Definition (2)) and meminductor (Definition (3)) from Riazza [10].

Definition 1. A fully nonlinear current-controller memristive system[1] is a device governed by the relations

$$v = \eta(z, i, t) \tag{3}$$

$$\dot{z} = f(z, i) \tag{4}$$

Systems in which the characteristics in Eq. (3) amount to $v = M(z, i)i$ describe the settings originally discussed by Chua and Kang [2]. Note also that the fully nonlinear form in Eqs. (3) and (4) makes it possible to accommodate physical devices that display memristive effects but whose characteristic does not arise as the time derivative of a $\phi - q$ relation, contrary to Chua's memristor [10].

Definition 2. A fully nonlinear voltage-controlled memcapacitor is governed by the relations

$$q = \omega(z, v, t) \tag{5}$$

$$\dot{z} = f(z, v) \tag{6}$$

[1] A voltage-controlled fully nonlinear memristive system is analogously defined.

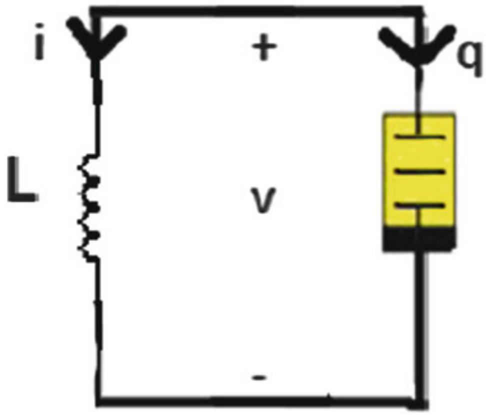

Fig. 2 An inductor in parallel with a fully nonlinear memcapacitor. The inductor provides one state variable whereas the memcapacitor provides two state variables. Hence we get a total of three state variables, the minimum required for chaotic behavior in a continuous time smooth dynamical system

Definition 3. A fully nonlinear current-controlled meminductor is governed by the relations

$$\phi = \theta(z, i, t) \tag{7}$$

$$\dot{z} = f(z, i) \tag{8}$$

Again, with $\omega(z, v, t) = C(z, v)v$ and $\theta(z, i, t) = L(z, i)i$, we get the systems proposed by Di Ventra et al. [4]. A physical instance of a fully nonlinear memcapacitor arising in a Josephson junction model is discussed in [10].

Now we will utilize these fully nonlinear devices to design chaotic and hyper chaotic circuits that utilize two elements in series-parallel configuration. This is possible because the internal state of a memory device need not be simply charge or flux-linkage. Rather, as mentioned earlier, it could be a n-dimensional set.

3 Two Element Chaotic Circuit

Consider the schematic shown in Fig. 2. This circuit is a modified version of the three element chaotic circuit from [5]. The memory device could be implemented on a FPGA and the inductor could be an external physical device. The interface to the FPGA is discussed in Sect. 5. Equations (9) through (11) describe the circuit. We have used x_C to denote the internal state of the memcapacitor.

