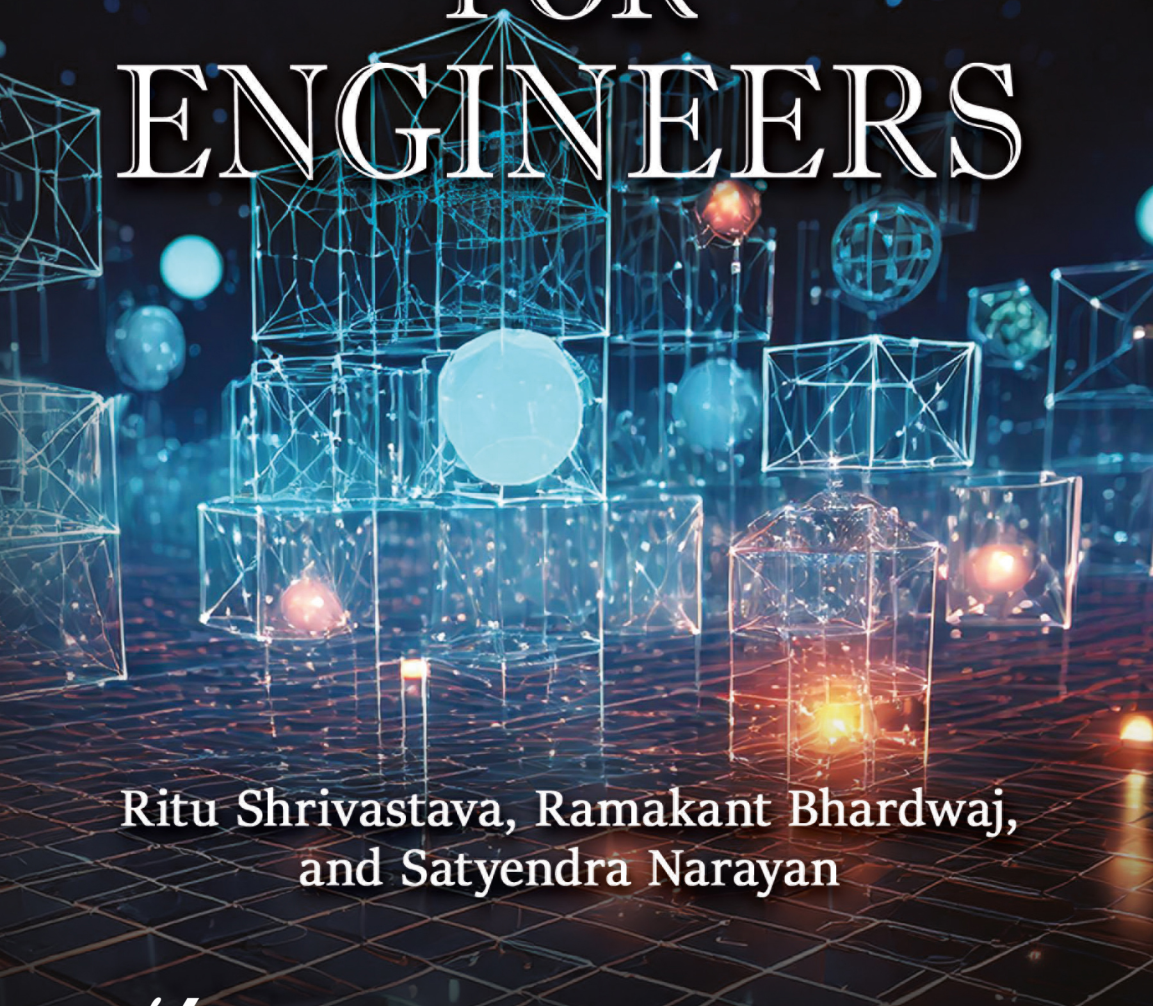
MATHEMATICS IN COMPUTATIONAL SCIENCE AND ENGINEERING

MATHEMATICS FOR ENGINEERS



Ritu Shrivastava, Ramakant Bhardwaj, and Satyendra Narayan

Scrivener Publishing

WILEY

Mathematics for Engineers

Scrivener Publishing
100 Cummings Center, Suite 541J
Beverly, MA 01915-6106

Publishers at Scrivener
Martin Scrivener (martin@scrivenerpublishing.com)
Phillip Carmical (pcarmical@scrivenerpublishing.com)

Mathematics for Engineers

Ritu Shrivastava
Ramakant Bhardwaj
and
Satyendra Narayan

Scrivener Publishing

WILEY

This edition first published 2025 by John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, USA and Scrivener Publishing LLC, 100 Cummings Center, Suite 541J, Beverly, MA 01915, USA

For more information about Scrivener publications please visit www.scrivenerpublishing.com.

Wiley Global Headquarters
111 River Street, Hoboken, NJ 07030, USA

For details of our global editorial offices, customer services, and more information about Wiley products visit us at www.wiley.com.

Library of Congress Cataloging-in-Publication Data

ISBN 9781394214587

Front cover images supplied by Adobe Firefly
Cover design by Russell Richardson

Set in size of 11pt and Minion Pro by Manila Typesetting Company, Makati, Philippines

Printed in the USA

10 9 8 7 6 5 4 3 2 1

Contents

Preface

Mathematics without practice is incomplete. Practice of mathematics without understanding theory is meaningless. Theory and practice are two sides of the same coin. Like the two sides of a coin, theory and practice have their differences, and they are inseparable. *Mathematics for Engineers* is designed to explain theory and practice together in a meaningful way, providing engineers with a comprehensive understanding of fundamental mathematical concepts crucial for their academic and professional endeavors. The depth and breadth of this textbook offers comprehensive coverage of all the basic mathematical tools needed for first-year engineering students, as well as a reference for veteran engineers and industry professionals.

This textbook is originated from a one-semester course dedicated to introductory engineering mathematics taught in many universities and engineering colleges over the past several years primarily to first-year engineering students. This covers a wide range of syllabus requirements globally. In particular, the textbook is highly suitable for the latest National Certificate and Diploma courses and Vocational Certificate of Education in science & engineering programs. First year undergraduates who need some remedial mathematics may also find it helpful in meeting their needs. Overall, the textbook provides a solid foundation in mathematical principles, which may enable students to solve mathematical, scientific, and associated engineering principles. Key Features of this book are highlighted below.

Chapter 1: Fractions, Decimals, and Percentages lays the groundwork by revisiting fundamental arithmetic concepts, such as fractions, decimals, and percentages. From understanding the basics of fractions to performing algebraic operations and mental math tricks, it equips leaners with essential skills for more complex mathematical endeavors.

Chapter 2: Ratios and Proportions exploring ratios, rates, and proportions delves into the principles governing the relative magnitudes of quantities. They play pivotal roles in engineering, influencing everything from structural design to financial analysis. Learners will learn techniques for simplifying ratios, finding equivalent ratios, and solving problems involving direct and inverse proportions.

Chapter 3 and 4: 2D & 3D Geometry are fundamental to engineering, and it covers essential concepts related to 2D and 3D shapes, perimeter, area, surface area, and volume calculations. Understanding geometric principles is important, crucial, and required for designing structures and analyzing spatial relationships in engineering applications.

Chapter 5: Algebra and Graphs provides algebraic skills that are indispensable for engineers. It covers equations, expressions, and graphical representations of mathematical relationships. From linear to quadratic equations and beyond, learners will explore various algebraic techniques required for problem-solving in engineering contexts.

Chapter 6: Exponentials & Logarithms are the powerful mathematical tools. They are used in a wide range of applications in engineering problems. It elucidates the rules governing these concepts, their graphical representations, and their role in modeling growth and decay phenomena.

Chapter 7: Trigonometry is indispensable for analyzing angles, distances, and relationships within geometric structures. It covers trigonometric ratios, Pythagoras' theorem, trigonometric equations, and applications such as finding angles of elevation and depression.

