Willful Ignorance

The Mismeasure of Uncertainty

HERBERT I. WEISBERG

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I have known that thing the Greeks knew not–uncertainty.…Mine is a dizzying country in which the Lottery is a major element of reality.

Jorge Luis Borges[1]

This fundamental requirement for the applicability to individual cases of the concept of classical probability shows clearly the role of subjective ignorance as well as that of objective knowledge in a typical probability statement.

Ronald Aylmer Fisher[2]

To a stranger, the probability that I shall send a letter to the post unstamped may be derived from the statistics of the Post Office; for me those figures would have but the slightest bearing on the question.

John Maynard Keynes[3]

NOTES

1. Jorge Luis Borges (1941). La Loteria en Babilonia (The Lottery in Babylon) in Ficciones (1944). Buenos Aires: Editorial Sur; Translated by Anthony Bonner and published as Ficciones (1962). New York: Grove Press.

2. Ronald A. Fisher (1959b). Statistical Methods and Scientific Inference, 2nd ed. Edinburgh: Oliver and Boyd, p. 33.

3. Keynes, John Maynard (1921). A Treatise on Probability. London: Macmillan, p. 71.

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PREFACE

> *The History of Science has suffered greatly from the use by teachers of second-hand material, and the consequent obliteration of the circumstances and the intellectual atmosphere in which the great discoveries of the past were made*.
>
> R. A. Fisher[1]

Sir Ronald A. Fisher, the founder of modern statistics, was certainly correct to point out how much is lost by abstracting major scientific developments from the context in which they evolved. However, it is clearly impractical for all but a few specialists to delve into original source material, especially when it is technical (or in Latin). In this book, I have attempted to convey some of the "circumstances and intellectual atmosphere" that have led to our modern idea of probability. I believe this is important for two reasons. First, to really appreciate what probability is all about, we must understand the *process* by which it has come about. Second, to transcend the limitations our current conception imposes on us, we must demystify probability by recognizing its inadequacy as the sole yardstick of uncertainty.

Willful Ignorance: The Mismeasure of Uncertainty can be regarded as two books in one. On one hand, it is a history of a big idea: how we have come to *think* about uncertainty. On the other, it is a prescription for change, especially with regard to how we perform research in the biomedical and social sciences. Modern probability and statistics are the outgrowth of a convoluted process that began over three centuries ago. This evolution has sharpened, but also narrowed, how we have come to reason about uncertainty.

Willful ignorance entails simplifying our understanding in order to quantify our uncertainty as mathematical probability. Probability theory will no doubt continue to serve us well, but only when it satisfies Einstein's famous maxim to “make everything as simple as possible but not simpler.” I believe that in many cases, we now deploy probability in a way that is simpler than it needs to be. The mesh through which probability often filters our knowledge may be too coarse. To reengineer probability for the future, we must account for at least some of the complexity that is now being ignored.

I have tried to tell the story of probability in 12 chapters. Chapter 1 presents the problem that needs to be addressed: the dilemma faced by modern research methodology. Chapter 2 is a whirlwind tour of the book's main themes. After these two introductory chapters, the next five are rich in historical detail, covering the period from 1654 to around 1800 during the time mathematical probability developed. Those readers who are more interested in current issues than history, might wish to skip ahead to read Chapter 12, in which I propose a “solution,” before circling back to the historical chapters.

Chapter 8 is a mix of history and philosophy, sketching the diversity of interpretations that have been attached to the basic concept of probability. In Chapter 9, with help primarily from Fisher, I attempt to cut through the massive confusion that still exists about probability. Chapter 10 discusses the origins of modern statistical methodology in the twentieth century, and its impact on scientific research. In Chapter 11, I explore how mathematical probability has come to dominate and in certain respects limit our thinking about uncertainty. The final chapter offers a suggestion for adapting statistical methodology to a new world of greatly expanded data and computational resources.

Previous historical writing about probability has focused almost exclusively on the mathematical development of the subject. From this point of view, the story is one of steady progress leading to a mature intellectual achievement. The basic principles of probability and statistics are well established. Remaining advances will be mainly technical, extending applications by building on solid foundations. The fundamental *creative* work is behind us; the interesting times are over.

There is, however, an all but forgotten flip side of the story. This *non*-mathematical aspect pertains to a fundamental question: what is probability? If we interpret probability as a measure of uncertainty in its broadest sense, what do we really *mean* by probability? This conceptual, or philosophical, conundrum was effectively put aside many decades ago as an unnecessary distraction, or even impediment, to scientific progress. It was never resolved, leaving the future (us) with an intellectual debt that would eventually come due.