$$\dot{x_C} = -v - \alpha x_C + v x_C \tag{9}$$

$$\dot{v} = \frac{-1}{C}\left(i + \beta(x_C^2 - 1)v\right) \tag{10}$$

$$\dot{i} = \frac{1}{L}v \tag{11}$$

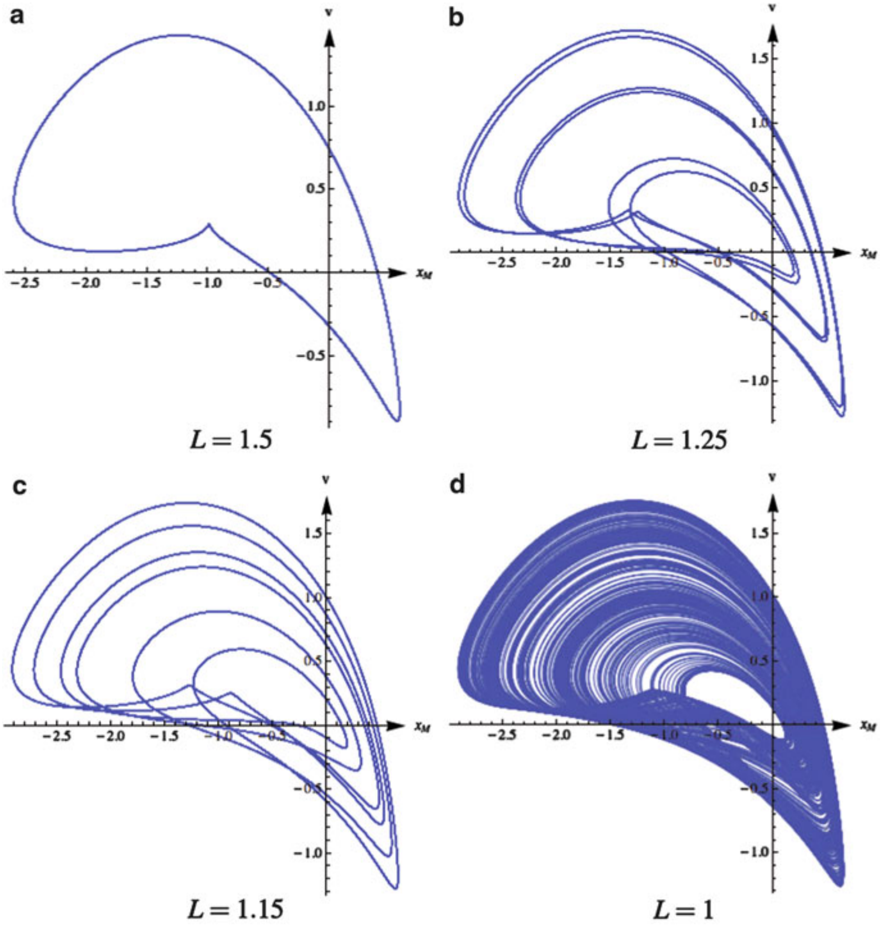

Fig. 3 Period-doubling route to chaos in the two element chaotic circuit. Note that we are using a dimensionless formulation of the equations, hence units are not specified. Parameter values are $\alpha = 0.6, \beta = 1.5, C = 3$. L is the bifurcation parameter. Initial conditions are $(0.1, 0, 0.1)$. The system is simulated for 10,000 time units, plot is only from 5,000 to 10,000 time units to minimize transient effects

The fully nonlinear memcapacitor is described by Eqs. (12) and (13).

$$q = Cv + \beta\left(\frac{x_C^3}{2} - x_C\right) \tag{12}$$

$$\dot{x_C} = -v - \alpha x_C + v x_C \tag{13}$$

Figure 3 shows a simulation of the period-doubling route to chaos. Mathematica simulation code is in appendix.

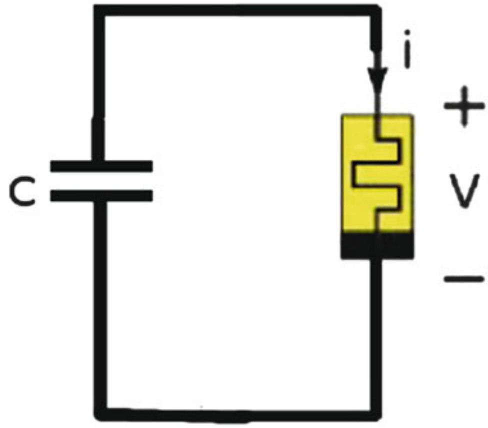

Fig. 4 A capacitor in parallel with a memristive device. This circuit models a higher order Rössler system [11]

The numerically calculated Lyapunov exponents are (0.029,0,–0.47) [5]. Note how we have one positive Lyapunov exponent and the sum of the Lyapunov exponents is negative. This indicates steady-state chaotic behavior.

4 Two Element Hyperchaotic Circuit

By simply increasing the number of internal state variables in the memory device, one could get hyperchaotic behavior. Consider the schematic shown in Fig. 4. Equations (14) through (17) describe the circuit.

$$\dot{x} = -y - z \tag{14}$$

$$\dot{y} = z + ay + v \tag{15}$$

$$\dot{z} = b + xz \tag{16}$$

$$\dot{v} = \frac{1}{C}(dv - ez) \tag{17}$$

The memristive system is described by Eqs. (18) through (21).

$$\dot{x} = -y - z \tag{18}$$

$$\dot{y} = z + ay + v \tag{19}$$

$$\dot{z} = b + xz \tag{20}$$

$$i = dv - ex \tag{21}$$

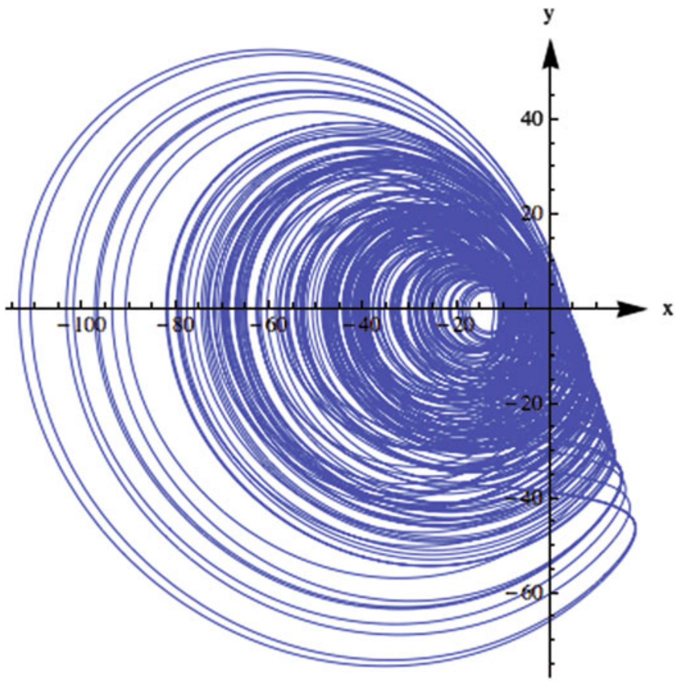

Fig. 5 The hyperchaotic attractor. Parameter values are $a = 0.25, b = 3, C = 1, d = 0.05, e = 0.5$. Initial conditions are (−6,0,0.5,14) [11]. Simulation is done for 10,000 time units, plot is only from 9,000 to 10,000 time units to minimize errors due to transient response

Figure 5 shows a phase plot from the simulation of the two element hyperchaotic circuit. We have omitted the simulation code and any bifurcation phenomenon since they are very similar to the two element chaotic circuit.

Lyapunov exponents are (0.1120, 0.0211,0,−24.9312) [11]. We now have two positive Lyapunov exponents but sum of the exponents is still negative. The two positive exponents indicate that stretching dynamics occur in two directions [11], indicating the presence of hyperchaos.

5 FPGA Based Physical Emulation Platform

Figure 6 shows the period-doubling route to chaos in a physical realization of the two element circuit. We are currently working on the physical realization of the hyperchaotic circuit.

The FPGA implements implements the nonlinear relations in Eqs. (3) through (8). Figure 6 shows the physical realization and the various analog sub-components that sample either the voltage or current into the FPGA. We utilize the circuit in

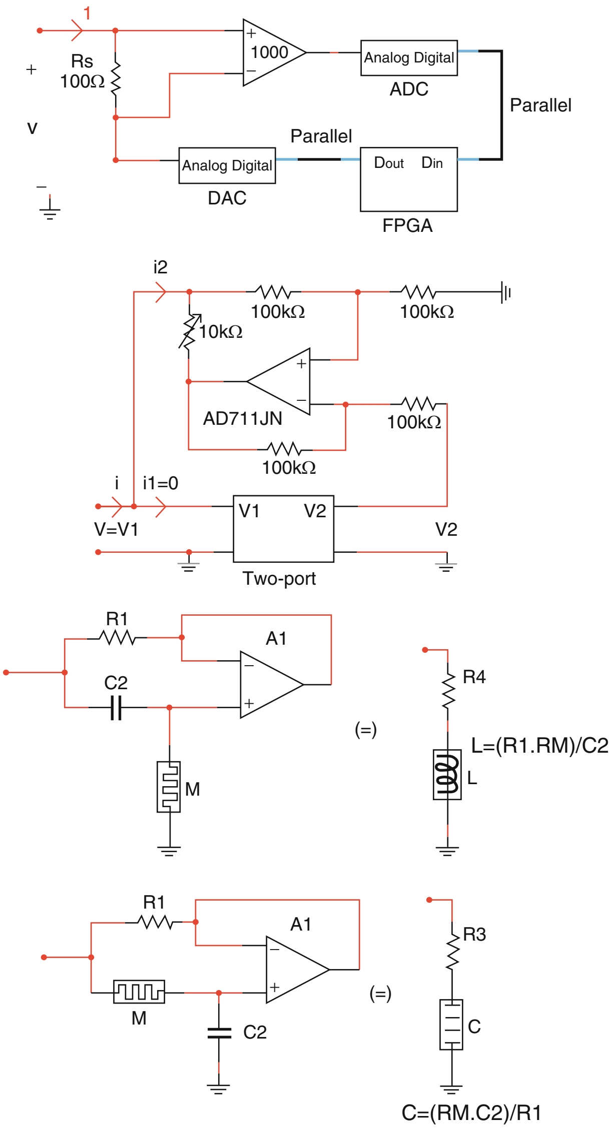

Fig. 6 The various components of the FPGA based memory devices emulator. The FPGA used was a Cyclone II, the development platform is a DE2 board from Terasic. All amplifiers are the AD711. The analog-to-digital converter (ADC) is the ADS7804 and the digital-to-analog converter (DAC) is the AD7245. 27-bit fixed point was used to emulate the memory devices on the FPGA. We used 27-bits because the Cyclone II FPGA has embedded 9-bit multipliers and hence the synthesis tool will be able to utilize the hardware multipliers instead of using FPGA logic elements for multiplication

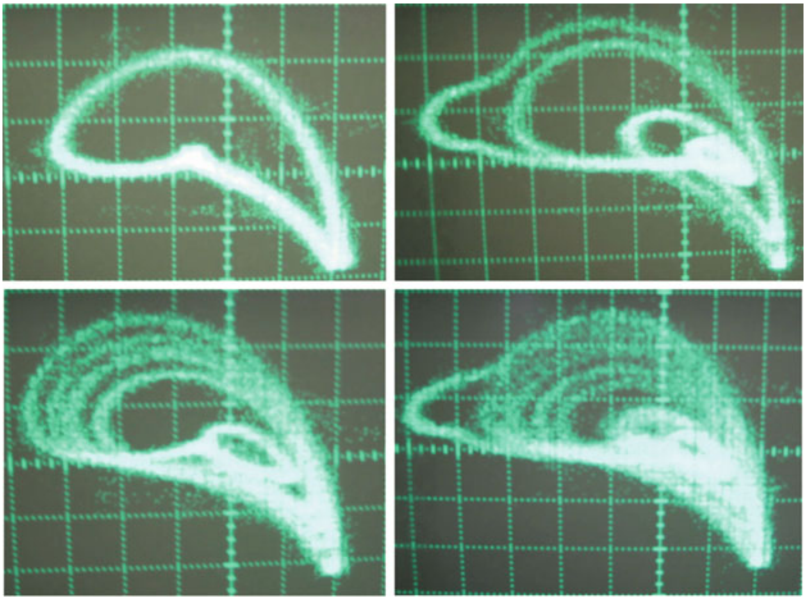

Fig. 7 Period-doubling route to chaos from the physical emulator

[3] along with the memristor to memcapacitor or meminductor converter proposed in [4]. Note the differential equations implemented on the FPGA may have to be amplitude scaled to match the analog subsystems.

Note that compared to a microcontroller emulation [8], the FPGA is not easy to "program". However the difficulty in using the FPGA is offset by the advantage gained in operational frequency of the memory device and the arbitrary size of the datapath.

6 Conclusions and Future Work

In this work, we have proposed the use of a mixed analog-digital emulation of memory devices. Currently, we are working on realizing the memory device characteristics for hyperchaos. We are also developing a robust user-interface for ease of FPGA development (Figs. 7 and 8).

We are also migrating our design to use sigma-delta and delta-sigma converters, instead of the current parallel ADCs and DACs. The reasoning is simplicity of interface. Nevertheless, sigma-delta and delta-sigma converters process one-bit at a time and hence the operational frequency of the circuit is reduced.

```
twoElementChaos =
 NDSolve[{i'[t] == v[t]/L, v'[t] == -1/C * (i[t] + β * (xM[t]^2 - 1) * v[t]), xM'[t] == -v[t] - α * xM[t] + v[t] * xM[t],
    i[0] == 0.1, v[0] == 0, xM[0] == 0.1} /. {L → 1, C → 3, β → 1.5, α → 0.6}, {i, v, xM},
   {t, 0, 10000}, MaxSteps → Infinity]

ParametricPlot[Evaluate[{xM[t], v[t]} /. twoElementChaos], {t, 5000, 10000},
 AxesLabel → {"x_M", "v"}, LabelStyle → Directive[Medium], AspectRatio → 1,
 AxesStyle → Arrowheads[0.05], PlotRange → All, PlotPoints → 10000]
```

Fig. 8 Mathematica 8 simulation code. NDSolve command simulates the differential equations and ParametricPlot command displays the phase plot

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Appendix

Mathematica simulation code for two element chaotic circuit. Simulation parameters are for chaotic behavior.

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RETRACTED CHAPTER: Lempel–Ziv Model of Dynamical-Chaotic and Fibonacci-Quasiperiodic Systems

Alireza Heidari and Mohammadali Ghorbani

Abstract Here we show that how the LZ-complexity concept connects to the concepts such as Lyapunov exponent and K-entropy and has an application in the theory of dynamical systems regardless of its main origin in the information theory. Furthermore, selecting the Fibonacci sequence as a sample of evolutionary arrays, it is proved that these systems' LZ complexity represents its long-range order.

1 Introduction

Chaos is complex and disordered [1]. However, the outstanding attribute of such behavior is that the state of the system cannot be predicted for a long period of time. This limitation occurs when one withdraws infinite accuracy from the common attitude toward the Newton dynamical system. Chaotic systems are very sensitive to initial conditions. It means that every slight change in initial conditions has a major impact on the final output [2–4].

2 LZ Complexity as a Dynamic Index

Here we consider the LZ complexity of the phase-space snapper of dynamic systems, which is coded to specific messages. Based on this formalism, the LZ complexity has been proposed as an index in the analysis of dynamic systems' behavior. Following this idea, by coding the dynamic observables, the Henon system is as follows:

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$$H\colon X_{n+1} = 1 - aX_n^2 + Y_n; \qquad Y_{n+1} = bX_n, \tag{1}$$

and the driven dissipative oscillating continuous system with the motion equation:

$$P\colon \ddot{X} + K\dot{X} + \sin X = g\cos(\omega_d t). \tag{2}$$

The LZ complexity has been calculated for the specific values of control parameters of these two systems. In the method of array-code generation for Henon mapping, the system's phase space is divided into four regions:$(X, -Y), (X, Y), (-X, Y), (-X, -Y)$. Next, when the asymptotic response of the dynamic system (X_n, Y_n) is placed on the first and third quarters for a specific value of a, it be-comes the symbol 1 in the array, otherwise it becomes 0. The resulted array is input to the LZ-complexity-calculation program. This process is iterated and calculated for each a in the interval [1, 4] at step size $\Delta a = 0.001$ and $b = 0.3$. The calculation of LZ complexity for a driven dissipative oscillating system has been carried out as follows. Here the control parameters are the motive angular velocity ($\omega_d = 2/3$), friction factor ($K = 0.5$), and the motive force amplitude (g), which changes in the range $0.9 \leq g \leq 1.5$ by the step size $\Delta g = 0.001$. The array length is considered $n = 20000$. The generation of array codes for a specific motive force amplitude in the above interval has been performed based on a simple procedure such that whenever the oscillator angular velocity ($\dot{x}$) is larger than zero, it is considered the number corresponding to the single array, and otherwise zero. The complexity analysis of each array has been done by the computer program. The normalized LZ complexity, $\{C(n)/b(n)\}$, is near zero for the periodic trajectories and bifurcation points of both systems. It means that despite the fact that new bifurcations form and the periodic models become more complex, there is not a clear and easy-to-observe effect of these changes on the LZ complexity. This matter may result from the fact that data existing in the phase-space snapper is mapped to two-alphabet array symbols. At this level of calculation, it seems that we can just distinguish the determinable dynamic system's chaotic behavior $(0 < C(n)/b(n) < 1)$ from ordered behavior $(C(n)/b(n) = 0)$. Despite this difference in the periodic behavior region, the correspondence of these two indexes is obvious in the consistent explanation of the nature evolution of Henon and oscillator systems. This fact suggests the existence of a profound link between these two concepts. The correlation between Lyapunov exponent and LZ complexity was also proposed by Kasper and Shuster in 1987. This correlation can be explicitly understood. The quantities such as algorithmic complexity, LZ complexity, statistical complexity, and Rissanen complexity tend to the value of K-entropy at n $\rightarrow \infty$. On the other hand, the ith positive Lyapunov exponent (λ_i^+) determines the data generation rate under the system evolution along the ith coordinate of phase space. K-entropy is the total data generation rate in that dynamic process:

$$h_\mu(X) = \Sigma_i D_i \lambda_i^+, \tag{3}$$

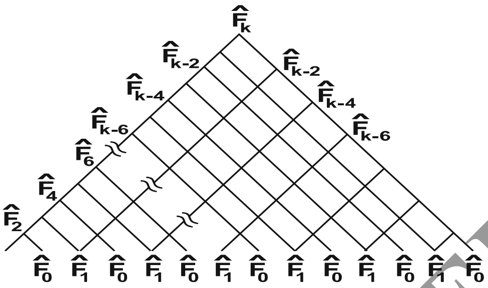

Fig. 1 The correlation between Fibonacci blocks and subblocks

where D_i is the data density for each bit of the ith coordinate of phase space. For directions that are dynamically unstable, although they are determinable, we cannot predict every single bit of system-evolution data. Therefore, D_i equals one for these directions. Consequently:

$$h_\mu(X) = \Sigma_i \lambda_i^+. \tag{4}$$

Accordingly, in chaotic modes, the LZ complexity directly relates to the sum of positive Lyapunov exponent. For the systems such as Henon mapping and dissipative driven oscillator in which there is only a positive Lyapunov exponent the LZ complexity corresponds with that only positive Lyapunov exponent.

3 LZ Complexity of Fibonacci Sequence

The subarray $A_i A_{i+1} \ldots A_{i+m}$ with length $l \geq m$ from the sequence $A_1 A_2 \ldots A_l \ldots A_{l+m} \ldots A_L$ is called a block.

Fibonacci block: The Fibonacci sequence $\{f_n\}_n^N = 0$ in which $f_n = f_{n-1} + f_{n-2}, \;\; n \leq 2$ is considered. The Fibonacci block is a finite array whose formation results from a command based on the formation of Fibonacci sequence, which is due to the sort order operation ($\oplus$) of two previous blocks. The simplest block is called zero or one block. Figure 1 shows each block's relation with its constitutive subblocks. Furthermore, the correlation between each block and basic blocks F_0 and F_1 is clear. For example, the block F_6 is composed of F_4 and F_5. On the other hand, F_6 consists of a subblock F_5, two subblocks F_4, three subblocks F_3, and five subblocks F_2. The calculation of these blocks' LZ complexity is of importance. The following simple example shows a procedure for calculating the LZ complexity.

Trick 1: The LZ complexity of the third Fibonacci block (F_2) is equal to two.

It is proved by using the description of LZ complexity:

$$S' = F_2 = 10;\ 1)S^2 = 1/;\ 2)Q = 0; SQ = 1/0; V(S\hat{Q}) = 1; \because 1 \varepsilon V(S\hat{Q}); \therefore$$
$$S' = S = 1/0 \Rightarrow C(S') = C(F_2) = 2. \tag{5}$$

Trick 2: If we add each block of S with length $n - r$ to the sequence S with length r ($\oplus$), the LZ complexity of the resulted sequence (S') will be:

$$C(S') = C(S) + 1, \tag{6}$$

$$S' = S \oplus S'', \quad S'' \subset_B S; \quad L(S') = L(S \oplus S'') = n, \tag{7}$$

$$S'' = s_1 s_{i+1} \dots s_j = s_{r+1} s_{r+2} \dots s_n, 1 \le i \le j \le r;$$

$$S' = s_1 s_2 \dots s_r s_{r+1} \dots s_n. \tag{8}$$

Using the description, we calculate the LZ complexity of S'.

$$\begin{aligned}
&j)\ S' = S\backslash = s_1 s_2 \dots s_r \backslash; Q = s_{r+1}; SQ = S\backslash s_{r+1}; S\hat{Q} = S\backslash;\\
&S'' \subset_B S \Rightarrow (Q = s_{r+1}) \in S; \therefore Q \in V(S\hat{Q})\\
&j+1)\ Q = s_{r+1}s_{r+2}; SQ = S\backslash s_{r+1}\ s_{r+2}; S\hat{Q} = S\backslash s_{r+1};\\
&S'' \subset_B S \Rightarrow (Q = s_{r+1}s_{r+2}) \in S; \therefore Q \in V(S\hat{Q}).
\end{aligned} \tag{9}$$

Accordingly, this process is iterated up to the last term ($j = r$). For the last step, we write:

$$\begin{aligned}
&j = r)\ Q = s_{r+1}s_{r+2} \dots s_n; SQ = S\backslash s_{r+1}s_{r+2} \dots s_n; S\hat{Q} = S\backslash s_{r+1}s_{r+2} \dots s_{n-1}\\
&S'' \subset_B S \Rightarrow Q \subset_B S; \therefore Q \in V(S\hat{Q}); \text{COPY } Q = S''; \therefore S' = S/Q = S/S''; \therefore\\
&C(S') = C(S) + 1.
\end{aligned} \tag{10}$$

4 Conclusion

The calculation of Lyapunov exponent for large-scale systems faces some problems such as lack of convergence in numerical solutions. Herein it is shown that the LZ complexity can be also utilized for systems more complex than one-dimensional mappings such as logistic mappings. Therefore, by selecting the appropriate coding method or more alphabets (more than two alphabets), the LZ complexity, which is simpler and less time-consuming in terms of calculation, can be used as a dynamical index equivalent to Lyapunov exponent.

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