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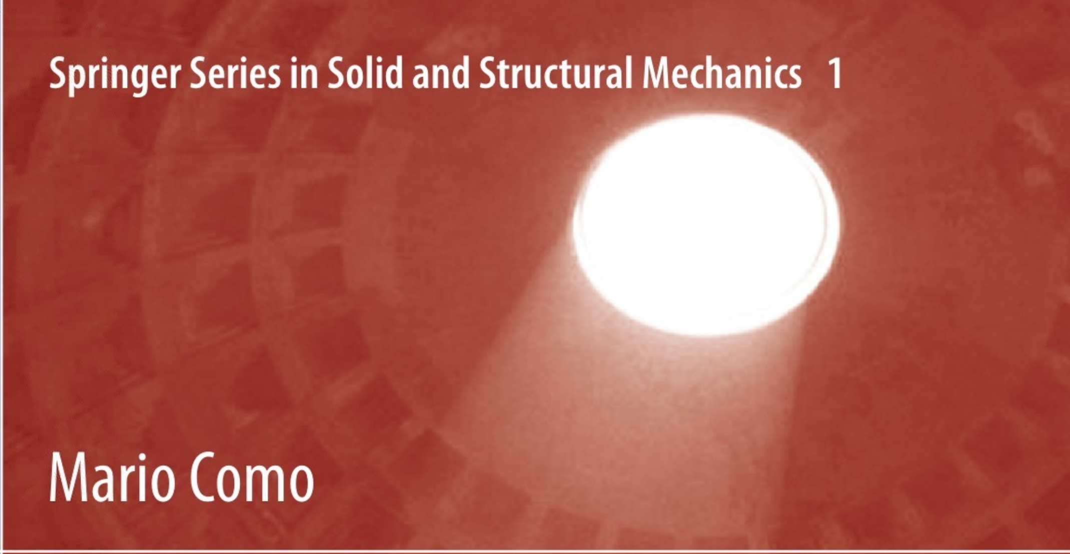

Mario Como

Statics of Historic Masonry Constructions

Springer

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To Ida

Preface

Masonry constructions are the great majority of the buildings in Europe's historic centers and the most important monuments in its architectural heritage. Given the age of much of these constructions, the demand for safety assessments and restoration projects is pressing and constant. Nevertheless, there is a lack of a widely accepted approach to studying the statics of masonry structures. Simple linear elastic models, which form the foundation of common structural analyzes, cannot in fact be applied to masonry because of its inherent, widely differing response to tension and compression.

The ingenious Heyman no-tension model well interprets the masonry behavior and is widely used and fruitfully applied in analyzing the statics of systems of arches. However, completely different assumptions are commonly used for other types of masonry structures in other contexts, for example, strength evaluations of masonry buildings under seismic forces, which is rather perplexing, given that a masonry arch, a vault and a building wall are all still made of the same material. Moreover, most masonry studies approach strength evaluations of structures through Limit Analysis, forgoing any study of the construction's actual state.

This book aims to help fill these gaps in the study of masonry structures by formulating a new comprehensive, unified theory of statics of masonry constructions extending the Heyman model to the analysis of the masonry continuum. The book features complete mathematical derivation of all the given results and, through an interdisciplinary approach combining engineering, architecture and a bit of history, advances from the simple to the complex, while striving, above all, for clarity.

The book is the result of thirty years of research and professional experience. It is divided into nine chapters, each of which begins with historical notes and an introduction highlighting the main aspects of the topics covered.

The strength and deformability of masonry materials are addressed in the first chapter. The second chapter deals with the deformation and equilibrium of masonry solids. The kinematics of strains and crackings, as well as internal stress states are analyzed. The fundamental concepts of admissible equilibrium and the parameters governing collapse strength are examined in detail to highlight the strict relation between structural geometry and strength. The notion of minimum thrust, by a static and kinematic approach, is then introduced – an aspect of masonry structural behavior that extends the field of application of Limit Analysis – to include study of the actual stress states of masonry constructions. The third and fourth chapter examine the static behavior of the main basic masonry structures, such as arches and vaults. By way of example, static analysis are conducted of a number of renowned examples from the world's architecture heritage, such as ancient Mycenaean domes, the Rome Pantheon, the large cross vaults of the Baths of Diocletian, and the domes of Santa Maria del Fiore in Florence and Saint Peter's in Rome. The fifth chapter turns to a detailed analysis of the statics of the Rome Colosseum and examines the reasons for its actual state of damage. The sixth chapter describes and analyzes the statics of cantilevered stairways, a typical element whose structural behavior is still somewhat unknown. Chapter seven then takes up the structural analysis of walls, piers and towers under vertical loads. The stability of such structures is heavily affected by the non-linear interactions between the destabilizing effects of the axial loads and masonry's no-tension response. The instability of towers, leaning towers in particular, is addressed in a specific section of the chapter. In this regard, a detailed stability analysis is conducted of the famous leaning Tower of Pisa, which has recently undergone a successful restoration work. The eighth chapter then analyzes the statics of Gothic cathedrals, with particular reference to analysis of their resistance to wind actions. The 1294 collapse of the Beauvais cathedral is also examined in depth. The last chapter deals with the seismic behavior of historic masonry buildings and crucial issues regarding their conservation. The latter part of the chapter regards, in particular, the analysis of the transmission of seismic forces between the various constituents of a building, together with the out-of-plane and in-plane strengths evaluations of multi-story walls with openings.