As a result, we have inherited a serious problem. The main symptoms of this problem are confusion and stagnation in the biomedical and social sciences. There is enormously more research than ever before, but precious little useful insight being generated. Most important, there is a serious disconnect between quantitative research methodology and clinical practice. I believe that our stunted understanding of uncertainty is in many ways responsible for this gap.

I have proposed that willful ignorance is the central concept that underlies mathematical probability. In a nutshell, the idea is to deal effectively with an uncertain situation, we must filter out, or ignore, much of what we know about it. In short, we must simplify our conceptions by reducing ambiguity. In fact, being able to frame a mathematical probability implies that we have found some way to resolve the ambiguity to our satisfaction. Attempting to resolve ambiguity fruitfully is an essential aspect of scientific research. However, it always comes at a cost: we purchase clarity and precision at the expense of creativity and possibility.

For most scientists today, ambiguity is regarded as the enemy, to be overcome at all cost. But remaining open-minded in the face of ambiguity can ultimately generate deeper insights, while *prematurely* eliminating ambiguity can lead to intellectual sterility. Mathematical probability as we know it is an *invention*, a device to aid our thinking. It is powerful, but not natural or inevitable, and did not even exist in finished form until the eighteenth century.

The evolution of probability involved contributions by many brilliant individuals. To aid in keeping track of the important historical figures and key events, I have provided a *timeline*. This timeline is introduced at the beginning, and lists all of the major "landmarks" (mostly important publications) in the development of the concept of probability. At several points in later chapters, a streamlined version of the timeline has been inserted to indicate exactly when the events described in the text occurred. Of course, the question of which landmarks to include in the timeline can be debated. Scholars who are knowledgeable about the history of probability may disagree with my selections, and indeed I have second-guessed myself.

Please keep in mind that my focus is not on the mathematics of probability theory, but on the *conception* of probability as the measure of our uncertainty. In this light, it is noteworthy that my timeline ends in 1959. Perhaps that is because I lack perspective on the more recent contributions of my contemporaries. However, I believe that it underscores the lack of any profound advances in thinking about the quantification of uncertainty. There have certainly been some impressive attempts to broaden the *mathematics* of probability (e.g., belief functions, fuzzy logic), but none of these has (yet) entered the mainstream of scientific thought.

Some readers may feel that I have given short shrift to certain important topics that would seem to be relevant. For example, I had originally intended to say much more about the early history of statistics. However, I found that broadening the scope to deal extensively with statistical developments was too daunting and not directly germane to my task. I have similarly chosen not to deal with the theory of risk and decision-making directly. Mathematical probability is central to these disciplines, but entails aspects of economics, finance, and psychology that lie outside my main concerns. I would be delighted if someone else deems it worthwhile to explore the implications of willful ignorance for decision-making.

The time is ripe for a renewal of interest in the philosophical and psychological aspects of uncertainty quantification. These have been virtually ignored for half a century in the mistaken belief that our current version of probability is a finished product that is fully adequate for every purpose. This may have been true in the relatively data-poor twentieth century, but no longer. We need to learn how willful ignorance can be better applied for a more data-rich world. If this book can help stimulate a much-needed conversation on this issue, I will consider the effort in writing it to have been very worthwhile.

NOTE

1. R. A. Fisher's introduction, written in 1955, to Gregor Mendel's papers in Bennett, J. H. (ed.) (1965).

Experiments in Plant Hybridisation: Gregor Mendel.
Edinburgh: Oliver & Boyd, p. 6.

ACKNOWLEDGMENTS

Researching and writing this book has been a long journey of exploration and self-discovery. It was on one hand a foray into the realm of historical research, which was new to me. On the other hand, it was a lengthy meditation on questions that had puzzled me throughout my career. I feel extremely fortunate to have had this opportunity. A major part of this good fortune is owed to many family members, friends, and colleagues who have made it possible by supporting me in various ways.

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Ten years ago, this book could not have been written (at least by me). The cost and effort involved in obtaining access to original source material would have been prohibitive. Today, thanks to the internet, these limitations have been largely alleviated. Two resources have been especially valuable to me in my research. One is Google Books, which allowed free access to photocopies of many original publications. The second is the incredibly useful and comprehensive website: *Sources on the History of Probability and Statistics*, maintained by Prof. Richard J. Pulskamp of Xavier University at http://www.cs.xu.edu/math/Sources/. He has assembled numerous original documents pertaining to the history of probability and statistics, including many of his own translations into English of documents originally in other languages.

