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*Dedicated to our wives Azar, Xinming, and  
Mojgan*



*Trying is not doing; doing is trying.*





# Preface

The main scope of this book is to present the established analytical and experimental techniques to address the dynamic responses of elastic as well as porous half-space media when they are subjected to dynamic loads and the related topics in a concise and suitable manner. The book introduces the reader to the dynamic response of the surface of an elastic half-space excited by concentrated vertical or tangential force. Based on the presented analyses, it also addresses the dynamic response of a rigid massless footing of arbitrary shape resting on the surface of an elastic half-space medium for three modes of vertical, horizontal, and rocking vibrations. The book also presents solutions to the three pure modes of vibration for massive rectangular foundations by employing the impedance matching technique and provides design charts for these modes of vibrations. The solution for these modes is extended to develop a solution to the dynamics of simultaneous horizontal and rocking motions of a rectangular foundations resting on the surface of elastic half-space medium. Moreover, the book presents the required theoretical background needed for analysis of interaction of two rectangular foundations founded on the surface of an elastic half-space. In addition to the theoretical topics, the book describes a finite model to simulate an elastic half-space and introduces experimental techniques to verify the presented solution. Furthermore, experimental methods are presented to determine the two important elastic properties of shear modulus and Poisson's ratio for the medium. In order to verify the present theoretical results some experiments, procedures, and results are also provided.

This book presents the required theoretical background needed to develop mathematical models and their solutions for the above topics. Furthermore, it offers the engineering information and quantitative data needed for design analysis and applications of the presented analytical procedures for different disciplines such as: mechanical, civil, and bioengineering. The book in its entirety constitutes as an extensive guidance for its reader. It also provides a systematic solution for the dynamic analysis of elastic, porous, and layered half-space media. It also extends the provided analytical solution to address a variety of practical problems in engineering and to determine the essential elastic properties of the medium. The book is intended to lay the foundation for understanding mathematical modeling, vibration analysis,

and the design of engineering systems which can be modeled by a half-space medium in a complete and succinct manner. Throughout the book, an attempt has been made to provide a conceptual framework that includes exposure to the required background in mathematics and the fundamentals of the theory of elasticity. The knowledge of the presented topics will enable the reader to pursue further advances in the field.

## **Level of the Book**

The primary audience of this book is the graduate students in mechanical engineering, engineering mechanics, civil engineering, bioengineering, ocean engineering, mathematics, and science disciplines. In particular, it is geared toward the students interested in enhancing their knowledge by taking the second graduate course in the areas of vibration of continuous systems, application of wave propagation, and soil dynamics. The presented topics have been prepared to serve as an aid to engineering designers. It can also be utilized as a guide for professional engineers in research and industry who are seeking to expand their expertise and are expected to extend their knowledge for setting design specifications and ensuring their fulfillment.

## **Organization of the Book**

The book is presented in ten chapters: introduction, fundamentals of elasticity, vibration analysis for single-layer cylinders, modal analysis for single-layer cylinders, vibration of multilayer thick cylinders, constrained-layer damping for cylindrical structures, and vibration of thick cylindrical panels. Furthermore, it offers helpful and significant tabulated results, which can be used as design guidelines for these structures.

To make effective use of the presented topics, the following procedure is suggested. The realization of the topics may require a review of certain theoretical concepts and methods which can be achieved through references in Chaps. 1 through 5. To become acquainted with the state of the art in this particular field and learn about the historical background on this topic, the reader should begin with Chap. 1, which lists an extensive number of key references with brief discussions on their methodology, required assumptions, and their achievements. Chapter 2 reviews the succinct fundamental theoretical background and concepts needed from the theory of elasto-dynamics, which will enable the reader to follow the derivation of the required governing equations and their solutions in Chaps. 3 and 4. Chapter 5 is intended to present numerical results for the non-dimensional frequency responses of rigid rectangular foundations resting on an elastic half-space for three modes of vertical, horizontal, and rocking vibrations, as well as coupled horizontal and rocking vibrations. Chapter 6 presents a finite size experimental model for a

semi-infinite elastic half-space model and the experimental procedures for verifying the theoretical results. It also presents available techniques for determining the dynamic properties of the medium needed for analytical analysis. Chapters 7 and 8 provide analytical method to determine dynamic response of a rigid foundation subjected to a distance blast and to identify position of a vertical exciting force on the surface of an elastic half-space medium using sensor fusion, respectively. Chapter 9 presents an overview of techniques established for analyzing Surface Vibration of a multilayered elastic medium due to harmonic concentrated force. Chapter 10 will cover the three-dimensional wave propagation in porous media.

## **Method of Presentation**

The scope of each chapter is clearly outlined and the governing equations are derived with an adequate explanation of the procedures. The covered topics are logically and completely presented without unnecessary overemphasis. The topics are presented in a book form rather than in the style of a handbook. Tables, charts, equations, and references are used in abundance. Proofs and derivations are often emphasized and the physical model and final results are accompanied with illustrations and interpretations. Certain specific information that is required in carrying out the design analysis in detail has been stressed.

## **Prerequisites**

The book is written for graduate students, so the assumption is that the readers are familiar with the fundamentals of differential equations, as well as a basic knowledge of linear algebra, Fourier transform, and numerical methods. The presented topics are aimed to establish a conceptual framework that enables the reader to pursue further advances in the field. Although the governing equations will be derived with adequate explanations of the procedures, it is assumed that the readers have a working knowledge of theory of elasticity, fluid–structure interaction, and vibration engineering.

## **Unit System**

Through the chapters, for the sake of generality, computed results and the required parameters are provided in non-dimensional forms. Nevertheless, the system of units adopted for case studies is, unless otherwise stated, the British Gravitational system of units (BG). The units of degree (deg) or radian (rad) are utilized for variables representing angular quantities.

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# Chapter 1

## Introduction

**Abstract** This chapter will discuss some of the issues of dynamics of soils and foundations from a practical point of view. Since this topic is quite broad, a brief description of methodology will be outlined, while details will be given for a few procedures that have proven to be effective and accurate. One of the main objectives of this chapter is to survey different available techniques for solving the dynamic response of foundations when subjected to harmonic loadings. Special attention is directed to the dynamic response of the surface of the medium due to concentrated dynamic loads, response of foundations, coupled vibrations of foundations, interaction between two foundations, experimental aspects of soils and foundations, and laboratory simulations.

**Keywords** Surface forces • Concentrated forces • Contact stress distribution • Mixed boundary value problems • Elastic medium

The possible occurrence of extreme dynamic excitation, either natural or man-made, has a major influence on the design of buildings and machine foundations. A primary concern in designing foundations is the knowledge of how they are expected to respond when subjected to dynamic loadings. The validity of the mathematical analysis depends entirely on how well the mathematical model simulates the behavior of the real foundation. Over the past decades our ability to analyze mathematical models for dynamics of foundations has been improved by the use of different analytical and numerical techniques. In most of these analyses it is common to assume that the footing is rigid and the medium is a homogeneous elastic half-space. Extensive efforts have been confined in the development of procedures and computer simulations to tackle some practical problems that arise in this field, while other important problems have been neglected. It should be noted that interaction between foundations for noncircular footings was not treated in a satisfactory manner and significant deficiencies remain in most of the previous analyses.

This chapter will discuss some of the issues of dynamics of soils and foundations from a practical point of view. Since this topic is quite broad, a brief description of methodology will be outlined, while details will be given for a few procedures that have proven to be effective and accurate. One of the main objectives of this chapter is to survey different available techniques for solving the dynamic response of foundations when subjected to harmonic loadings. Special attention is directed to the dynamic response of the surface of the medium due to concentrated dynamic loads, response of foundations, coupled vibrations of foundations, interaction between two foundations, experimental aspects of soils and foundations, and laboratory simulations.

Before addressing the abovementioned problems in soil dynamics, it is essential to define the soil medium. To the authors' knowledge, diverse dynamic analyses for the soil–foundation interaction have been conducted using the following modeling approaches for the soil medium: subgrade reaction model, Winkler foundations, elastic half-space medium, which can be elastic isotropic or viscoelastic, Gibson soil model, layered medium, porous medium, and medium with particulate materials.

Among these models the subgrade reaction model can only be used for lumped systems and is developed using experimental results on different soils. Detailed information about this model are provided by Terzaghi (1955) and Barkan (1962). Winkler's foundation (1987) is a weak model which cannot account for the geometrical damping of the half-space medium. The half-space model is the most realistic model which can be extended to the viscoelastic one by considering complex elastic moduli for the medium. This model is adopted throughout Chaps. 2–6. The half-space medium can also be considered to be porous. Gibson's model (1967) allows for variation of the shear modulus within the depth of the elastic half-space. This model has rarely been used for real design analysis. In the layered soil medium model, the medium is divided into thin layers and each thin layer considered to be an elastic or viscoelastic medium with specific mechanical properties. See Kausel and Roesset (1981). The porous half-space medium represents volumetrically interacting solid–fluid aggregates, which can be modeled using continuum porous media theories by allowing for both solid-matrix deformation and fluid flow (Morand and Ohayon 1955). To study the mechanical behavior of sand and other granular soil medium, Tavaréz and Plesha (2007) used the discrete element method to account for the discontinuous characteristic of some geomaterials. It should be noted that this book also addresses the porous and the layered stratum in different chapters.

## 1.1 Surface Response Due to Concentrated Forces

In the field of propagation of disturbances on the surface of an elastic half-space, the first mathematical attempt was made by Lamb (1904). He gave integral representations for the vertical and radial displacements of the surface of an elastic half-space due to a concentrated vertical harmonic force. Evaluation of these integrals involves considerable mathematical difficulties, due to the evaluation of

a Cauchy principal integral and certain infinite integrals with oscillatory integrands. Nakano (1930) considered the same problem for a normal and tangential force distribution on the surface. Barkan (1962) presented a series solution for the evaluation of integrals for the vertical displacement caused by a vertical force on the surface, which was given by Shekhter (1948). Pekeris (1955a, b) gave a greatly improved solution to this problem when the surface motion is produced by a vertical point load varying with time, like the Heaviside function. Elorduy et al. (1967) developed a solution by applying Duhamel's integral to obtain the harmonic response of the surface of an elastic half-space due to a vertical harmonic point force. Heller and Weiss (1967) studied the far field ground motion due to an energy source on the surface of the ground.

Among the investigators who considered the three-dimensional problem for a tangential point force, Chao (1960) presented an integral solution to this problem for an applied force varying with time, like the Heaviside unit function. Papadopoulos (1963) and Aggarwal and Ablow (1967) have presented solutions, in integral expressions, to a class of three-dimensional pulse propagation in an elastic half-space. Johnson (1974) used Green's functions for solving Lamb's problem, and Apsel (1979) employed Green's functions to formulate the procedure for layered media. Kausel and Roesset (1975) reported an explicit solution for dynamic response of layered media. Davies and Banerjee (1983) used Green's functions to determine responses of the medium due to forces which were harmonic in time with a constant amplitudes. The solution was derived from the general analysis for impulsive sources. Kobayashi and Nishimura (1980) utilized the Fourier transform to develop a solution for this problem and expressed the results in terms of the full-space Green's functions, which include infinite integrals of exponential and Bessel's function products. Banerjee and Mamoon (1990) provided a solution for a periodic point force in the interior of a three-dimensional, isotropic elastic half-space by employing the methods of synthesis and superposition. The solution was obtained in the Laplace transform as well as the frequency domain.

Hamidzadeh (1978) presented mathematical procedures for determination of the dynamic response of surface of an elastic half-space subjected to harmonic loadings and provided numerical results for displacement of any point on the surface in terms of properties of the medium and of the exciting force. The solution was analytically formulated by employing double Fourier transforms and was presented by integral expressions. Hamidzadeh (1987) and Hamidzadeh and Chandler (1991) provided dimensionless response for an elastic half-space and compared their results with other available approximate results. Verruijt (2010) provided mathematical procedures for analyzing the vertical displacement of the surface of an elastic half-space due to the surface point load, and moving vertical loads. Meral and Royston (2009) studied shear and surface wave motion in and on a viscoelastic material representative of biological tissue. They considered the surface wave motion on a half-space caused by a finite rigid circular disk located on the surface and oscillating normal to it and determined the compression, shear, and surface wave motion in a half-space generated by a subsurface finite dipole. In their study, they

assumed fractional order Voigt model of viscoelasticity in their theoretical analysis. They concluded that their theoretical results had a better agreement with their measured results over a limited frequency range.

## 1.2 Dynamic Response of Foundations

Advances in the development of solutions for soil–foundation interaction problems are categorized in the following sections based on the formulation procedures.

### 1.2.1 Assumed Contact Stress Distributions

The first attempt to solve the vertical vibration of a massive circular base on the surface of an elastic medium was made by Reissner (1936). He adopted Lamb's (1904) approach and developed a solution by assuming a uniform stress distribution on the surface of the medium. He established an estimated solution for determining the vertical steady state response of circular footings. He also calculated the displacement of the center of the base and introduced the amplitude of vibration in terms of non-dimensional parameters. It has been proven that his results over-estimated the amplitude due to his consideration of the displacement at the center of the base. Reissner and Sagoci (1944) presented a static solution for the torsional oscillation of a disc on the medium. Miller and Pursey (1954, 1955) considered the vertical response of a circular base due to a force uniformly distributed on the contact surface. Quinlan (1953) and Sung (1953) independently extended Reissner's (1936, 1944) approach to solve the problem of vertical vibration of circular and infinitely long rectangular footings. In their analyses they considered three different harmonic stress distributions: uniform, parabolic, and stress produced by a rigid base under static conditions. They showed that the vibration characteristics of semi-infinite media effectively vary with the type of stress distributions and elastic properties of the medium.

Arnold et al. (1955) considered four vibrational modes (vertical, horizontal, torsional, and rocking) for a circular base on the surface of elastic media. By assuming harmonic static stress distributions for all modes, they evaluated the dynamic responses using an averaging technique. They also verified this work with experimental results using a finite model for the infinite medium. Bycroft (1956, 1959) followed the same approach for four modes of vibration by determining complex functions to represent the in-phase and out-of-phase components of displacement of a rigid massless circular plate. Bycroft (1977) later carried out some tests to verify his previous theoretical work. Thomson and Kobori (1963) and Kobori et al. (1966a, b, 1968, 1970, 1971) considered the dynamic response of a rectangular base. They provided computational results for components of the complex displacement functions by assuming a uniform stress distribution on the