Chapter 8 and 9: Differential Calculus & Integration provide engineers with powerful tools for analyzing rates of change, optimization problems, and modeling dynamic systems. Differentiation, integration, and their applications in engineering contexts are covered completely.

Chapter 10: Probability plays an important and crucial role in engineering decision-making problems and risk assessment. This chapter introduces principles of counting, permutations, combinations, and various techniques for calculating probabilities of single and compound events.

Each chapter is dedicated to building a strong foundation, engineering applications, problem-solving skills, culturally relevant context, and practical problem-based learning (PBL) exercises and experiments to reinforce learner's understanding and application of mathematical concepts in real-world engineering scenarios.

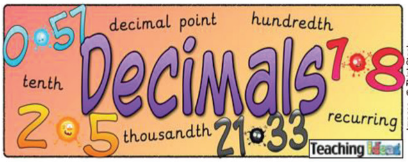
The authors hope this textbook serves as a valuable resource in the student's journey to mastering mathematics for engineering and as a reference for the veteran engineer or industry professional. It does not matter whether you are just embarking on your academic studies or a practicing engineer seeking to enhance your mathematical skills and proficiency. This book is designed to support each individual's educational and professional development.

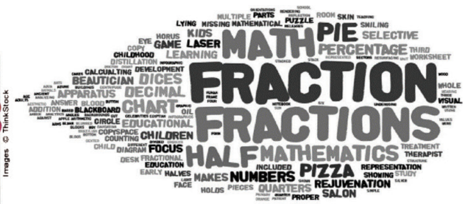
The authors of this book would like to acknowledge the use of freely available mathematical figures and images sourced from various online repositories. These resources have been invaluable in illustrating key concepts and enhancing the visual appeal of this book.

This book builds upon the foundational work of many brilliant mathematicians. Their groundbreaking discoveries have shaped our understanding of the world and continue to inspire generations of researchers.

1

Fractions, Decimals, and Percentages





Objectives: This chapter
Introduces terminology associated with Fractions, decimals, and percentages.
Explains the methods to convert Fractions, decimals, and percentages.

1.1 Introduction

You may frequently have seen discounts written as 50% off or $\frac{1}{2}$ rate but never as 0.5 times the original rate in malls and stores. Also, when we convert currency, the exchange rates never show us as a decimal or a percentage.

These presentations are mathematically equivalent and are different ways of saying the same results.

The percentage considered is out of 100. A fraction is considered a number less than a whole. A Decimal is a number where the 'whole' part of a number and the 'fractional' part is separated by a decimal point.

Ritu Shrivastava, Ramakant Bhardwaj and Satyendra Narayan. Mathematics for Engineers, (1–36)

1.2 Fractions

1.2.1 Definition of Fractions

A fraction is simply a value that represents part of a whole. The critical point that one must understand is that when we speak about a fraction, we generally refer to some whole thing. Whenever we divide a whole into two parts, both must be equal. We will always speak of some *whole* amount divided into a specified number of equally sized pieces. In short, we can think briefly about a fraction as given below.

Notation for a fraction is the slash (/) written between the two numbers. Another way of representing fractions is to keep a top number, the numerator, and a bottom number, the denominator. For example, we can write a fraction like $\frac{3}{5}$. "3," the top number, is the numerator, and "5," the bottom, is the denominator.

1.2.2 Types of Fractions

We can see many different fractions, but the most popular are three types of fractions in Mathematics.

Common Fraction or Proper Fraction

When a fraction representing a part of the whole is known as a proper fraction (For example: in the case of a proper fraction, the denominator is always greater than the fraction's numerator).

Look at the fraction $\frac{3}{8}$. Can you tell if this is a proper fraction or not?

This fraction is a proper fraction as the denominator '8' is greater than the numerator '3'.

$\frac{8}{3}$ is not a proper fraction as the numerator '8' is greater than the denominator '3'.

Improper Fraction

A fraction that is not a proper fraction is called an improper fraction (e.g., the denominator of the improper fraction is less than or equal to its numerator).

$\frac{5}{3}$ is an improper fraction as the numerator '5' is greater than the denominator '3'.

$\frac{3}{5}$ is not an improper fraction as the numerator '3' is less than the denominator '5'.

Mixed Fraction

Those fractions expressed as a sum of a whole and a proper fraction number are called mixed fractions.

$\frac{9}{4} = 2\frac{1}{4}$ is a mixed fraction, and it can also be written as $2+\frac{1}{4}$.

***How to Convert an Improper Fraction to a Mixed Fraction?**

Divide the numerator until the remainder becomes less than the denominator, then the mixed fraction will be in the format shown below.

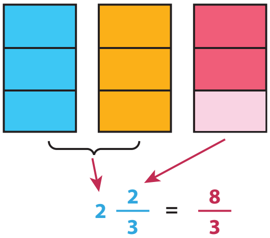
Integer part (Reminder/Denominator)

- **Step 1:** Divide the given fraction's numerator by its denominator and note its quotient and remainder.
- **Step 2:** The integer part you got by division will be the integer part for a mixed fraction.
- **Step 3:** Leave the denominator the same as the original.
- **Step 4:** Write the improper fraction into the mixed fraction.

For example: Convert $\frac{17}{8}$ into a mixed fraction.

- **Step 1:** $\frac{17}{8}$ has '2' as the quotient and '1' as the remainder.
- **Step 2:** In this case, '2' is the integer.
- **Step 3:** Here, the denominator is '8'.
- **Step 4:** Hence, $\frac{17}{8}$ is changed to a mixed fraction $2\frac{1}{8}$.

***How to Convert a Mixed Fraction to an Improper Fraction?**



- **Step 1:** Multiply the denominator of the given fraction with the integer.
- **Step 2:** Add the fraction's numerator to the result in step 1.
- **Step 3:** Leave the denominator the same as the original.
- **Step 4:** Write the mixed fraction as an improper fraction. Reduce the result to the simplest form.

For example: Convert $2\frac{1}{8}$ into Improper fractions.

- **Step 1:** Multiply 8 with 2 in, $2\frac{1}{8}$, $8 \times 2 = 16$
- **Step 2:** Result in step 1 = 16 and numerator = 1, 1+ 16 =17.
- **Step 3:** Here, the denominator is '8'.
- **Step 4:** Hence, the improper fraction obtained is $\frac{17}{8}$ which is already in its simplest form.

Complex Fractions

A fraction in which the numerator, denominator, or both are fractions is called a complex fraction. $\frac{3/5}{7}$, $\frac{5/7}{11}$ are examples of complex fractions.

Equivalent Fraction
Fractions that have the same final value are known as Equivalent Fractions. For example:

$\frac{4}{8}$	=	$\frac{2}{4}$	=	$\frac{1}{2}$
(Four-Eighths)		**(Two-Fourths)**		**(One-Half)**
	=		=	

For any problem, the final answer should be written in the simplest form.

Like Fractions
Like fractions are fractions that have the same denominator. $\frac{7}{15}, \frac{11}{15}$ etc., are like fractions.

Unlike Fractions
Unlike fractions are fractions that have different denominators. $\frac{2}{15}, \frac{5}{13}, \frac{7}{17}$ etc., are unlike fractions.

Unit Fractions
A fraction with '1' as the numerator and a "positive integer" as the denominator is a unit fraction. $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ are called unit fractions.

1.2.3 Simplest Form or Fraction in Lowest Term

A fraction in simplest forms is just a ratio, $\frac{p}{q}$, where p and q share no common prime factors.

- **Step 1:** Identify a number that will divide into both the numerator and the denominator.
- **Step 2:** Divide the numerator and denominator by this number.

For example:

$$\frac{10}{15} = \frac{10/5}{15/5} = \frac{2}{3} \text{(because 10 and 15 can both be divided by 5)}$$

Also,

$$\frac{4}{8} = \frac{4/4}{8/4} = \frac{1}{2} \text{(because 4 and 8 can both be divided by 4)}$$

If a fraction's numerator and denominator share only '1' as a common factor, it is said to be in its simplest form.

EXAMPLE 1: Convert $\frac{225}{180}$ to its lowest form.

SOLUTION: The factorization method determines the 225 and 180's highest common factor (HCF).

The factors of 225 are 1, 3, 5, 9, 15, 25, 45, 75, and 225.

The factors of 180 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60, 90 and 180.

The common factors of 225 and 180 are 1, 3, 5, 9, 15, and 45.

The highest common factor (HCF) of 225 and 180 is 45.

$$\frac{225}{180} = \frac{225/45}{180/45} = \frac{5}{4} \text{(In the simplest form)}$$

1.2.4 Comparing Fractions

Looking at two given fractions and then figuring out which one is greater is known as Comparing fractions.

Comparing Like Fractions
It is always easy to compare two or more like fractions because their denominators are the same. Compare only the numerators of the given like fractions to identify which is greater. If there are two like fractions, the fraction with a greater numerator is greater.

For example:

$$\frac{9}{13} > \frac{5}{13}$$

To arrange more than two like fractions in ascending or descending order, we must arrange their numerators in ascending or descending order, respectively.

Comparing Unlike Fractions
If denominators are not the same, follow these steps:

- **Step 1:** Find the LCM (Least Common Multiple) of their denominators for the given fractions.
- **Step 2:** Reduce each fraction to its equivalent fraction with a denominator like the LCM obtained in step 1.
- **Step 3:** Finally, arrange the fractions in ascending or descending order by arranging the numerator in ascending or descending order.

EXAMPLE 2: Sarah has three-fourths of a pizza, and Aisha has two-thirds. If both pizzas are the same size, which girl has more pizza?

SOLUTION: Fractions $\frac{3}{4}$ and $\frac{2}{3}$ have, unlike denominators, unlike numerators. We must change these fractions into equivalent fractions with a common denominator to simplify comparisons.

$$\text{Sarah: } \frac{3}{4} = \frac{3\times3}{4\times3} = \frac{9}{12}$$

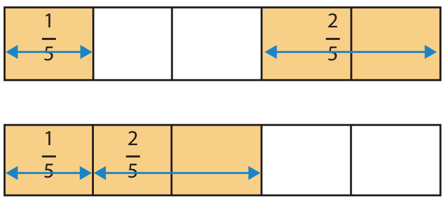
$$\text{Aisha: } \frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12}$$

$$\frac{3}{4} > \frac{2}{3} \text{ since } \frac{9}{12} > \frac{8}{12}$$

Since nine-twelfths is greater than eight-twelfths, three-fourths is greater than two-thirds. Therefore, Sarah had more pizza.

1.2.5 Algebra of Fractions

Addition & Subtraction of Fractions



$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

***How to Add and Subtract Fractions?**

- **Step 1:** Convert fractions to equivalent fractions with a common denominator.
- **Step 2:** Add or subtract the numerators only.
- **Step 3:** Leave the denominator the same.
- **Step 4:** Change the answer to the lowest terms

EXAMPLE 3: Add $\frac{1}{2}$ and $\frac{7}{8}$

SOLUTION: $\frac{1}{2} + \frac{7}{8}$ Here, the common denominator is "8" because both 2 and 8 are factors of 8

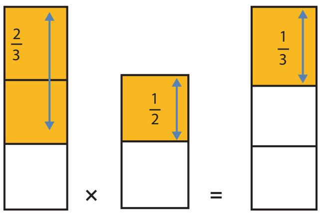
$$\frac{1}{2} = \frac{4}{8} \text{ and } \frac{7}{8} = \frac{7}{8}$$

$$\frac{4}{8} + \frac{7}{8} = \frac{11}{8}$$

Similarly,

$$\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$$

Multiplication of Fractions



$$\frac{2}{3} \times \frac{1}{2} = \frac{2 \times 1}{3 \times 2} = \frac{2}{6} = \frac{1}{3}$$

***How to Multiply Fractions?**

- **Step 1:** Multiply the numerators.
- **Step 2:** Multiply the denominators.
- **Step 3:** Reduce the answer to the lowest terms.

For example:

$$\frac{1}{7} \times \frac{4}{6} = \frac{4}{42} = \frac{2}{21}$$

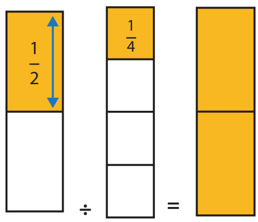
***How to Multiply Mixed Fractions?**

- **Step 1:** Convert the mixed numbers to improper fractions first.
- **Step 2:** Multiply the numerators.
- **Step 3:** Multiply the denominators.
- **Step 4:** Reduce the answer to the lowest terms.

For example:

$$2\frac{1}{3} \times 1\frac{1}{2} = \frac{7}{3} \times \frac{3}{2} = \frac{21}{6} \text{ which then reduces to } 3\frac{1}{2}$$

Division of Fractions



$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = 2$$

***How to Divide Fractions?**

- **Step 1:** Change the division sign to multiplication.
- **Step 2:** The fraction's inverse that follows the multiplication symbol (flip the fraction around so the old denominator is the new numerator, and the old numerator is the new denominator).
- **Step 3:** Multiply the numerators.
- **Step 4:** Multiply the denominators.
- **Step 5:** Change the answer to the lowest terms.

For example:

$$\frac{1}{8} \div \frac{2}{3} = \frac{1}{8} \times \frac{3}{2} = \frac{3}{16}$$

***How to Divide Mixed Fractions?**

- **Step 1:** Convert the mixed number to the improper fraction.
- **Step 2:** Reciprocate the dividend and multiply it with the divisor.
- **Step 3:** Multiply the numerators.
- **Step 4:** Multiply the denominators.
- **Step 5:** Change the answer to the lowest terms.

For example:

$$3\frac{3}{4} \div 2\frac{5}{6} = \frac{15}{4} \div \frac{17}{6} = \frac{15}{4} \times \frac{6}{17}$$

which, when solved, is $\frac{15 \times 6}{4 \times 17} = \frac{90}{68} = \frac{45}{34}$ which simplifies $1\frac{11}{34}$

***How to divide a whole number by a fraction, and a fraction by a whole number?**

- **Step 1:** Find the reciprocal of the fraction to be divided.
- **Step 2:** Multiply the number by the reciprocal of the fraction.
- **Step 3:** Simplify the resulting fraction if possible.
- **Step 4:** Check your answer: Multiply the result you got by the divisor and be sure it equals the original dividend.

*Note: *Division can be possible for only non-zero fractions.*
For example: to find a division of any whole number by a fraction.

$$3 \div \frac{5}{6} = 3 \times \frac{6}{5} = \frac{18}{5}$$

and to find a division of any fraction by a whole number

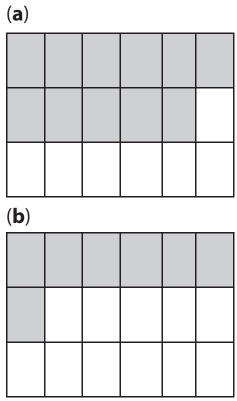
$$\frac{5}{6} \div 3 = \frac{5}{6} \times \frac{1}{3} = \frac{5}{18}$$

also, to find a division of any fraction by a fraction

$$\frac{3}{4} \div \frac{5}{6} = \frac{3}{4} \times \frac{6}{5} = \frac{18}{20} = \frac{9}{10}$$

EXAMPLE 4: (a) Shade $\frac{11}{18}$ (b) Shade $\frac{7}{18}$

SOLUTION:



EXAMPLE 5: Write down three equivalent fractions of $\frac{11}{18}$
SOLUTION:

$$\frac{11}{18} = \frac{11}{18} \times \frac{2}{2} = \frac{22}{36}$$
$$\frac{11}{18} = \frac{11}{18} \times \frac{3}{3} = \frac{33}{54}$$
$$\frac{11}{18} = \frac{11}{18} \times \frac{4}{4} = \frac{44}{72}$$

As in all the three fractions $\frac{22}{36}, \frac{33}{54}, \frac{44}{72}$, the simplest form is $\frac{11}{18}$ hence we can say that they are equal or equivalent fractions.

EXAMPLE 6: Convert mixed fraction $5\frac{6}{7}$ to an improper fraction.
SOLUTION:

- **Step 1:** $5\frac{6}{7} = 7 \times 5 = 35$
- **Step 2:** $6 + 35 = 41$
- **Step 3:** Leave the Denominator the same, i.e., 7.
- **Step 4:** The Improper fraction $= \frac{41}{7}$.

EXAMPLE 7: Convert mixed fractions $11\frac{1}{2}$ to improper fractions.
SOLUTION:

$$11\frac{1}{2} = \frac{(2\times 11)+1}{2} = \frac{(22+1)}{2} = \frac{23}{2}$$

EXAMPLE 8: Multiply the given Fractions $\frac{2}{3}$ and $\frac{5}{2}$
SOLUTION:

$$\frac{2}{3}\times\frac{5}{2} = \frac{2\times 5}{3\times 2} = \frac{10}{6} = \frac{5}{3}$$

EXAMPLE 9: Divide $3\frac{3}{4}$ by $2\frac{5}{6}$
SOLUTION:

$$3\frac{3}{4}\div 2\frac{5}{6} = \frac{15}{4}\div\frac{17}{6} = \frac{15}{4}\times\frac{6}{17}$$

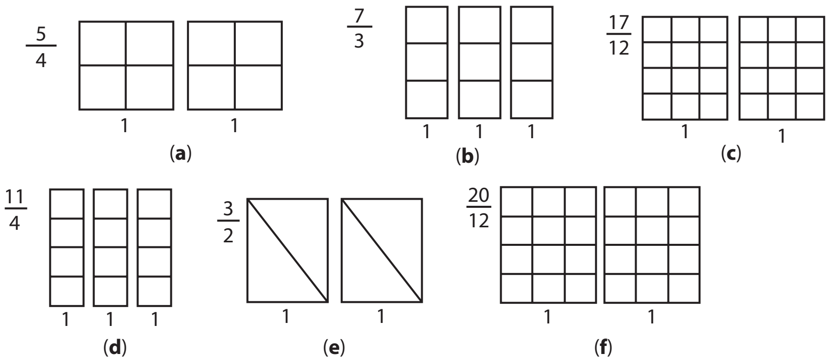
which, when solved, is $\frac{15\times 6}{4\times 17} = \frac{90}{68} = \frac{45}{34}$ which simplifies $1\frac{11}{34}$

EXAMPLE 10: Divide $\frac{1}{8}$ by $\frac{2}{3}$
SOLUTION:

$$\frac{1}{8}\div\frac{2}{3} = \frac{1}{8}\times\frac{3}{2} \text{ which, when solved, is } \frac{3}{16}$$

Level 1: Exercise 1.1 Practice Questions

1. Color the given shapes to create the following improper fractions. Remember, each shape is one whole.



2. Write down three fractions equivalent to each of the following.

   a. $\frac{2}{3}$

   b. $\frac{4}{9}$

   c. $\frac{7}{10}$

3. Write these fractions in their simplest form.

   a. $\frac{6}{10}$

   b. $\frac{10}{40}$

   c. $\frac{24}{36}$

4. Convert to mixed fractions.

   a. $\frac{12}{5}$

   b. $\frac{24}{7}$