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Franklin, James (2001). *The Science of Conjecture: Evidence and Probability before Pascal*. Baltimore, MD: Johns Hopkins University Press. pp. 74–76, 234–235, 275, 366. Reprinted with permission from Johns Hopkins University Press.

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HERBERT I. WEISBERG

CHAPTER 1
THE OPPOSITE OF CERTAINTY

During the past century, research in the medical, social, and economic sciences has led to major improvements in longevity and living conditions. Statistical methods grounded in the mathematics of probability have played a major role in much of this progress. Our confidence in these quantitative tools has grown, along with our ability to wield them with great proficiency. We have an enormous investment of tangible and intellectual capital in scientific research that is predicated on this framework. We assume that the statistical methods as applied in the past so successfully will continue to be productive. Yet, something is amiss.

New findings often contradict previously accepted theories. Faith in the ability of science to provide reliable answers is being steadily eroded, as expert opinion on many critical issues flip-flops. Scientists in some fields seriously debate whether a majority of their published research findings are ultimately overturned[1]; the *decline effect* has been coined to describe how even strongly positive results often fade over time in the light of subsequent study[2]; revelations of errors in the findings published in prestigious scientific journals, and even fraud, are becoming more common.[3] Instead of achieving greater certainty, we seem to be moving backwards. What is going on?

Consider efforts to help disadvantaged children through early childhood educational intervention. Beginning around 1970, the U.S. government sponsored several major programs to help overcome social and economic disadvantage. The most famous of these, Project Head

Start, aimed to close the perceived gap in cognitive development between richer and poorer children that was already evident in kindergarten. The aims of this program were admirable and the rationale compelling. However, policy debates about the efficacy and cost of this initiative have gone on for four decades, with no resolution in sight. Research on the impact of Head Start has been extensive and costly, but answers are few and equivocal.

Medical research is often held up as the paragon of statistical research methodology. Evidence-based medicine, based on randomized clinical trials, can provide proof of the effectiveness and safety of various drugs and other therapies. But cracks are appearing even in this apparently solid foundation. Low dose aspirin for prevention of heart attacks was gospel for years but is now being questioned. Perhaps the benefits are less and the risks, more than we previously believed. Hormone replacement therapy for postmenopausal women was considered almost miraculous until a decade ago when a landmark study overturned previous findings. Not a year goes by without some new recommendation regarding whether, how, and by whom, hormone replacement should be used.

These are not isolated instances. The ideal of science is an evolution of useful theory coupled with improved practice, as new research builds upon and refines previous findings. Each individual study should be a piece of a larger puzzle to which it contributes. Instead, research in the biomedical and social sciences is rarely cumulative, and each research paper tends to stand alone. We fill millions of pages in scientific journals with “statistically significant” results that add little to our store of practical knowledge and often cannot be replicated. Practitioners, whose clinical judgment should be informed by hard data, gain little that is truly useful to them.

TWO DEAD ENDS

If I am correct in observing that scientific research has contributed so little to our understanding of “what works” in areas like education, health care, and economic development, it is important to ask why this is the case. I believe that much of the problem lies with our research methodology. At one end of the spectrum, we have what can be called the quantitative approach, grounded in modern probability-based statistical methods. At the other extreme are researchers who support a radically different paradigm, one that is primarily qualitative and more subjective. This school of thought emphasizes the use of case studies and in-depth participatory observation to understand the dynamics of complex causal processes.

Both statistical and qualitative approaches have important contributions to make. However, researchers in either of these traditions tend to view those in the other with suspicion, like warriors in two opposing camps peering across a great divide. Nowadays, the statistical types dominate, because methods based on probability and statistics virtually define our standard of what is deemed “scientific.” The perspective of qualitative researchers is much closer to that of clinicians but lacks the authority that the objectivity of statistics seems to provide.

Sadly, each side in this fruitless debate is stuck in a mindset that is too restricted to address the kinds of problems we face. Conventional statistical methods make it difficult to think seriously about causal processes underlying observable data. Qualitative researchers, on the other hand, tend to underestimate the value of statistical generalizations based on patterns of data. One approach willfully ignores all salient distinctions among individuals, while the other drowns in infinite complexity.

The resulting intellectual gridlock is especially unfortunate as we enter an era in which the potential to organize and analyze data is expanding exponentially. We already have the ability to assemble databases in ways that could not even be imagined when the modern statistical paradigm was formulated. Innovative statistical analyses that transcend twentieth century data limitations are possible if we can summon the will and imagination to fully embrace the opportunities presented by new technology.