The book is addressed especially to researchers, engineers and architects operating in the field of masonry structures and of their consolidation and restoration, as well as to students of civil engineering and architecture. It is, for the most part, an English translation of a recent Italian book of mine "Statica delle Costruzioni Storiche in muratura". The English edition has however been revamped to address some new questions and, hopefully, improve on the original.

Many thanks go to colleagues Michel Frémond and Franco Maceri for their precious encouragement to prepare the book. Many thanks go also to Anthony Cafazzo, English Lecturer at the University of Pisa, who insightfully and patiently assisted me in revising the text.

I would also like to thank all the graduate and postdoctoral students, researchers, visiting scholars, external collaborators and students, who attended my courses at the Faculty of Engineering of the University of Rome Tor Vergata – all of whose contributions have been duly noted – for their invaluable assistance in the various research studies without which this book would not have been possible.

January 2012 Mario Como
Rome

Contents

Chapter 1
Masonry Strength and Deformability

Abstract. This chapter deals with the strength and deformability of masonry materials composing the structure of the so-called historic constructions.

After some historical introductory notes, special attention has been given to the analysis of various strength features of these materials and of their components, as bricks, stone blocks, and mortars. The common peculiarity of all the stone materials, a strength in tension much lower than in compression, is analyzed in detail and a suitable triaxial failure criterion is thoroughly discussed. These results are then applied to the strength evaluation of uniaxial compression strength of the masonry, composed by regular patterns of blocks and mortar courses, as function of the geometry and strength properties of its components. The study of the masonry deformations, both the instantaneous as the delayed, ends this chapter.

1.1 Brief Notes on the History of Masonry Constructions

Masonry constructions, whose oldest examples date back to about eight thousand years ago, developed during the beginning of the earliest urban civilizations, when more ancient techniques employing building materials such as wood, straw, and hides were gradually replaced by more advanced technologies, enabling the construction of stronger, longer lasting structures.

Initially, masonry walls were built by setting large rough-hewn stones one on the other, dry, without mortar, to form so-called Cyclopean masonry. During the Classic Age regularly shaped stone blocks with smooth outer faces were used to build walls or piers, still without the use of mortar. This technique was utilized in the construction of many of the temples of the Athens Acropolis and later the Roman Colosseum. Because of the scarcity of suitably hard rocks in the Mesopotamian area, the societies there developed techniques to produce artificial building blocks. Initially, bricks were sun-baked, friable, and unreliable over time. The use of kilns to harden the clay developed later. This allowed producing more resistant elements – fired bricks, a technique still in widespread use today.

The use of binders, substances that set and harden, in masonry construction is also an ancient technique. Over the course of history, various materials have been used as binders. The first mortars were made by mixing mud and clay. Then, the ancient Egyptians added gypsum as binder, while the ancient Persians used bitumen. The discovery of lime by the Etruscans was the last fundamental turning

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point in the evolution of masonry. It was discovered that limestone, when burnt and combined with water, produced lime that would harden with age. The mixture of lime with pozzolana, a volcanic ash that reacts with calcium hydroxide in the presence of water, improved the quality of mortars, which would set under water. Historically, constructions with pozzolanic mortar first appeared in Greece, though it was the Romans (Choisy, 1873, Giuliani, 1995) who developed this technique to its full potential. Over time, they defined a number of different types of *opus* (literally 'work') used at different times in different structures. These *opera* were remarkable for the construction procedures used and the different geometries of the masonry patterns achievable:

- *opus caementicium*: a construction technique using aggregates, water, and a binding agent. The aggregate, rubble of broken fragments of uncut stones or fist-sized tuff blocks (*caementa*), was mixed to lime and pozzolana mortar (Choisy, 1873);
- *opus incertum*, a crude masonry made up of irregularly shaped, uncut (or 'undressed') stones randomly inserted into a core of *opus caementicium*;
- *opus quadratum*, facings built with cut stone blocks laid in regular horizontal courses;
- *opus testaceum*, or *latericium*, brick-faced masonry with kiln-backed bricks, which prevailed throughout the Imperial Age;
- *opus reticulatum*, a Roman decorative design using small square slabs of stone or small bricks embedded into a regular, tightly knit diamond pattern;
- *opus mixtum*, masonry of reticulated material reinforced and/or intersected by brick bands or interlocked with bricks;
- opus *vittatum,* oblong (occasionally square) tuff blocks intersected by one or more brick bands at more or less regular intervals.

Typical Roman masonry walls were usually quite thick and made up of an inner rubble core of *opus caementicium* and two outer facings. In particular, a wall, or pier, made with *opus quadratum* had facings of large bricks placed along horizontal courses. In the Imperial Age, brick facings were built using square-shaped bricks (*opus testaceum*), as in the Baths of Diocletian. Wall facings were otherwise built using *opus reticulatum, opus vittatum,* or *opus testaceum.*(fig. 1.1).

Dead loads tend to pull the walls horizontally apart, causing vertical cracking. Roman masons devised a method to connect the facings and the inner rubble core. They sawed square bricks diagonally and laid these triangular half-bricks in the core with their hypotenuses outward to create a toothed bonding surface to the facings. Figure 1.1 illustrates this technique, showing the structure of a wall built *with opus testaceum.* Although a large variety of bricks were produced, they came in three main sizes:

- *bessales*, 8 in (19.7 cm) square;
- *sesquipedales*, 1.5 ft (44.4 cm) square;
- *bipedales*, 2 ft (59.2 cm) square.

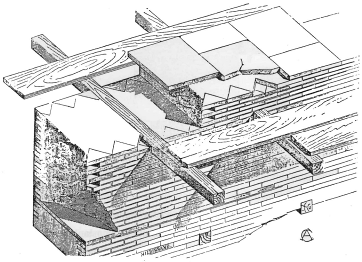


Fig. 1.1. An *opus testaceum* masonry wall (from A. Choisy, 1873).

Constructing a wall able to sustain loads and eventual settling of the foundation without severe damage was a difficult task. Greek architects first recognized the benefits of laying blocks with staggered vertical joints to achieve more compact walls (fig. 1.2.) (Giuffrè, 1990).

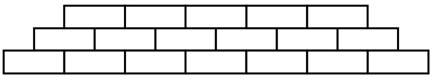


Fig. 1.2. Isodomic pattern of a masonry facing.

This technique also defined the positioning of the bricks within the wall's thickness

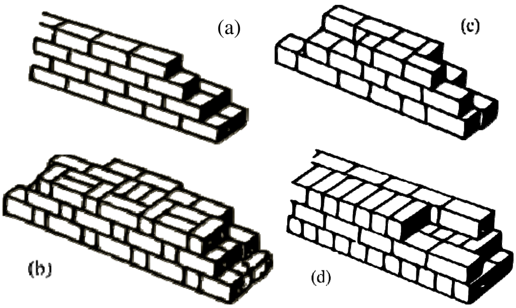


Fig. 1.3. First Greek and Romans patterns of the masonry texture: (*a*) isodomic Greek system; (*c*) archaic Roman system; (*b*) Greek system with alternating stretchers and headers (*d*) Roman system with courses of stretchers and headers

Initially, in walls laid according to Etruscan methods, some discontinuities occurred along the courses and some blocks had to be shaped differently from the others, as can be seen in some examples of walls built in ancient Etruscan towns and later in Rome (underground reservoirs, terracing walls, and temple podiums)

(Fig. 1.3 *a* and *c*). The Greeks later solved this problem (Sparacio, 1999) by laying blocks in alternating longitudinal and transverse rows. (fig. 1.3b). Finally, the Roman fashion, shown in figure 1.3 (*d*), enjoyed widespread application.

The disastrous economical conditions ensuing in Europe after the fall of the Roman Empire made it necessary for Romanesque builders to reduce transport costs and thus to use materials that were easily available locally, such as the marl from nearby quarries. Moreover, using small elements simplified loading and unloading and, at the same time, reduced the amount of mortar needed. It thus became very common to build walls with small-sized blocks of tuff or bricks and mortar. Such simple building procedures continued throughout the Middle Ages and into the Renaissance (Morabito, 2004).

1.2 The Masonry of Historic Buildings

A wide variety of types of masonry are present in historic buildings. Except for low-cost housing, whose walls were usually built with stone rubble, historic masonry is composed of mortar-cemented parallelepiped-shaped elements, usually bricks, whose standard size is 5.5cm × 12cm × 25 cm, though other types of elements are also common. Thus, according to the elements used, masonry can be subdivided into:

– *regular brickwork*: constructed with brick elements laid with mortar in horizontal courses with staggered vertical joints (fig. 1.4).

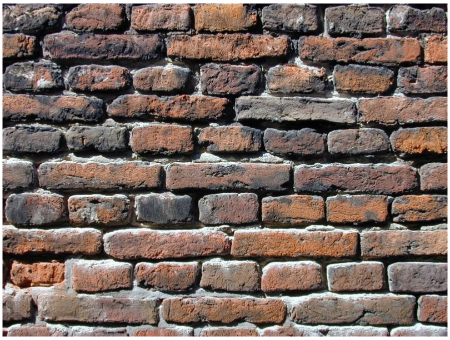


Fig. 1.4. Regular brickwork.

In this arrangement the bricks are named according to their placement in the wall. *A stretcher* is a brick laid horizontally flat, with its long side exposed on the outer face of the wall. *A header* is a brick laid flat across the wall's width with its short end exposed. Bricks may be laid in a variety of patterns, or bonds, of alternating headers and stretchers. Thinner walls are made using a stretcher bond, also known as a running bond, with stretchers forming the entire thickness of the wall, i.e., 5.5cm (excluding the plaster or stucco facing).

Others walls are constructed with a single row of stretchers, so that the wall is as thick as the brick head, 12 cm. There are many other types of bonds that use two or three headers in different alternating configurations with stretchers.

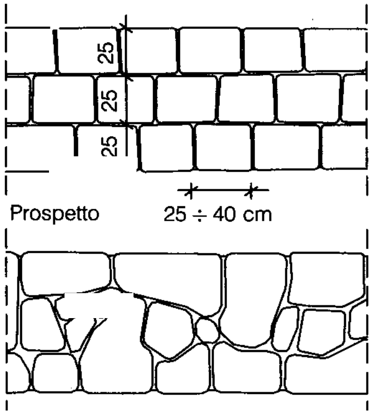


Fig. 1.5. Masonry built with tuff blocks.

– *regular brickwork with squared stone blocks*: built with tuff blocks bound by horizontal mortar and vertically staggered joints, as in *regular brick-work.* (fig. 1.5). Thick walls may present an inner and outer tuff facing over an internal rubble core.
– *brickwork with mixed stone and brick*: come in two different types. In the first, called *edged masonry,* the bricks are arranged in horizontal courses along the entire thickness of the wall at varying distances (80–160 cm) between the stone masonry. In the second, *mixed masonry with bricks,* single bricks are laid in various places to level the stone planes (fig. 1.6).

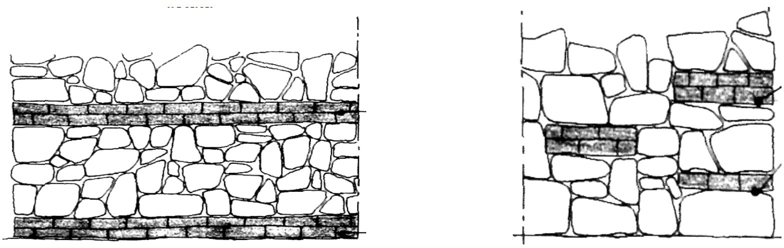


Fig. 1.6. Two examples of masonry with a mix of stones and bricks. *a*) edged masonry; *b*) mixed masonry with bricks.

– *ordinary brickwork with huddled stones*: obtained by mortaring irregularly shaped elements, such as chunks of bricks or stones, along roughly horizontal planes in such a way as to reduce the spaces between them. (Fig. 1.7). Such masonry, used frequently to build homes in small historical communities in southern Italy, is particular vulnerable to earthquakes.

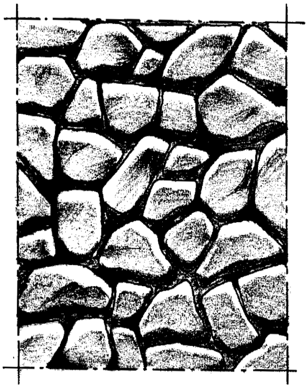


Fig. 1.7. Masonry built with huddled stones and mortar.

The Italian Department of Civil Defense (2002) has issued its own classification of masonry, according to which there are five classes, each with a number of subclasses:

*a*1) *rounded stone masonry:* built with small- or medium-sized cobblestones laid randomly or in ordered patterns (i.e. bonds):
- with neither courses nor regular bonding pattern;
- without courses, but presenting an orderly bonding pattern;
- stones with brick courses;

*a*2) *rough stone masonry*: generally built with irregularly shaped, undressed elements of varying sizes, such as chunks of brick or stone:
- with neither stone courses nor regular bonding pattern;
- without stone courses, but with an orderly bonding pattern;
- coursed with flat interlocking tiles and stones;
- with brick courses;

*b*1) *masonry with ribbon-like stones:* built with rocks that tend to split along horizontal planes:
- without courses;
- with courses;

*b*2) *semi-regular masonries*: built with semi-finished medium-sized elements:
- semi-finished limestone in courses;
- semi-finished limestone without courses;

*c*1) *squared stone masonry*: made of “dressed” or worked stones, also called *ashlars*.
- tuff ashlars without courses;
- tuff ashlars with courses.

1.3 Compression Strength of Brick and Stone Elements

1.3.1 Bricks

Bricks, or masonry units, are made of clay, shale, soft slate, and calcium silicate. Bricks are generally manufactured by extrusion. Masonry units come in standard sizes of: 5.5cm × 12cm × 25 cm.

The compression strength of fired bricks is about 200–250 kg/cm^2. However, when poorly fired, bricks may exhibit severely reduced compression strength, as low as 50 kg/cm^2. Standard compression tests are performed by cutting a brick in half and then gluing the two parts together with cement paste. These glued interfaces reproduce the effects of the mortar joints present in masonry. Four wet and four dry samples are then placed under platens of a so-called 'universal testing machine', which applies a compression load at a preset rate. The standard compression strength of a unit is obtained via the relation

$$f_b = f_{bm}(1-1.64\delta) \tag{1}$$

where f_{bm} is the mean strength of the three most consistent results, and $\delta = s/f_{bm}$ is the variation factor, with s the root-mean-square deviation. The tensile strength is assumed to be equal to about one-tenth of the compression strength.

Another compression test is performed by placing a single prism-shaped brick specimen directly between the platens of the universal testing machine and evaluating the corresponding compression strength. The failure pattern in this case is the so-called *hourglass* mode, typical of the failure of concrete specimens.

Alternatively, before the test, the platens of the machine are treated with wax or stearic acid. In this case, the specimen, which can freely expand laterally during the test, breaks through vertical cracking under lower compression stresses.

1.3.2 Stone Blocks

As discussed above, squared elements, hewn from stone quarried in many different sites, have been used in masonry constructions for centuries. The mechanical features of these stone elements thus depend heavily on the source of the rock.

1.3.2.1 Strength of Stone Materials

Table 1.1. shows a classification of stones into five types according to the compression strength of undamaged rock samples. Few rocks belong to *class* A, the most notable examples being quartzite and basalt. *Class* B includes most magmatic rocks, the more resistant metamorphic rocks, and few sedimentary rocks, as well as most lime stones and dolomites. *Class C* comprises many argillites, marls, and metamorphic rocks, such as shale. *Classes D* and *E* include many porous rocks, such as brittle sandstones, tuff, halite, etc.

Table 1.1. Classification of stones according to compression strength f_c.

Class	Strength	fc [MPa]
A	Very high	> 225
B	High	225÷112
C	Mean	112÷56
D	Low	56÷28
E	Very low	< 28

Another, simpler classification subdivides rocks into *soft*, *medium hard,* and *hard*. Tuff, of both volcanic and sedimentary origins, are *soft* rocks. Sandstone, limestone, and travertine are *medium-hard* rocks. Dolostone, trachyte, porphyry, gneiss, granite, and basalt are all classified as *hard*.

Table 1.2. Density, Elastic Modulus, and compression strength of some rocks.

	Density (g/cm³)	*compression strength* (kg/cm²)	*Elastic Modulus* (kg/cm² $\times 10^5$)
Igneous rocks			
Granite, Syenite	2.6–2.8	1600–2400	5–6
Diorite, gabbroid	2.8–3.0	1700–3000	8–10
Porphyry, quartz	2.6–2.8	1800–3000	5–7
Basalt	2.9–3.0	2000–4000	9–12
Pumice	0.5–1.1	50–200	1–3
Sedimentary Rocks			
Soft limestone	1.7–.6	200–900	3–6
Compact limestone	2.7–2.9	800–1900	4–7
Dolomite	2.3–2.8	200–600	2–5
Metamorphic Rocks			
Gneiss	2.6–3.0	1600–2800	3–4
Shale	2.7–2.8	900–1000	2–6
Marble	2.7–2.8	1000–1800	4–7
Quartzite	2.6–2.7	1500–3000	5–7

Table 1.2 gives the corresponding values of the uniaxial compression strengths f_c and the elastic modulus E_e, the latter measured as the tangent modulus on the σ - ε diagram at 50% compression strength. Rocks used in constructions are mainly those designated as B, C, D, E. For instance, travertine was used to build the piers and perimeter arcades of the Rome Colosseum. The Milan cathedral was instead built of

hewn marble blocks from quarries near Lake Maggiore in northern Italy. Hard sandstone was the main building material used for many Gothic cathedrals, and tuff is widespread in many types of historic architecture.

1.3.2.2 Tuff Blocks

The term tuff derives from the Latin name, *tuphos,* which was originally used to indicate both a pyroclastic rock formed by slow consolidation of volcanic materials, such as lapillus, ash, and sand, as well as sedimentary rocks, such as *Apulia* tuff. Actually, tuff properly refers only to the volcanic rock types, while the term *tufa* should be reserved for the sedimentary type. Both are considered soft rocks and were used without distinction in historic buildings. Tuff is frequently used in constructions because it is light and soft, and therefore easily worked. It is also quite porous, and thus its density is low compared to other rock materials, such as limestones, shales, and so forth, though it nonetheless offers fairly high compression strength. Standard tuff block dimensions are 30 cm × 40 cm × 13 cm. Thus, building with tuff blocks yields wall thicknesses ranging from 30 cm to 40 cm, or multiples thereof. Tuff blocks can also be laid together with bricks because their bases are about the same size (13 cm). Some mechanical parameters of tuff:

- Poisson's ratio: $\nu = 0{,}15$;
- Elastic modulus: 30,000–150,000 kg/cm^2;
- Unit weight: (volcanic tuff) 1,100–1,700 kg/m^3;
- Compression strength: ~ 40–50 kg/cm^2.
- Tensile strength: ~ 1/15 of compression strength.

1.4 Mortars

Mortar is a workable paste used to bind masonry blocks together and fill the gaps between them. Mortar becomes hard when it sets, resulting in a rigid aggregate structure. Modern mortars are typically made from a mixture of sand, a binder such as cement or lime, and water.

1.4.1 Binders

Binders used in mortar preparation are:

- gypsum;
- lime;
- hydraulic lime;
- cement.

Gypsum, the oldest binder, was used in the first Egyptian pyramids. It is present in alabaster, a decorative stone used in Ancient Egypt. It is obtained by baking the gypsum stone, made up of calcium sulfate, at a temperature of 110–200 °C, after which the stone turns to powder. Mixed with water, the powder hardens rapidly, though it has very low strength.

Calcium oxide is the main component of lime. Lime is produced through a two-step process: firing and slaking. First the limestone is burnt at a temperature of 850–900 °C, to produce the so-called *quicklime*, which is able to absorb large quantities of water. The quicklime is then combined with water and crushed into powder, giving rise to *slaked lime* or calcium hydroxide. The slaked lime is then used to produce either simple lime mortar, by mixing it with sand and more water, or hydraulic mortar, by mixing it with pozzolana. Simple lime will only set in contact with the air.

Hydraulic limes, which will instead even set in water, are made from marly limestone or mixtures of limestone and clayey materials.

Cement is made by grinding together its main raw materials, which are (a) argillaceous, such as clay and shale, and (b) calcareous, such as limestone, chalk, and marls. The mixture is then burnt in rotary kilns at temperatures between 1400 and 1500 °C to form clinkers. These are ground to a powder and mixed with gypsum to create the gray flour-like substance known as cement. When water is added to cement, a chemical process occurs as it hydrates, allowing it to harden anywhere, even under water.

Cement, patented in 1824 by Joseph Aspdin in the UK, was called *Portland* because this artificial stone resembled the Portland stone. As cement began to be used only toward the end of the nineteenth century, cement mortars are generally not found in the masonry of historic buildings.

1.4.2 Aggregates

Aggregates are classified as fine or coarse. Sands are used as fine aggregates, while gravel or crushed rocks represent coarse aggregates. Sand, whose grain dimensions range from 0.5 to 1 mm, is generally used to prepare masonry mortars. Sand is the mineral skeleton of the mortar: it increases the volume of the paste and facilitates penetration of carbon dioxide within the mixture to improve setting. Moreover, sand reduces shrinkage and the consequent cracking that may occur during setting and hardening of the paste. Romans used *Caementa*, irregular pieces of stone or brick, as aggregate in preparing *opus caementicium* masonry.

1.4.3 Mortars of Lime

Mixtures of lime, water, and sand form the mortar paste, which sets and hardens.

1.4.3.1 Roman Mortars

Roman mortar contained pozzolana, a volcanic ash that added a useful property lacking in the simple lime mortars used by the Greeks: *hydraulicity,* that is, the ability to set underwater. o the material called *pulvis puteolanus*, discovered in ancient times in the Bay of Naples. Pozzolana was also produced in the volcanic districts to the south of Rome, where it was termed *harena fossicia*, a volcanic sand with similar water-setting features to *pulvis puteolanus*, though it was less effective in practice than the latter. Table 1.3 gives the compositions of some Roman mortars quoted in Vitruvius' treatise *De Architectura*. Table 1.3 also gives the proportions for producing *cocciopesto*, or *opus signinum*, a mortar made with crushed terracotta. *Cocciopesto* is the material most commonly used to line cisterns and to protect the extrados of vaults exposed to the elements.

Table 1.3. Roman mortars (J.P. Adam, 1988).

Binder	*Aggregates*	*Water*
1 part lime	3 parts *harena fossicia* (Vitruvius, II, V, 5)	15–20%
1 part lime	2 parts *river sand* (Vitruvius, II, 5,6)	15–20%
1 part lime	1 part *terra cotta* (Vitruvius, II, V, 7)	15–20%
1 part lime	2 parts *pulvis puteolanus* (Vitruvius, V, XII, 8–9, sea works)	15–20%

Table 1.4. Composition of mortars according to Italian building codes.

Class	*Mortar*	*Cement*	*Simple lime*	*Hydraulic lime*	*Sand*	*Pozzolana*
M_4	hydraulic	–	–	1	3	–
M_4	pozzolanic	–	1	–	–	3
M_4	composite	1	–	2	9	–
M_3	composite	1	–	1	5	–
M_2	cementitious	1	–	0.5	4	–
M_1	cementitious	1	–	–	3	–

1.4.3.2 Mortars of Historic Masonries

The mortars present in the masonry of historic buildings are as a rule composed of simple or hydraulic limes. They can be subdivided into:

- simple mortars;
- hydraulic mortars;
- composite mortars.

Italian building codes provide for dividing mortars into the types: *cementitious*, classified as M_1 and M_2, according to the cement content; *composite,* indicated as M_3, containing both lime and cement; and *hydraulic* or *pozzolanic*, indicated as M_4, containing only hydraulic lime or lime and pozzolana. For instance, a mix designated as M_1 has three parts sand by volume and one part cement, while an M_4 mix has three parts sand by volume and one part hydraulic lime (Table 1.4).

Simple limes were in widespread use in the past because of the efficiency and easy workability of quicklime. They harden slowly and weaken in the air. Their compression strength is very low, about 5 kg/cm^2.

Hydraulic mortars are prepared with mixtures of hydraulic limes, water, and sand. The standard composition of a hydraulic mortar is given in Table 1.4: three parts sand and one part hydraulic lime by volume. The compression strength of hydraulic mortar is about 25 kg/cm^2, lower than that of composite mortars (about 50 kg/cm^2), and much lower than that of cementitious mortars (at least 120 kg/cm^2). As a rule, the strength of mortar is less than that of concrete.

1.5 Tests on Rock and Mortar Specimens

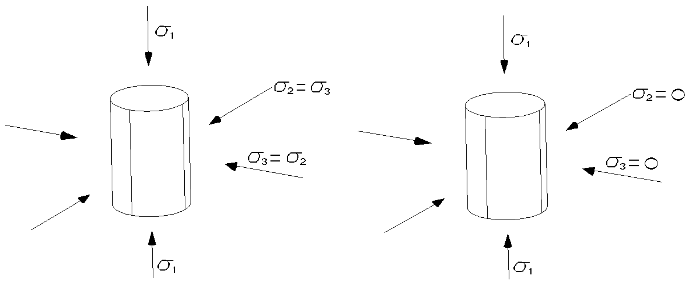
The most common experiments carried out on specimens of rocks or mortars are:

a) uniaxial tension and compression tests
b) multi-axial tests
c) torsion tests;
d) flexural tests.

1.5.1 Tests on Rock Specimens

Cylindrical specimens are used. In multiaxial testing, three stresses ($\sigma_1, \sigma_2, \sigma_3$) and three strains ($\varepsilon_1, \varepsilon_2, \varepsilon_3$) are measured. Usually, σ_1 is the major principal stress, generally vertical, and $\sigma_2 = \sigma_3$ the intermediate lateral stresses. (Fig. 1.8).

In a uniaxial tension test, a tensile stress σ_1 is applied by means of pincers or a metal plate glued to the specimen. There are also indirect splitting tests, such as the so-called Brazilian and Flexural tests. In a standard uniaxial compression test a load is applied to a cylindrical specimen by means of steel loading platens. The friction strength between the rigid platens of the testing machine and the specimen heads prevents lateral expansion of the specimen. Shears (Fig. 1.9) are thus superimposed on the vertical compression causing a three-dimensional stress state leading to cracks splitting the specimen diagonally). Figure 1.10 shows the typical *hourglass* failure mode of a concrete specimen obtained through a standard compression test. Mortar specimens exhibit the same behavior.



a) , *b*)

Fig. 1.8. Triaxial tests: *a*) *b*) uniaxial test.

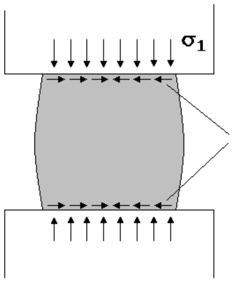


Fig. 1.9. Shear friction stresses

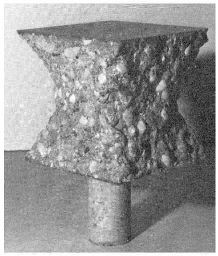


Fig. 1.10. Hourglass failure

Figure 1.11 presents the results obtained by Brown testing marble prisms under biaxial compression at a constant ratio σ_2/σ_1. The strengths in the diagram are presented as ratios σ_2/σ_c and. σ_1/σ_c ,where σ_c is the corresponding uniaxial compression strength. The presence of the lateral compression σ_2 yields an increase in strength of no greater than 15%.

The effects of the loading conditions are significant. Figure 1.12 shows the biaxial failure domain of concrete according to the test results obtained by Kupfer (1973). These results show that the behavior of concrete is similar to the marble specimen tested by Brown. These results will be reconsidered in the following sections.

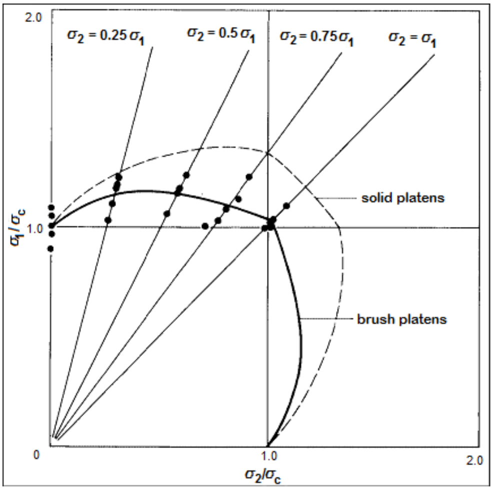


Fig. 1.11. Biaxial compression tests on marble prisms (Brown, 1974).

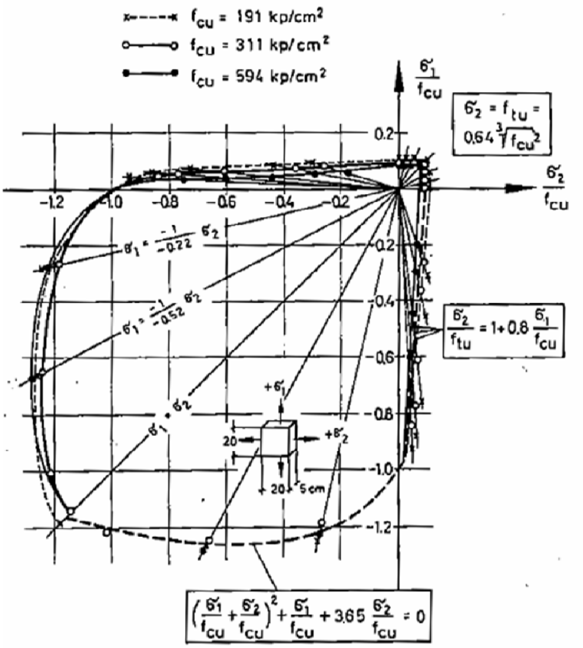


Fig. 1.12. The biaxial failure domain of concrete (Kupfer, 1973).

1.5.2 Uniaxial Compression Tests on Mortar Specimens

Italian regulations call for measuring compression strength by testing three prismatic specimens of dimensions 40 × 40 × 160 mm. The three mortar specimens are first cast in metal molds from which they are removed after 24 hours and cured in a humid environment at a constant temperature of 20 °C. A specimen is then placed on side supports and loaded with a central point load until bending failure is reached. The bending failure stress is evaluated simply as

$$f_{mf} = \frac{3}{2}\frac{PL}{b^3} \tag{2}$$

where P is the applied collapse load, L = 100 mm the distance between the supports, and b the length of the side of the specimen's square cross-section. Six simple compression tests are then performed on the remaining half prisms and the average compression strength $f_m = Q/b^2$ is obtained from the failure force Q.

The compression strength of mortar is quite low: M_1 cement mortar has a strength of about 120 kg/cm^2; while the strength of M_4 lime mortar does not exceed 20 kg/cm^2. The strength of the different types of mortars used in historic masonries can be considered similar to that of M_4 type mortar, or lower.

1.5.3 Stress-Strain Diagrams for Stone and Mortar Materials

Many rocks, when loaded in uniaxial compression, exhibit the typical load deformation response plotted in figure 1.13.

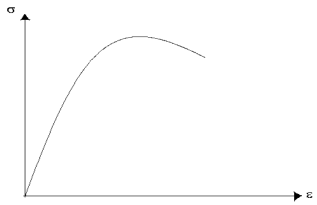


Fig. 1.13. A typical compression stress-strain diagram for rock material

An ascending branch is followed by a softening one. The peak represents the compression strength of the rock. The initial segment of the ascending branch is more or less straight up to a stress level equal to about 60 % peak stress. The slope of the diagram at the origin measures the rock's Young's modulus. A stiff testing machine is necessary to trace the full extent of the descending branch of the stress-strain curve.

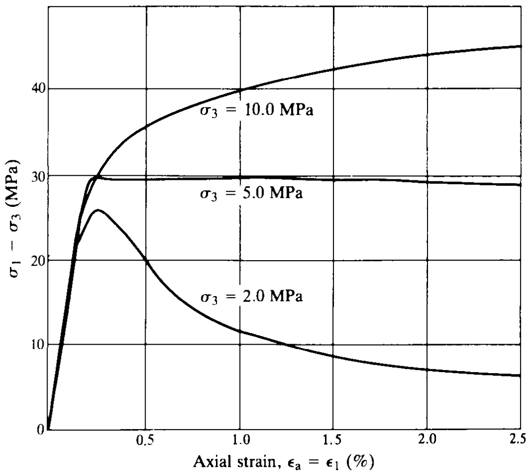


Fig. 1.14. σ -ε diagrams of a limestone in triaxial tests with various cell pressures (Brady and Brown, 2004)

Stress-strain diagrams are heavily influenced by the test conditions. In triaxial tests three conditions are of paramount importance:

a) the cell pressure value;
b) the temperature;
c) the load application velocity.

Figure 1.14 shows the influence of the cell pressure $\sigma_2 = \sigma_3$ on the stress-strain diagram of a rock specimen. Increasing the cell pressure increases both the compression strength and ductility of the material.

1.6 Formulation of a Triaxial Failure Criterion for Stone Materials

1.6.1 Preliminary Considerations

There are many failure criteria for stone materials.(Bazant and Jirasek, 2002). These criteria, generally, adapt the Plasticity theory to fit, more or less, the properties of the experimentally determined failure surfaces, to get the heart of the problem. In the next following sections, on the contrary, we will present a simple criterion, (Como, Luciano, 2006, 2007) founded on very different assumptions, able to describe the basic aspects of the question. Stone, together with brick, is the basic material of masonry constructions. Knowing the strength behavior of stone materials is thus essential to understand the statics of masonry structures.