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Andreas Johann
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Stephan Schmitz *Editors*

Recent Trends in Dynamical Systems

Proceedings of a Conference in Honor of Jürgen Scheurle

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Editors

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To Jürgen Scheurle

Preface

In January 2012 the International Conference *Recent Trends in Dynamical Systems* was held in Munich on the occasion of Jürgen Scheurle's 60th birthday. As parts of this conference, a scientific colloquium took place at the Carl Friedrich von Siemens Stiftung in Munich from 11th to 13th of January and also a Festkolloquium at the Technische Universität München in the afternoon of January 13th. Besides numerous posters on recent advances in the field of dynamical systems, 25 highly recognized scholars gave plenary talks that were grouped according to the following themes:

– Stability and bifurcation
– Geometric mechanics and control theory
– Invariant manifolds, attractors, and chaos
– Fluid mechanics and elasticity
– Perturbations and multiscale problems
– Hamiltonian dynamics and KAM theory

These themes reflect the broad scientific interests of Jürgen Scheurle and his fascination of applying mathematics to real world situations, in particular from physics and mechanics. The volume at hand is an outgrowth of this conference, containing research articles about exciting new developments in the multifaceted subject of dynamical systems as well as survey articles. We are very happy that the authors accepted the invitation to contribute to this volume in honour of Jürgen Scheurle and we are sure that their exciting articles will be of interest not only to experts in the field of dynamical systems but also to graduate students and scientists from many other fields, including engineering. This is in the spirit of Jürgen Scheurle, who, besides his research activities, always puts a lot of emphasis on conveying the beauty of the Theory of Dynamical Systems and its applicability to real world problems in extremely well-prepared, beautiful lectures.

Munich, Germany
January 2013

Andreas Johann
Hans-Peter Kruse
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Short Curriculum Vitae of Jürgen Scheurle

Jürgen Scheurle was born on September 26, 1951, in Schwäbisch Gmünd, Baden-Württemberg. He received his professional education at the University of Stuttgart, where he studied mathematics, physics, and computer science from 1970 until 1974, and finished his diploma degree in mathematics with a thesis entitled "Ein Antikonvergenzprinzip". Some months later, in 1975, he completed his doctorate under the guidance of Klaus Kirchgässner. The title of his Ph.D. thesis is "Ein selektives Iterationsverfahren und Verzweigungsprobleme". In 1981 he presented his Habilitation thesis on "Verzweigung quasiperiodischer Lösungen bei reversiblen dynamischen Systemen".

From 1974 to 1985 Jürgen Scheurle held positions as a postdoctoral researcher, senior researcher, and assistant professor, at the University of Stuttgart. In 1982 he was visiting professor at the Department of Mathematics, University of California, Berkeley (USA), and in 1983 at the Division of Applied Mathematics, Brown University, Providence (USA). In 1985 Jürgen Scheurle moved to Fort Collins (USA), where he became an associate and later full professor at Colorado State University. In 1987 he accepted a full professorship and the Chair of Theory and Applications of Partial Differential Equations at the University of Hamburg. In 1996 Jürgen Scheurle was appointed full professor at the Technische Universität München (TUM) and since then holds the Chair of Advanced Mathematics and Analytical Mechanics. Notable predecessors at this chair were Felix Klein, Walter von Dyck, and Robert Sauer, see Fig. 1, which illustrates the special responsibility of Jürgen Scheurle for the mathematical education of engineering students.

He was the founding director of the Center for Mathematics at TUM and later dean of the Faculty of Mathematics. As dean, he continued the reform-oriented politics of his predecessors. During his term in office, the faculty voluntarily conducted a peer assessment and was awarded the title "Reformfakultät" by the "Stifterverband der Deutschen Wissenschaft". Such assessments are common nowadays but were completely novel 10 years ago. Moreover, far ahead before such procedures were put into law, the Bavarian Ministry of Research and Teaching allowed the faculty to introduce an "Experimentierklausel" to assess prospective for the admission of students.

Jürgen Scheurle was responsible for the introduction of the "Master of Science in Industrial & Financial Mathematics" at the off-shore campus of TUM in Singapore. He was a member of the planning team for the new mathematics building at the research campus Garching and in charge of the relocation from downtown Munich to Garching in 2002. Finally, Jürgen Scheurle was and is member of numerous expert committees appointed by the president of the TUM and the faculty of mathematics. Inter alia he is representative of the "Bayerische Eliteakademie", member of the "Hurwitz-Gesellschaft zur Förderung der Mathematik an der TU München" and its president since 2011.

Jürgen Scheurle authored and co-authored several pioneering publications, and among them the following are highly influential articles:

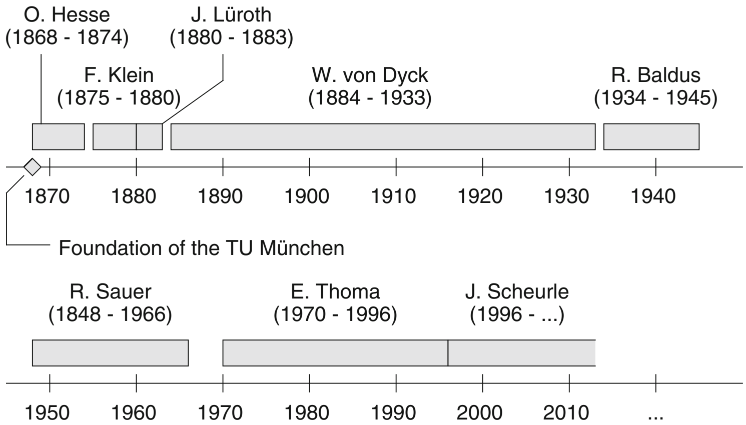

Fig. 1 Genealogy of the chair "Analytische Mechanik und Angewandte Mathematik" at the Technische Universität München

- *On the bounded solutions of a semilinear elliptic equation in a strip* (together with K. Kirchgässner). J. Diff. Equat. 32 (1) (1979), 119–148.
- *Smoothness of bounded solutions of non-linear evolution equations* (together with J. Hale). J. Diff. Equat. 56 (1) (1985), 142–163.
- *Chaotic solutions of systems with almost periodic forcing*. ZAMP 37 (1986), 12–26.
- *The construction and smoothness of invariant manifolds by the deformation method* (together with J. Marsden). SIAM J. Math. Anal. 18 (5) (1987), 1261–1274.
- *Exponentially small splittings of separatrices in KAM theory and degenerate bifurcations* (together with P. Holmes and J. Marsden). Cont. Math. 81 (1988), 213–243.
- *Existence of perturbed solitary wave solutions to a model equation for water waves* (together with J. Hunter). Physica D 32 (1988), 253–268.
- *Lagrangian reduction and bifurcations of relative equilibria of the double spherical pendulum* (together with J. Marsden). ZAMP 44 (1993), 17 - 43.
- *The reduced Euler-Lagrange equations* (together with J. Marsden). Fields Inst. Comm. 1 (1993), 139–164.
- *Pattern evocation and geometric phases in mechanical systems with symmetry* (together with J. Marsden), Dyn. and Stab. of Systems 10 (1995), 315–338.
- *Discretization of homoclinic orbits and "invisible" chaos* (together with B. Fiedler). Memoirs of the AMS vol. 119, nb. 570 (3), Providence 1996.
- *Reduction Theory and the Lagrange-Routh equations* (together with J. Marsden and T. Ratiu). J. Math. Phys. 41(6) (2000), 3379–3429.
- *The orbit space method* (together with M. Rumberger). In Ergodic Theory, Analysis and Efficient Simulation of Dynamical Systems, B. Fiedler edt., Springer-Verlag 2001, 649–689.

- *On the generation of conjugate flanks for arbitrary gear geometries* (together with A. Johann). GAMM-Mitt. 32, No. 1, 2009, 61–79.

His teaching covers a wide spectrum of subjects, ranging from mathematics for engineering students, functional analysis, ordinary differential equations and partial differential equations to dynamical systems, bifurcation theory, hamiltonian dynamics, geometric mechanics, mathematical methods in continuum mechanics, and mathematical modeling in biology and ecology. He supervised more than 20 dissertations and habilitations in these areas.

Jürgen Scheurle was a member of the advisory board of the book series *Dynamics Reported* and an executive editor of the *International Journal of Nonlinear Mechanics*. He is currently a member of the editorial board of the *Journal of Nonlinear Science*, *Nonlinear Science Today*, *Journal of Applied Mathematics and Mechanics* (*ZAMM*), and *Journal of Geometric Mechanics*.

Conference photo in the garden of the Carl Friedrich von Siemens Stiftung at the Schloß Nymphenburg, Munich

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- Jörg-Stefan Bock
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- Francesco Fasso
- Peter Giesl
- Christoph Glocker
- John Guckenheimer
- Thomas Hagen
- Heinz Hanßmann
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- Oliver Junge
- Hansjörg Kielhöfer
- Peter E. Kloeden
- Thorsten Knott
- Peter Koltai
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Contents

Part II Hamiltonian Dynamics, Geometric Mechanics and Control Theory

Part III Continuum Mechanics: Solids, Fluids and Other Materials

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Part I
Stability, Bifurcation and Perturbations

Chapter 1
The Birth of Chaos

John Guckenheimer

1.1 Introduction

The word ***chaos*** has become firmly embedded in the literature on dynamical systems. Indeed, James Gleick's book, Chaos Theory [17], established that term as a description of the entire subject in the public mind. Nonetheless, there is no authoritative technical meaning of "chaos" in dynamical systems. Li and Yorke first used the word in the title of their paper "Period three implies chaos" [31], but it does not appear in the text. They refer to trajectories that are "nonperiodic and might be called 'chaotic'." Ruelle and Takens [45] used the longer phrase "sensitive dependence to initial conditions" and the two terms have largely been regarded as synonyms [15, 57]. The informal definition of sensitive dependence to initial conditions is that nearby initial conditions separate; the technical definition is that there are sets of trajectories with positive Lyapunov exponents [15] that measure the exponential rate of separation of nearby trajectories. What is not often specified in the definition is ***how many*** trajectories have positive Lyapunov exponents. For example, if a dynamical system has a saddle point, this point has a positive Lyapunov exponent, but the presence of a single saddle point (or even more complicated normally hyperbolic sets) does not make the system chaotic. There appears to be little consensus on the minimal requirements for sets of trajectories with positive Lyapunov exponents that make a system chaotic, but there is a sufficient criterion formulated by Smale [48] that is often used as a practical test: namely, that the system possesses a "transversal intersection of stable and unstable

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manifolds of a periodic orbit." This concept is explained below. Such ***homoclinic*** orbits were first discovered by Poincaré in 1890 in a prize winning essay [43] motivated by the question, is the solar system stable? The intriguing history of Poincaré's discovery has been studied and recounted by Barrow-Green [3]. The work of Poincaré and later Birkhoff was directed at ***conservative*** dynamical systems arising in celestial mechanics. Within the setting of systems that preserve a symplectic structure, they investigated the presence of transversal intersections of the stable and unstable manifolds of periodic orbits. The first mathematical analysis of transversal homoclinic orbits in the context of dissipative systems that are not conservative was carried out by Cartwright and Littlewood, beginning during World War II [9–12] and culminating in Littlewood's long two part paper of 1957 [32–34]. The personal aspects of the Cartwright–Littlewood collaboration are also fascinating and have been described by McMurran and Tattersall [37, 38] as well as by Cartwright herself [8, 50].

The initial presentations of significant mathematical discoveries seldom appear in full clarity. The path to a new discovery is often tortuous, so reformulation is typically needed to distill the essence of new insights. This has been true in dynamical systems theory: the papers of Poincaré and Littlewood cited above are excellent examples. The work of Cartwright–Littlewood has a dual character, containing detailed analysis of the forced Van der Pol differential equation as well as a description of the dynamical consequences of transversal homoclinic orbits in dissipative systems. There was a long period of abstraction and simplification of the arguments of Cartwright–Littlewood that led to piecewise linear vector fields studied by Levinson [30] and later Levi [29], the geometric discrete time Smale horseshoe [47, 49] and the concept of ***hyperbolic invariant sets*** [46]. Figure 1.1 illustrates the horseshoe. These developments provided tremendous insight into chaotic dynamics, but they draw upon only a small portion of the Cartwright–Littlewood analysis of the forced Van der Pol differential equation. Thus, there is a disparity between mathematical awareness of these two aspects of the Cartwright–Littlewood discovery of chaos in dissipative systems. The horseshoe and its symbolic dynamics are a beautiful geometric example of chaotic dynamics, simple enough to be included routinely in undergraduate courses. Littlewood's analysis of the forced Van der Pol equation remains obscure despite its central role in the book of Grasman [18]. This paper visualizes horseshoes in the forced Van der Pol equation from the perspective of ***geometric singular perturbation theory*** and describes recent extensions of the work of Cartwright–Littlewood by myself and collaborators [6, 19, 20] that culminated in the thesis of Radu Haiduc [22, 23]. Haiduc proved that there are parameter values for which the forced Van der Pol equation is structurally stable and possesses a chaotic invariant set. This paper gives an extended outline of this work, presenting the key geometric constructions used in the analysis of the forced Van der Pol equation.

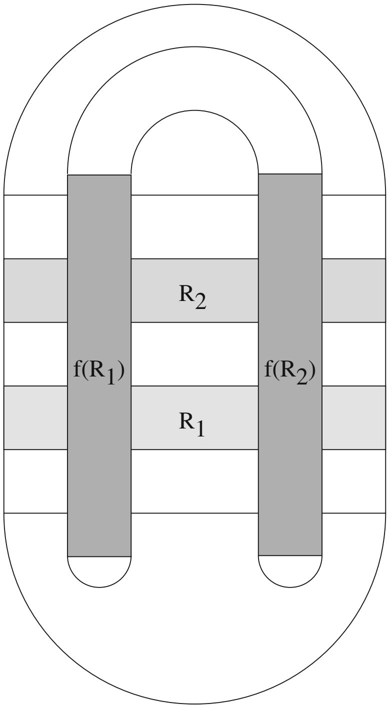

Fig. 1.1 The horseshoe is an invariant set Λ of the discrete map f depicted in this figure. The map f stretches the background oval vertically, compresses it horizontally, and maps it back into itself. The rectangles R_1 and R_2 shaded in *light gray* are mapped rectilinearly into their images shaded in *dark gray*. Inside the intersection $(R_1 \cup R_2) \cap (f(R_1) \cup f(R_2))$, there is an invariant Cantor set Λ consisting of points whose f-trajectories (both forward and backward) remain inside the intersection. The vertical distance between points that lie on different horizontal lines increases until one of the points lands in R_1 at the same time that the other lands in R_2. This expresses the ***sensitivity to initial conditions*** of this map. There is a one-to-one correspondence between points of Λ and bi-infinite sequences of 1 and 2 that encode which rectangle R_j each iterate lies in

1.2 The Forced Van der Pol Equation

The main object of this paper is analysis of the system of differential equations

$$\begin{aligned} \varepsilon\dot{x} &= y + x - \frac{x^3}{3} \\ \dot{y} &= -x + a\sin(2\pi\theta) \\ \dot{\theta} &= \omega \end{aligned} \tag{1.1}$$

where the variable $\theta \in S^1 = \mathbb{R}/\mathbb{Z}$, so we identify θ and $\theta + 1$. We are interested in the parameter regime where $\varepsilon > 0$ is small. The limit $\varepsilon = 0$ produces a system