Unfortunately, as statistical methodology has matured, it has grown more timid. For many, the concept of scientific method has been restricted to a narrow range of approved techniques, often applied mechanically. The result is to limit the scope of individual creativity and inspiration in a futile attempt to attain virtual certainty. Already in 1962, the iconoclastic statistical genius John Tukey counseled that data analysts "must be willing to err moderately often in order that inadequate evidence shall more often *suggest* the right answer."[4]

Instead, to achieve an illusory pseudo-certainty, we dutifully perform the ritual of computing a significance level or confidence interval, having forgotten the original purposes and assumptions underlying such techniques. This "technology" for interpreting evidence and generating conclusions has come to replace expert judgment to a large extent. Scientists no longer trust their own intuition and judgment enough to risk modest failure in the quest for great success. As a result, we are raising a generation of young researchers who are highly adept technically but have, in many cases, forgotten how to think for themselves.

ANALYTICAL ENGINES

The dream of “automating” the human sciences by substituting calculation for intuition arose about two centuries ago. Adolphe Quetelet's famous treatise on his statistically based “social physics” was published in 1835, and Siméon Poisson's masterwork on probability theory and judgments in civil and criminal matters appeared in 1837.[5, 6] It is perhaps not coincidental that in 1834 Charles Babbage first began to design a mechanical computer, which he called an *analytical engine*.[7] Optimism about the potential ability of mathematical analysis, and especially the theory of probability, to resolve various medical, social, and economic problems was at its zenith.

Shortly after this historical moment, the tide turned. The attempt to supplant human judgment by automated procedures was criticized as hopelessly naïve. Reliance on mathematical probability and statistical methods to deal with such subtle issues went out of favor. The philosopher John Stuart Mill termed such uses of mathematical probability “the real opprobrium of mathematics.”[8] The famous physiologist Claude Bernard objected that “statistics teach absolutely nothing about the mode of action of medicine nor the mechanics of cure” in any particular patient.[9] Probability was again relegated to a modest supporting role, suitable for augmenting our reasoning. Acquiring and evaluating relevant information, and reaching final conclusions and decisions remained human prerogatives.

Early in the twentieth century, the balance between judgment and calculation began to shift once again. Gradually, mathematical probability and statistical methods based on it came to be regarded as more objective, reliable, and generally “scientific” than human theorizing and subjective weighing of evidence. Supported by rapidly developing computational capabilities, probability and statistics were increasingly viewed as methods to generate definitive solutions and decisions. Conversely, human intuition became seen as an outmoded and flawed aspect of scientific investigation.

Instead of serving as an adjunct to scientific reasoning, statistical methods today are widely perceived as a corrective to the many cognitive *biases* that often lead us astray. In particular, our naïve tendencies to misinterpret and overreact to limited data must be countered by a better understanding of probability and statistics. Thus, the genie that was put back in the bottle after 1837 has emerged in a new and more sophisticated guise. Poisson's ambition of rationalizing such activities as medical research and social policy development is alive and well. Mathematical probability, implemented by modern analytical engines, is widely perceived to be capable of providing scientific evidence-based answers to guide us in such matters.

Regrettably, modern science has bought into the misconception that probability and statistics can arbitrate truth. Evidence that is “tainted” by personal intuition and judgment is often denigrated as merely descriptive or “anecdotal.” This radical change in perspective has come about because probability appears capable of objectively *quantifying* our uncertainty in the same unambiguous way as measurement techniques in the physical sciences. But this is illusory:

> Uncertain situations call for probability theory and statistics, the mathematics of uncertainty. Since it was precisely in those areas where uncertainty was greatest that the burden of judgment was heaviest, statistical tools seemed ideally suited to the task of ridding first the sciences and then daily life of personal discretion, with its pejorative associations of the arbitrary, the idiosyncratic, and the subjective. Our contemporary notion of objectivity, defined largely by the absence of these elements, owes a great deal to the dream of mechanized inference. It is therefore not surprising that the statistical techniques that aspire to mechanize inference should have taken on a normative character. Whereas probability theory once aimed to describe judgment, statistical inference now aims to replace it in the name of objectivity. … Of course, this escape from judgment is an illusion. … No amount of mathematical legerdemain can transform uncertainty into certainty, although much of the appeal of statistical inference techniques stems from just such great expectations. These expectations are fed … above all by the hope of avoiding the oppressive responsibilities that every exercise of personal judgment entails.[10]

Probability by its very nature entails ambiguity and subjectivity. Embedded within every probability statement are unexamined simplifications and assumptions. We can think of probability as a kind of devil's bargain. We gain practical advantages by accepting its terms but unwittingly cede control over something fundamental. What we obtain is a special kind of knowledge; what we give up is conceptual understanding. In short, by willingly remaining *ignorant*, in a particular sense, we may acquire a form of useful knowledge. This is the essential paradox of probability.

