ERNST ZERMELO

Collected Works
Gesammelte Werke

VOLUME II
BAND II

Calculus of Variations,
Applied Mathematics,
and Physics

Variationsrechnung,
Angewandte Mathematik
und Physik

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Collected Works
Gesammelte Werke

Zermelo around 1910

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Set Theory,
Miscellanea

Mengenlehre,
Varia

VOLUME II
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BAND II

Calculus of Variations, Applied Mathematics, and Physics
Variationsrechnung, Angewandte Mathematik und Physik

Edited by
Herausgegeben von

Heinz-Dieter Ebbinghaus,
Akihiro Kanamori

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Preface to the Zermelo edition

This is a complete edition of the published works of Ernst Zermelo which moreover includes selected correspondence and unpublished manuscripts. Zermelo is generally acknowledged for his pioneering work in axiomatic set theory and for introducing the axiom of choice as a basic principle of mathematics. In contrast, his work in applied mathematics and physics, despite its originality, is hardly recognized or has even been attributed to others. This edition of Zermelo's collected works provides a picture of the entire mathematician. It appears in two volumes. The first volume comprises Zermelo's published papers in set theory and the foundation of mathematics together with isolated papers of an algebraic, analytic, or number-theoretic character. The second volume is dedicated to Zermelo's work in the calculus of variations, mathematical physics, and fluid dynamics. Both volumes are supplemented by selected notes and manuscripts, mainly from Zermelo's *Nachlass*, which throw additional light on his papers, reflect his point of view, or are unpublished continuations of published work. To the best judgment of the editors, the selected notes and manuscripts fully and faithfully represent the essential unpublished writings of Zermelo concerning mathematics. Nevertheless, a possible edition of a third volume comprising further unpublished notes and letters from the *Nachlass* has *expressis verbis* been left open.

Both volumes contain some writings by other authors which include contributions actually written by Zermelo or which react to criticism Zermelo had made. Details are given in the prefaces to the respective volumes.

In order to provide access to a wider audience, the original papers are printed face to face with English translations. As both versions use the same layout, it is easy to go from the translation to the original version and vice versa. The layout itself tries to preserve the appearance of the original papers. For details we refer to the editorial information below.

Each paper or coherent group of papers is preceded by an introductory note which comments on contents, motivation, aims, and influence of the paper(s) concerned. Written by an expert in the field, it came to its final form in discussions with the editors.

Each volume contains a full bibliography of Zermelo together with a schematic *curriculum vitae* which will enable the reader to become acquainted with the personal circumstances from which a paper arose. In addition, Volume I starts with a more detailed biographical sketch of Zermelo's life and work.

Many of these features found their inspiration in the exemplary edition of Kurt Gödel's collected works by Solomon Feferman, John W. Dawson, Jr., and others.

The edition of Zermelo's collected works has a prehistory. Already as early as 1912, at the age of 41 and faced with a serious recurrence of his tuberculosis,

Zermelo conceived plans for an edition of his collected papers, but did not pursue them when his health improved. In 1949, under likewise deplorable personal circumstances, he tried again, this time approaching several publishers, among them Springer-Verlag. But the difficult situation in post-war Germany precluded such an enterprise. Immediately after Zermelo's death, in 1953, the historian of mathematics Helmuth Gericke and the philosopher Gottfried Martin, who had gotten to know Zermelo in the 1930s in Freiburg, started work on a two-volume edition, in 1956 gaining Paul Bernays as a third editor. Support was provided by the Kant-Gesellschaft. However, the plans were not realized; in 1962 work on the edition came to a definite end.

When in early 2004 new plans for an edition of Zermelo's collected works became more concrete, they found the enthusiastic support of Martin Peters of Springer-Verlag. In discussions with him it became clear very quickly that the edition should provide English translations and detailed comments. As Zermelo had been a member of the Heidelberger Akademie der Wissenschaften, the editors turned to the academy for financial support. The application found warm backing of Hans Günter Dosch, then Sekretar of the class for mathematics and the sciences of the academy. The application was successful. Even more, besides providing generous funding, the academy offered to let the edition appear in its regular series of publications of the class for mathematics and the sciences published by Springer-Verlag.

The editors wish to express deep gratitude to the Heidelberg academy for their ideal, financial support and to Springer-Verlag for their open-minded cooperation. In particular, many thanks go to Hans Günter Dosch and Martin Peters.

Freiburg, Toronto, and Boston
September 2009

Heinz-Dieter Ebbinghaus
Craig G. Fraser
Akihiro Kanamori

Preface to volume II

This second volume concludes the edition of Ernst Zermelo's collected works. The volume focuses on his contributions mainly to analysis and physics. Except for an excursion into physical chemistry (*Riesenfeld and Zermelo 1909*), the papers come from the decade around 1900 when Zermelo was in Berlin and Göttingen and about two years around 1930 when he was in Freiburg. They are accompanied by three items found in Zermelo's and in David Hilbert's *Nachlass*. For orientation especially about the personal circumstances accompanying the genesis of the papers, the volume starts with Zermelo's *curriculum vitae*, the one given in volume I.

Zermelo's works of an applied character may hold pioneering ideas and insights, but they did not receive the attention they deserved. One reason may be the sheer diversity of topics he treated. Of course, one should also take into consideration that starting soon after the turn of the century his mathematical work shifted elsewhere for more than two decades, to set theory and mathematical logic, research in these disciplines leading him to his most influential scientific achievements.

The Berlin-Göttingen period comprises three topics: the calculus of variations, the kinetic theory of gases, and hydrodynamics.

The engagement with the calculus of variations started with Zermelo's Ph. D. thesis (*1894*), written at the University of Berlin under the guidance of Hermann Amandus Schwarz.

The engagement in the kinetic theory of gases started in 1896, also in Berlin, when Zermelo became an assistant to Max Planck. It lasted for about ten years. Its best-known part, a controversy with Ludwig Boltzmann, is described and analyzed here in full with the inclusion of *Boltzmann 1896, 1897*.

Zermelo's interest in meteorology led him to hydrodynamics, work that culminated in his 1899 *Habilitation* thesis (*1902a, s1902b, s1902c*) in Göttingen.

In the late 1920s, Zermelo came back to his "old, even though mostly unhappy love for the 'applications' ". The starting paper (*1928*) on the evaluation of chess tournaments, with its early use of the maximum likelihood method, was to remain unknown until several decades later other people rediscovered his methods and results. Motivated by the circumnavigation of the earth by the airship Graf Zeppelin in August 1929, Zermelo wrote two papers (*1930c, 1931a*) on optimal steering methods of airships. Soon, however, this return to mathematics of an applied character came to an end when Zermelo got involved in a serious foundational debate which fully occupied what strength was left him after a serious illness.

The introductory notes are a crucial part of the Zermelo edition. Those who agreed to comment on a paper or a group of papers in this volume generously

shared their experience and knowledge with us and the potential reader. We at times had involved discussions toward securing the most informative and accurate presentations, and we appreciate the professionalism that was brought to bear.

The translations of the original papers were carried out by Enzo de Pellegrin. We again admire his extraordinary care and his feeling for both languages when handling Zermelo's style with its richness in nuances and its involved sentential structures. The introductory notes of Rüdiger Thiele were translated by David Kramer who with diligence and care successfully mirrored the style of the original German.

We express our gratitude to all who have supported us during our work. In this connection we would like to mention Ruth Allewelt from Springer-Verlag, Andrea Köhler and Petra Möws of Le-Tex Publishing Services, and Marlies Würth, the librarian of the Freiburg Mathematical Institute.

Again, Martin Peters of Springer-Verlag was ready to offer valuable help and advice.

We appreciate that Craig Fraser, while not being able to continue with his participation in the edition, was ready to contribute two substantial introductory notes.

Freiburg and Boston — Heinz-Dieter Ebbinghaus
December 2012 — Akihiro Kanamori

Editorial information

Layout. The layout of the texts as well as of the translations mirrors the layout of the originals. Emphasized words, i.e., words in italics or words spaced out or consisting of small capitals, are given in italics. Original pagebreaks are indicated in the texts by "|", and the number of the new original page beginning there is given on the margin.

Editorial annotations. These are set in double square brackets "⟦" and "⟧".

Misprints and errors. Small textual errors in the originals are tacitly corrected; larger ones are corrected with the corrections commented on in editorial annotations.

Wrong words or words missing in the originals have been replaced or added in double square brackets.

Misprints in mathematical expressions in the originals are not corrected in the texts. They are, however, corrected in the translations and noted by an editorial annotation.

References. In the texts Zermelo's references to the literature are not altered. Translations as well as introductory notes refer to the main bibliography at the end of the volume instead and have the form *author(s) year of appearance*, followed by an additional index *a*, *b*, *c*, ... if necessary. An example: *Hahn and Zermelo 1904*. If the authors are clear from the context, their names may be omitted; in such a case, *1904* may be short for *Hahn and Zermelo 1904*. References to page numbers are kept in both the texts and the translations; they can be traced via the original pagebreaks and the original page numbers provided in the texts.

Footnotes. Whereas the translations use natural numbers in ascending order as footnote marks, the texts preserve the original marks. It may thus happen that a page of the text may contain identical footnote marks. In such cases the original page numbers on the margin allow for quick correlation of mark and footnote.

Figures. Whenever possible, a figure is located at the same position as in the original. If this is not possible for a figure, say Fig. n, then its original position is indicated on the margin by "Fig. n" and the figure itself appears as close as possible, at worst on the top of the next page.

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Akademie der Wissenschaften zu Göttingen (*1899a*);

Niedersächsische Staats- und Universitätsbibliothek Göttingen (*s1899b*);

S. Hirzel Verlag (*1900, Riesenfeld and Zermelo 1909*);

B. G. Teubner (*1902a, 1902d, Hahn and Zermelo 1904, 1906, 1930c*);

Universitätsarchiv Freiburg (*s1902b, s1902c*);

Springer Verlag (*1904a, 1928*).

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Corrections to volume I

pp. xxiii–xxiv: Some English titles have been slightly changed.

p. xxiv, line 12: Replace "*Zermelo 1903*" by "*Zermelo 1904a*".

p. 239, footnote 4: Replace "art. 64" by "art. 66" and delete the part in brackets.

p. 364, lines 21/22 and p. 365, lines 23/24: Instead of

> "indem sie sich nicht sowohl auf die einzelnen 'definiten' Sätze p als auf ihre *Gesamtheit* P bezieht"

Zermelo means

> "indem sie sich nicht auf die einzelnen 'definiten' Sätze p, sondern vielmehr auf ihre *Gesamtheit* P bezieht".

Hence, instead of

> "since it does not refer both to the individual 'definite' propositions p and to their *totality* P"

the translation should be changed to

> "since it does not refer to the individual 'definite' propositions p, but rather to their *totality* P".

p. 367, line 1: Delete "0".

p. 537, line -18: Replace "*Husserl 1928*" by "*Husserl 1922*".

p. 539, line -1: Replace "partial ground relation" by "partial justification relation".

p. 540, line 6: Add a right parenthesis after "Sect. 1".

p. 541, line -3: Replace "the opening paragraph of Sect. 1" by "the paragraph preceding Sect. 1".

p. 642, line 11: Replace "1903" by "1904a" and shift the newly named item *1904a* behind item *1904*.

Ernst Zermelo

Collected Works
Gesammelte Werke

Volume II
Band II

Calculus of Variations,
Applied Mathematics,
and Physics

Variationsrechnung,
Angewandte Mathematik
und Physik

Ernst Zermelo's *curriculum vitae*

Heinz-Dieter Ebbinghaus

1871

27 July: Zermelo is born in Berlin as the second child and the only son of the *Gymnasialprofessor* Theodor Zermelo and his wife Auguste neé Zieger.

1878

3 June: Death of Zermelo's mother.

1880

April: Zermelo enters the Luisenstädtisches Gymnasium in Berlin.

1889

24 January: Death of Zermelo's father.
1 March: Zermelo finishes school. Remarks in his leaving certificate show that he suffers from physical fatigue.
Summer semester – summer semester 1890: Zermelo studies mathematics and physics at the University of Berlin with, among others, Lazarus Fuchs and Johannes Knoblauch.

1890

Winter semester 1890/91: Zermelo studies at the University of Halle-Wittenberg with, among others, Georg Cantor and Edmund Husserl.

1891

Summer semester 1891: Zermelo studies at the University of Freiburg, with his subjects including, as in Berlin and Halle, philosophy and psychology.
Winter semester 1891/92 – summer semester 1897: Zermelo studies again at the University of Berlin with, among others, Ferdinand Frobenius, Max Planck, Hermann Amandus Schwarz, and Wilhelm Wien.

1894

23 March: Zermelo applies to begin the Ph. D. process.
6 October: Zermelo obtains his Ph. D. degree. His dissertation *Untersuchungen zur Variations-Rechnung* was supervised by Hermann Amandus Schwarz.
December – September 1897: Zermelo is an assistant to Max Planck at the Institute for Theoretical Physics of the University of Berlin.

1895

December: Zermelo completes his first paper, *1896a,* which sets out his opposition to Ludwig Boltzmann's statistical theory of heat.

1896

Summer: Zermelo applies for an assistantship at the Deutsche Seewarte in Hamburg, but then decides to pursue an academic career.

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15 September: Zermelo completes his second paper, *1896b,* opposing Boltzmann.

1897

German translation *Glazebrook 1897* of *Glazebrook 1894.*
2 February: Zermelo passes the state exam for *Gymnasiallehrer* (high school teachers) that allowed him to teach mathematics and physics as main subjects and chemistry, geography, and mineralogy as additional subjects. According to the reports Zermelo exhibited a broad knowledge in German literature, philosophy, and religion.
19 July: Zermelo asks Felix Klein in Göttingen for support for his *Habilitation.*
Winter semester 1897/98: Zermelo continues his studies at Göttingen with, among others, David Hilbert, Felix Klein, and Arthur Schoenflies.

1899

3 February: David Hilbert presents Zermelo's first paper in applied mathematics, *1899a,* to the Königliche Gesellschaft der Wissenschaften zu Göttingen; it treats differential equations with inequalities.
Zermelo initiates the *Habilitation* process with the *Habilitation* thesis "Hydrodynamische Untersuchungen über die Wirbelbewegungen in einer Kugelfläche" the first part of which is published as *1902a.* The second part (*1902b, 1902c*) remained unpublished; it contains a solution of the 3-vortex problem on the sphere.
4 March: Zermelo gives his *Habilitation* address, *1900,* which proposes an alternative probabilistic approach to Bolzmann's in the latter's work in statistical mechanics. He is granted the *venia legendi* for mathematics at the University of Göttingen.

Around 1900

Beginning of the cooperation with Hilbert on the foundations of mathematics. Zermelo formulates the Zermelo-Russell paradox.

1900

Winter semester 1900/01: Zermelo gives his first course on set theory, the main topic being the Cantorian theory of cardinals.

1901

9 March: David Hilbert presents Zermelo's result on the addition of cardinals, *1901,* to the Königliche Gesellschaft der Wissenschaften zu Göttingen. The proof uses the axiom of choice.

1902

12 May: Zermelo gives a talk on Frege's foundation of arithmetic before the Göttingen Mathematical Society.
Summer semester – winter semester 1906/07: Zermelo receives a *Privatdozenten* grant.

Publication of *1902d,* the first paper on the calculus of variations after the Ph.D. dissertation. It treats shortest lines of bounded steepness with or without bounded torsion.

1903

June: Zermelo is under consideration for an extraordinary professorship of mathematics at the University of Breslau. He is shortlisted in the second position after Gerhard Kowalewski, Franz London, and Josef Wellstein who are shortlisted *aequo loco* in the first position.
1 December: Zermelo completes his second paper on the calculus of variations, *1904a.* It gives two simple proofs of a result of Paul du Bois-Reymond on the range of the method of Lagrange.

1904

Beginning of a life-long friendship with Constantin Carathéodory.
Together with Hans Hahn, Zermelo writes a contribution on the calculus of variations, *Hahn and Zermelo 1904,* for the *Encyklopädie der mathematischen Wissenschaften.*
August: Third International Congress of Mathematicians at Heidelberg. Julius König gives a flawed refutation of Cantor's continuum hypothesis. The error is detected by both Zermelo and Felix Hausdorff.
24 September: Zermelo informs Hilbert about his proof of the well-ordering theorem and the essential role of the axiom of choice. The letter is published as *1904.*
15 November: During a meeting of the Göttingen Mathematical Society, Zermelo defends his well-ordering proof against criticism by Julius König, Felix Bernstein, and Arthur Schoenflies.

1905

January: Zermelo falls seriously ill. In order to recover, he spends spring and early summer in Italy.
German translation *Gibbs 1905* of *Gibbs 1902.*
Spring: Zermelo works on the theory of finite sets which finally results in *1909a* and *1909b.*
21 December: Zermelo receives the title "Professor". The application had been filed by Hilbert in December 1904.

1906

Early that year: Zermelo catches pleurisy.
Zermelo works on a book on the calculus of variations together with Carathéodory.
Zermelo publishes a final criticism of Boltzmann's statistical interpretation of the second law of thermodynamics in the review *1906* of *Gibbs 1902.*
Summer semester: Zermelo lectures on "Mengenlehre und Zahlbegriff". He formulates an axiom system of set theory which comes close to the one published by him in 1908.
June: Medical doctors diagnose tuberculosis of the lungs.

Summer: Zermelo spends a longer time at the seaside.
Autumn: Zermelo is under discussion for a full professorship of mathematics at the University of Würzburg. The professorship is given to the extraordinary Würzburg professor Georg Rost. According to Hermann Minkowski Zermelo's difficulties in obtaining a professorship are rooted in his "nervous haste".
Winter 1906/07 – winter 1907/08: Several extended stays in Swiss health resorts for lung diseases.

1907

March: Zermelo applies for a professorship at the Academy of Agriculture in Poppelsdorf without success.
May: During a stay in Montreux Zermelo finishes his paper *1909a.*
14 July and 30 July: During a stay in the Swiss alps Zermelo completes his papers on a new proof of the well-ordering theorem and on the axiomatization of set theory, *1908a* and *1908b,* respectively.
20 August: Following an application by the Göttingen Seminar of Mathematics and Physics, the ministry commissions Zermelo to give lecture courses in mathematical logic and related matters, thus installing the first official lectureship for mathematical logic in Germany.

1908

April: Fourth International Congress of Mathematicians in Rome. Zermelo presents his work on finite sets, *1909b.* He becomes acquainted with Bertrand Russell. Together with Gerhard Hessenberg and Hugo Dingler he conceives plans for establishing a quarterly journal for the foundations of mathematics. The project fails because of diverging views between the group and the Teubner publishing house.
Summer semester: Zermelo gives a course on mathematical logic in fulfilment of his lectureship for mathematical logic and related topics.

1909

July: Zermelo is under consideration for an extraordinary professorship of mathematics at the University of Würzburg. He is shortlisted in the first position. The professorship is given to Emil Hilb shortlisted in the second position.
September: Completion of *Riesenfeld and Zermelo 1909.*

1910

24 January: The board of directors of the Göttingen Seminar of Mathematics and Physics applies to the minister to appoint Zermelo an extraordinary professor.
21 January: Zermelo, being under consideration for a full professorship of mathematics at the University of Zurich, is shortlisted in the first position.
24 February: The *Regierungsrat* of the Canton Zurich approves the choice of Zermelo.
15 April: Zermelo is appointed a full professor at the University of Zurich for an initial period of six years.

1911

28 January: Zermelo applies for leave for the coming summer semester because of a worsening of his tuberculosis.
February and March: Together with a partner, Zermelo applies for several patents concerning, for example, a regulator for controlling the revolutions of a machine.
Zermelo is awarded the interest from the Wolfskehl prize, Hilbert being chairman of the Wolfskehl committee of the Gesellschaft der Wissenschaften zu Göttingen.
Summer semester – winter semester 1911/12: Leave for a cure because of tuberculosis.

1912

January: Serious worsening of tuberculosis diagnosed.
Beginning of the cooperation with Paul Bernays who completes his *Habilitation* with Zermelo in 1913 and stays at the University of Zurich as an assistant to Zermelo and later as a *Privatdozent* until 1919.
August: Fifth International Congress of Mathematicians in Cambridge. Following an invitation by Bertrand Russell, Zermelo gives two talks, one on axiomatic and genetic methods in the foundation of mathematical disciplines and one on the game of chess. The second one results in the paper *1913* which may be considered the first paper in game theory.
Faced with the seriousness of his illness, Zermelo conceives plans for an edition of his collected papers.

1913

Spring: Zermelo is considered for a full professorship in mathematics at the Technical University of Breslau. He is shortlisted in the first position. The professorship is given to Max Dehn, shortlisted in the second position together with Issai Schur.
December: Zermelo completes his paper *1914* on subrings of whole transcendental numbers of the field of the real numbers and the complex numbers, respectively; it makes essential use of the axiom of choice.

1914

Early that year: Zermelo has regular discussions with Albert Einstein.
March: Operation of the thorax by Ferdinand Sauerbruch, the pioneer of thorax surgery.

Around 1915

Zermelo develops a theory of ordinal numbers where the ordinals are defined as by John von Neumann in 1923.

1915

Spring: A new serious outbreak of tuberculosis forces Zermelo to take a one-year leave.
July: Waldemar Alexandrow completes his Ph. D. thesis *Alexandrow 1915.* It is the only thesis guided by Zermelo alone. Kurt Grelling's thesis *Grelling*

1910, which extends Zermelo's theory of finite sets, was officially supervised by David Hilbert, but guided by Zermelo.
Autumn: Several surgical treatments of a tuberculosis of the vocal chords.

1916

21 March: As his illness is expected to extend into the summer semester, Zermelo is urged to agree to retire.
5 April: Zermelo agrees to retire.
15 April: Zermelo retires from his professorship.
31 October: Zermelo is awarded the annual Alfred Ackermann-Teubner prize of the University of Leipzig for the promotion of the mathematical sciences. Later prize winners include, for example, Emil Artin and Emmy Noether.
1 November – February 1917: Zermelo stays in Göttingen.
7 November 1916: Zermelo presents his theory of ordinal numbers to the Göttingen Mathematical Society.

1917

March – October 1919: Zermelo stays in various health resorts in the Swiss alps.

1919

July: First draft of the paper *1928* wherein Zermelo develops a procedure for evaluating the result of a tournament by using a maximum likelihood method.
November – March 1921: Zermelo stays at Locarno, Switzerland.

1921

Spring: Zermelo stays in Southern Tyrol and has correspondence with Abraham Fraenkel.
6 May 1921: Fraenkel informs Zermelo about a gap he has discovered in Zermelo's 1908 axiom system of set theory.
10 May: In his answer to Fraenkel, Zermelo proposes a second-order version of the axiom of replacement in order to close the gap, at the same time criticizing it because of its non-definite character.
17 July (?): Zermelo formulates his "infinity theses" (*s1921*) where he describes the aims of his research in infinitary languages and infinitary logic as carried out in the early 1930s.
22 September: Fraenkel announces his axiom of replacement in a talk delivered at the annual meeting of the Deutsche Mathematiker-Vereinigung. Zermelo agrees in principle, but maintains a critical attitude because of a deficiency of definiteness.
1 October: Zermelo settles in Freiburg, Germany.

1923

Winter semester 1923/24: Zermelo attends Edmund Husserl's course "Erste Philosophie".
– 1929: Cooperation with Marvin Farber on the development of a semantically based logic system, in 1927 leading to plans for a monograph on logic.