WHAT IS PROBABILITY?

Among practical scientists nowadays, the true *meaning* of probability is almost never discussed. This is really quite remarkable! The proper interpretation of mathematical probability within scientific discourse was a hotly debated topic for over two centuries. In particular, questions about the adequacy of mathematical probability to represent fully our uncertainty were deemed important. Recently, however, there has been virtually no serious consideration of this critical issue.

As late as the 1920s, a variety of philosophical ideas about probability and uncertainty were still in the air. The central importance of probability theory in a general sense was recognized by all. However, there was wide disagreement over how the basic concept of probability should be defined, interpreted, and applied. Most notably, in 1921 two famous economists independently published influential treatises that drew attention to an important theoretical distinction. Both suggested that the conventional concept of mathematical probability is incomplete.

In his classic, *Risk, Uncertainty and Profit*, economist Frank Knight described the kind of uncertainty associated with ordinary probability by the term *risk*.[11] The amount of risk can be deduced from mathematical theory (as in a game of chance) or calculated by observing many outcomes of similar events, as done, for example, by an insurance company. However, Knight was principally concerned with probabilities that pertain to another level of uncertainty. He had particularly in mind a typical business decision faced by an entrepreneur. The probability that a specified outcome will result from a certain action is ordinarily based on subjective judgment, taking into account all available evidence.

According to Knight, such a probability may be entirely intuitive. There may be no way, even in principle, to verify this probability by reference to a hypothetical reference class of similar situations. In this sense, the probability is completely subjective, an idea that was shared by some of his contemporaries. However, Knight went further by suggesting that this subjective probability also carries with it some sense of how much *confidence* in this estimate is actually entertained. So, in an imprecise but very important way, the numerical measure of probability is only a *part* of the full uncertainty assessment. “The action which follows upon an opinion depends as much upon the confidence in that opinion as upon the favorableness of the opinion itself.” This broader but vaguer conception has come to be called Knightian uncertainty.

Knightian uncertainty was greeted by economists as a new and radical concept, but was in fact some very old wine being unwittingly rebottled. One of the few with even an inkling of probability's long and tortuous history was John Maynard Keynes. Long before he was a famous economist,[12] Keynes authored *A Treatise on Probability*, completed just before World War I, but not published until 1921. In this work, he probed the limits of ordinary probability theory as a vehicle for expressing our uncertainty. Like Knight, Keynes understood that some “probabilities” were of a different character from those assumed in the usual theory of probability. In fact, he conceived of probability quite generally as a measure of *rational belief* predicated on some particular *body of evidence*.

In this sense, there is no such thing as a unique probability, since the evidence available can vary over time or across individuals. Moreover, sometimes the evidence is too weak to support a firm numerical probability; our level of uncertainty may be better represented as entirely or partly *qualitative*. For example, my judgment about the outcome of the next U.S. presidential election might be that a Democrat is somewhat less likely than a Republican to win, but I cannot reduce this feeling to a single number between zero and one. Or, I may have no idea at all, so I may plead *complete ignorance*. Such notions of a non-numerical degree of belief, or even of complete ignorance (for lack of any relevant evidence), have no place in modern probability theory.

The mathematical probability of an event is often described in terms of the odds at which we should be willing to bet for or against its occurrence. For example, suppose my probability that the next president will be a Democrat is 40%, or 2/5. Then for me, the fair odds at which to bet on this outcome would be 3:2. So I will gain 3 dollars for every 2 dollars wagered if a Democrat actually wins, but lose my 2-dollar stake if a Republican wins. However, a full description of my uncertainty might also reflect how confident I would be about these odds. To force my expression of uncertainty into a precise specification of betting odds, as if I *must* lay a wager, may be artificially constraining.

Knight and Keynes were among a minority who perceived that uncertainty embodies something more than mere "risk." They understood that uncertainty is inherently *ambiguous* in ways that often preclude complete representation as a simple number between zero and one. William Byers eloquently articulates in *The Blind Spot* how such ambiguity can often prove highly generative and how attempts to resolve it completely or prematurely have costs.[13]

As a prime example, Byers discusses how the ancient proto-concept of "quantity" evolved over time into our current conception of numbers: