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Fourth Edition

Analysis of Electric Machinery and Drive Systems

Paul C. Krause, Oleg Wasynczuk,
Scott D. Sudhoff, Steven D. Pekarek

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Preface

This book is written for graduate students and engineers interested in machines and drives analysis. Chapter 1 covers some basic concepts that are common to books in this area. This fourth edition differs from previous editions in several ways. For example, the transformation for both the q and d variables is obtained from the expression of the rotating magnetomotive force or mmf. This is a very straightforward approach that provides an analytic origin of the transformation. Also, the analysis of each machine is focused on motor action to set the stage for electric drives, although generator action is considered in the case of the synchronous machine. Also, since for analysis purposes the stators of the AC machines considered in this text are the same, the stators are considered once in Chapter 2 rather than repeating the analysis for each machine. However, the rotors are different and are treated separately for each machine. This reduces the work considerably.

The induction machine is considered in Chapter 3. Most induction motors have squirrel-cage rotors. However, if the stator has sinusoidally distributed windings, the rotor may also be considered as having sinusoidally distributed windings even though the rotor may consist of solid bars. The transformation of the rotor variables to the q and d axes differs only in that the rotor windings are rotating relative to the stator. The permanent-magnet AC machine and the synchronous generator are considered in Chapters 4 and 5, respectively. In Chapter 4, we treat the brushless DC machine with $L_d = L_q$. Three different values of angle between $\tilde{V}_{as}$ and $\tilde{E}_a$, or ϕ_v, are considered. These are: $\phi_v = 0$, which is the most common operating mode, $\phi_v = \phi_{v,M\mathrm{T/V}}$ or maximum torque per volt, and $\phi_v = \phi_{v,M\mathrm{T/A}}$ or maximum torque per ampere. In this case, the permanent-magnet rotor is considered to be magnetized sinusoidally.

The first part of Chapter 5 is devoted to motor action of a synchronous machine. The second part is devoted to generator action with positive current assumed out of the machine. This latter mode of operation was treated by Park in his classic paper written in 1929. The basic analysis of AC machines covered in this text ends with Chapter 5. Power systems engineers could continue with Chapters 6, 7, and 8. The drives engineer would not cover these chapters, but would skip to Chapters 10 through 14, and would likely omit some of the material in Chapter 5.

In Chapter 6, the concept of neglecting stator transients is treated. This chapter would be of most interest to the power systems engineer since it deals with the basis of transient stability programs used in stability studies for power systems. Both power systems and drives engineers could find Chapter 9 interesting. Drives engineers would want to study Chapter 10, as it describes the most commonly used modulation strategies. Chapter 11 deals with DC drives. This chapter is brief but relevant to electric drive engineering.

In Chapter 12, the torque control of permanent-magnet AC and synchronous reluctance machines are considered. The analysis of the permanent-magnet machine is similar to the material in Chapter 4. The difference is that $L_d \neq L_q$ and a reluctance torque exists. The parameters of the machine considered are representative of electric drive motors used in hybrid and electric vehicles. The synchronous reluctance machine is considered with the permanent magnets removed, whereby only a reluctance torque exists. Synchronous reluctance machines are also considered as viable candidates as electric drive motors in hybrid and electric vehicles. It is shown that with power-electronic-based current control, the electric transients are so fast that they may be neglected when considering the mechanical dynamics.

Induction motor control is considered in Chapter 13, including the volt-per-hertz, constant-slip, and field-oriented control methods. Each is considered in substantial detail. Finally, the control of permanent-magnet AC machines is considered in Chapter 14.

Although this is a graduate text, the first six or seven chapters could be used at the senior-level with the remaining chapters used as a graduate text.

February 2025

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Acknowledgments

To Our Families

About the Companion Website

This book is accompanied by a companion website:

www.wiley.com/go/krause_aem4e

The website includes Solution Manuals.

1

Introductory Concepts

1.1 Introduction

This chapter is a review for most since the material is covered in undergraduate courses in the analysis of electromechanical devices [1]. The material is presented to start everyone with the same background. The chapter begins with coupled circuits (transformers) where the phasor equivalent circuit is established. Since phasors are not always taught the same, they are covered briefly in Appendix B to make sure everyone understands the concept of phasors as used in this text. Although we will give several approaches for the calculation of torque of electric machines; Section 1.1-3 sets forth a method of calculating force and torque that is generally taught at the undergraduate level.

Some instructors may choose to skip some material and/or select topics that were not covered in undergraduate courses at their school. As mentioned, the material will be a review for most and can be covered rather fast. On the other hand, Chapter 2 dives into machine analysis that contains new material and can be taught at a much slower pace.

1.2 Stationary Magnetically Coupled Circuits

Magnetically coupled electric circuits are central to the operation of transformers and electromechanical motion devices. In the case of transformers, stationary circuits are magnetically coupled for the purpose of changing the ac voltage and current levels. The two windings shown in Fig. 1.2-1 consist of turns N_1 and N_2, and they are wound on a common core, which is a ferromagnetic material with a permeability large relative to that of air. The magnetic core is illustrated in two dimensions.

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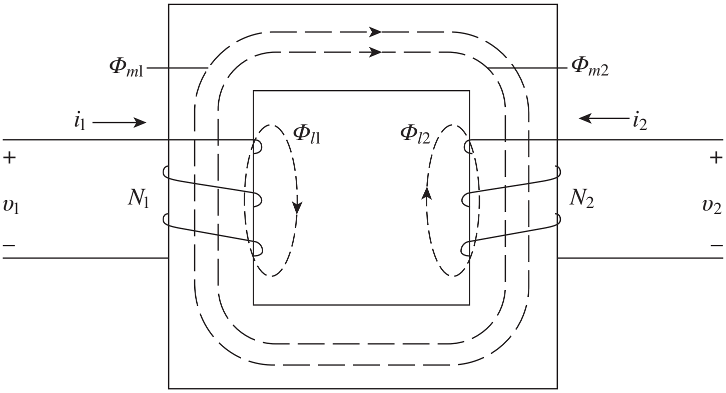

Figure 1.2-1 Magnetically coupled circuits.

The flux produced by each winding can be separated into two components: a leakage component denoted by the subscript l and a magnetizing component denoted by the subscript m. Each of these components is depicted by a single streamline with the positive direction determined by applying the right-hand rule to the directions of current flow in the winding. The leakage flux associated with a given winding links only that winding, whereas the magnetizing flux, whether it is due to current in winding 1 or winding 2, links both windings.

The flux linking of each winding may be expressed as

$$\boldsymbol{\Phi}_1 = \boldsymbol{\Phi}_{l1} + \boldsymbol{\Phi}_{m1} + \boldsymbol{\Phi}_{m2} \tag{1.2-1}$$

$$\boldsymbol{\Phi}_2 = \boldsymbol{\Phi}_{l2} + \boldsymbol{\Phi}_{m2} + \boldsymbol{\Phi}_{m1} \tag{1.2-2}$$

The leakage flux $\boldsymbol{\Phi}_{l1}$ is produced by current flowing in winding 1, and it links only the turns of winding 1. Likewise, the leakage flux $\boldsymbol{\Phi}_{l2}$ is produced by current flowing in winding 2, and it links only the turns of winding 2. The flux $\boldsymbol{\Phi}_{m1}$ is produced by current flowing in winding 1, and it links all turns of windings 1 and 2. Similarly, the magnetizing flux $\boldsymbol{\Phi}_{m2}$ is produced by current flowing in winding 2, and it also links all turns of windings 1 and 2. Both $\boldsymbol{\Phi}_{m1}$ and $\boldsymbol{\Phi}_{m2}$ are called *magnetizing fluxes*. With the selected positive directions of current flow and the manner in which the windings are wound, the magnetizing flux produced by positive current flowing in one winding can add to or subtract from the magnetizing flux produced by positive current flowing in the other winding. Thus, the mutual inductance can be positive or negative. In Fig. 1.2-1, it is positive.

It is appropriate to point out that this is an idealization of the actual magnetic system. It seems logical that all of the leakage flux will not link all the turns of the winding producing it; hence, $\boldsymbol{\Phi}_{l1}$ and $\boldsymbol{\Phi}_{l2}$ are "equivalent" leakage fluxes.

Similarly, all of the magnetizing fluxes of one winding may not link all of the turns of the other winding.

The voltage equations may be expressed as

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt} \tag{1.2-3}$$

$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt} \tag{1.2-4}$$

In matrix form,

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \tag{1.2-5}$$

The resistances r_1 and r_2 and the flux linkages λ_1 and λ_2 are related to windings 1 and 2, respectively. Since it is assumed that Φ_1 links the equivalent turns of winding 1 (N_1) and Φ_2 links the equivalent turns of winding 2 (N_2), the flux linkages may be written as

$$\lambda_1 = N_1 \Phi_1 \tag{1.2-6}$$

$$\lambda_2 = N_2 \Phi_2 \tag{1.2-7}$$

where Φ_1 and Φ_2 are given by (1.2-1) and (1.2-2), respectively.

If we assume that the magnetic system is magnetically linear (i.e., core losses and saturation are neglected), we may apply Ohm's law for magnetic circuits to express the fluxes. Thus, the fluxes may be written as

$$\Phi_{lk} = \frac{N_k i_k}{\mathfrak{R}_{\mathrm{lk}}} \tag{1.2-8}$$

$$\Phi_{mk} = \frac{N_k i_k}{\mathfrak{R}_m} \tag{1.2-9}$$

where $k = 1$ or 2 and $\mathfrak{R}_{l1}$ and $\mathfrak{R}_{l2}$ are the reluctances of the leakage paths, and $\mathfrak{R}_m$ is the reluctance of the path of magnetizing fluxes. Typically, the reluctances associated with leakage paths are much larger than the reluctance of the magnetizing path. The reluctance associated with an individual leakage path is difficult to determine exactly, and it is usually approximated from test data or by using the computer to solve the field equations numerically. On the other hand, the reluctance of the magnetizing path of the core shown in Fig. 1.2-1 may be computed with sufficient accuracy.

For the iron

$$\mathfrak{R}_i = \frac{l_i}{\mu_r \mu_0 A_i} \tag{1.2-10}$$

where l_i is the length of the path in iron, μ_r is the relative permeability of iron, μ_0 is the permeability of free space, and A_i is the cross-sectional area of the flux in the iron. In electromechanical devices, we will find that the magnetizing flux must transverse air gaps and

$$\mathfrak{R}_m = \mathfrak{R}_i + \mathfrak{R}_g \tag{1.2-11}$$

Substituting (1.2-8) and (1.2-9) into (1.2-1) and (1.2-2) yields

$$\Phi_1 = \frac{N_1 i_1}{\mathfrak{R}_{l1}} + \frac{N_1 i_1}{\mathfrak{R}_m} + \frac{N_2 i_2}{\mathfrak{R}_m} \tag{1.2-12}$$

$$\Phi_2 = \frac{N_2 i_2}{\mathfrak{R}_{l2}} + \frac{N_2 i_2}{\mathfrak{R}_m} + \frac{N_1 i_1}{\mathfrak{R}_m} \tag{1.2-13}$$

Substituting (1.2-12) and (1.2-13) into (1.2-6) and (1.2-7) yields

$$\lambda_1 = \frac{N_1^2}{\mathfrak{R}_{l1}} i_1 + \frac{N_1^2}{\mathfrak{R}_m} i_1 + \frac{N_1 N_2}{\mathfrak{R}_m} i_2 \tag{1.2-14}$$

$$\lambda_2 = \frac{N_2^2}{\mathfrak{R}_{l2}} i_2 + \frac{N_2^2}{\mathfrak{R}_m} i_2 + \frac{N_2 N_1}{\mathfrak{R}_m} i_1 \tag{1.2-15}$$

When the magnetic system is linear, the flux linkages are generally expressed in terms of inductances and currents. We see that the coefficients of the first two terms on the right-hand side of (1.2-14) depend on N_1 and the reluctance of the magnetic system, independent of the existence of winding 2. An analogous statement may be made regarding (1.2-15) with the roles of winding 1 and winding 2 reversed. Hence, the self-inductances are defined as

$$L_{11} = \frac{N_1^2}{\mathfrak{R}_{l1}} + \frac{N_1^2}{\mathfrak{R}_m} = L_{l1} + L_{m1} \tag{1.2-16}$$

$$L_{22} = \frac{N_2^2}{\mathfrak{R}_{l2}} + \frac{N_2^2}{\mathfrak{R}_m} = L_{l2} + L_{m2} \tag{1.2-17}$$

where L_{l1} and L_{l2} are the leakage inductances and L_{m1} and L_{m2} are the magnetizing inductances of windings 1 and 2, respectively. From (1.2-16) and (1.2-17), it follows that the magnetizing inductances may be related as

$$\frac{L_{m2}}{N_2^2} = \frac{L_{m1}}{N_1^2} \tag{1.2-18}$$

which is $1/\mathfrak{R}_m$.

The mutual inductances are defined as the coefficient of the third term on the right-hand side of (1.2-14) and (1.2-15). In particular,

$$L_{12} = \frac{N_1 N_2}{\mathfrak{R}_m} \tag{1.2-19}$$

$$L_{21} = \frac{N_2 N_1}{\mathfrak{R}_m} \tag{1.2-20}$$

We see that $L_{12} = L_{21}$ and, with the assumed positive direction of current flow and the manner in which the windings are wound as shown in Fig. 1.2-1, the mutual inductances are positive. If, however, the assumed positive directions of the current or the direction of the windings were such that $\boldsymbol{\Phi}_{m1}$ opposed $\boldsymbol{\Phi}_{m2}$, then the mutual inductances would be negative.

The mutual inductances may be related to the magnetizing inductances. Comparing (1.2-16) and (1.2-17) with (1.2-19) and (1.2-20), we see that

$$L_{12} = \frac{N_2}{N_1} L_{m1} = \frac{N_1}{N_2} L_{m2} \tag{1.2-21}$$

The flux linkages may now be written as

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \tag{1.2-22}$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2 \tag{1.2-23}$$

where L_{11} and L_{22} are defined by (1.2-16) and (1.2-17), respectively, and L_{12} and L_{21} by (1.2-19) and (1.2-20), respectively. The self-inductances L_{11} and L_{22} are always positive; however, the mutual inductances $L_{12}(L_{21})$ may be positive or negative, as previously mentioned.

Although the voltage equations given by (1.2-3) and (1.2-4) may be used for purposes of analysis, it is customary to perform a change of variables that yields the well-known equivalent T circuit of two windings coupled by a linear magnetic circuit. To set the stage for this derivation, let us express the flux linkages from (1.2-22) and (1.2-23) as

$$\lambda_1 = L_{l1} i_1 + L_{m1} \left(i_1 + \frac{N_2}{N_1} i_2 \right) \tag{1.2-24}$$

$$\lambda_2 = L_{l2} i_2 + L_{m2} \left(\frac{N_1}{N_2} i_1 + i_2 \right) \tag{1.2-25}$$

With λ_1 in terms of L_{m1} and λ_2 in terms of L_{m2}, we see two logical candidates for substitute variables, in particular, $(N_2/N_1)i_2$ or $(N_1/N_2)i_1$. If we let

$$i_2' = \frac{N_2}{N_1} i_2 \tag{1.2-26}$$

then we are using the substitute variable i_2', which, when flowing through winding 1, produces the same mmf as the actual i_2 flowing through winding 2; $N_1 i_2' = N_2 i_2$. This is said to be referring the current in winding 2 to winding 1 or to a winding with N_1 turns, whereupon winding 1 becomes the reference or primary winding and winding 2 is the secondary winding and i_2' is negative. On the other hand, if we let

$$i_1' = \frac{N_1}{N_2} i_1 \tag{1.2-27}$$

then i_1' is the substitute variable that produces the same mmf when flowing through winding 2 as i_1 does when flowing in winding 1; $N_2 i_1' = N_1 i_1$. This change of variables is said to refer to the current of winding 1 to winding 2 or to a winding with N_2 turns, whereupon winding 2 becomes the reference or primary winding and winding 1 the secondary with i_1'.

We will demonstrate the derivation of the equivalent T circuit by referring the current of winding 2 to a winding with N_1 turns; thus, i_2' is expressed by (1.2-26). We want the instantaneous power to be unchanged by this substitution of variables. Therefore,

$$v_2' i_2' = v_2 i_2 \tag{1.2-28}$$

Hence,

$$v_2' = \frac{N_1}{N_2} v_2 \tag{1.2-29}$$

Flux linkages, which have the units of V · s, are related to the substitute flux linkages in the same way as voltages. In particular,

$$\lambda_2' = \frac{N_1}{N_2} \lambda_2 \tag{1.2-30}$$

Now, replace $(N_2/N_1) i_2$ with i_2' in the expression for λ_1, given by (1.2-24). Next, solve (1.2-26) for i_2 and substitute it into λ_2 given by (1.2-25). Now, multiply this result by N_1/N_2 to obtain λ_2' and then substitute $(N_2/N_1)^2 L_{m1}$ for L_{m2} in λ_2'. If we do all this, we will obtain

$$\lambda_1 = L_{l1} i_1 + L_{m1} \left(i_1 + i_2'\right) \tag{1.2-31}$$

$$\lambda_2' = L_{l2}' i_2' + L_{m1} \left(i_1 + i_2'\right) \tag{1.2-32}$$

where

$$L_{l2}' = \left(\frac{N_1}{N_2}\right)^2 L_{l2} \tag{1.2-33}$$

The flux linkage equations given by (1.2-31) and (1.2-32) may also be written as

$$\lambda_1 = L_{11} i_1 + L_{m1} i_2' \tag{1.2-34}$$

$$\lambda_2' = L_{m1} i_1 + L_{22}' i_2' \tag{1.2-35}$$

where

$$L_{22}' = \left(\frac{N_1}{N_2}\right)^2 L_{22} = L_{l2}' + L_{m1} \tag{1.2-36}$$

and L_{22} is defined by (1.2-17).

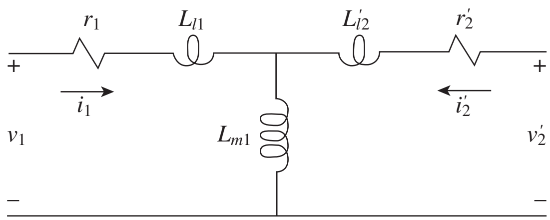

Figure 1.2-2 Equivalent *T* circuit with winding 1 selected as reference winding.

If we multiply (1.2-4) by N_1/N_2 to obtain v_2', the voltage equations become

$$\begin{bmatrix} v_1 \\ v_2' \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2' \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_1 \\ \lambda_2' \end{bmatrix} \tag{1.2-37}$$

where

$$r_2' = \left(\frac{N_1}{N_2}\right)^2 r_2 \tag{1.2-38}$$

The previous voltage equations, (1.2-37), together with the flux linkage equations, (1.2-34) and (1.2-35), suggest the equivalent *T* circuit shown in Fig. 1.2-2. This method may be extended to include any number of windings wound on the same core.

Example 1A ***The equivalent* T *circuit.***
It is instructive to illustrate the method of deriving an equivalent *T* circuit from open- and short-circuit measurements. When winding 2 of the two-winding transformer shown in Fig. 1.2-2 is open circuited and a 60 Hz voltage of 110 V (rms) is applied to winding 1, the average power supplied to winding 1 is 6.66 W. The measured current in winding 1 is 1.05 A (rms). Next, with winding 2 short-circuited, the current flowing in winding 1 is 2 A (rms) when the applied voltage is 30 V at 60 Hz. The average input power is 44 W. If we assume $L_{l1} = L_{l2}'$, an approximate equivalent *T* circuit can be determined from these measurements with winding 1 selected as the reference winding.

With $\tilde{V}_1 = |\tilde{V}_1| \underline{/\theta_{ev}(0)}$ and $\tilde{I}_1 = |\tilde{I}_1| \underline{/\theta_{ei}(0)}$ then the average power supplied to winding 1 may be expressed as

$$P_1 = |\tilde{V}_1||\tilde{I}_1| \cos \phi_{pf} \tag{1A-1}$$

where

$$\phi_{pf} = \theta_{ev}(0) - \theta_{ei}(0) \tag{1A-2}$$

Here, $\tilde{V}_1$ and $\tilde{I}_1$ are phasors with the positive direction of $\tilde{I}_1$ taken in the direction of the voltage drop, and $\theta_{ev}(0)$ and $\theta_{ei}(0)$ are the phase angles of $\tilde{V}_1$ and $\tilde{I}_1$, respectively. Phasors are covered in Appendix B. Solving for ϕ_{pf} during the open-circuit test, we have

$$\phi_{pf} = \cos^{-1}\frac{P_1}{|\tilde{V}_1||\tilde{I}_1|} = \cos^{-1}\frac{6.66}{(110)(1.05)} = 86.7^\circ \tag{1A-3}$$

Although $\phi_{pf} = -86.7^\circ$ is also a legitimate solution of (1A-3), the positive value is taken since $\tilde{V}_1$ leads $\tilde{I}_1$ in an inductive circuit. With winding 2 open-circuited, the input impedance of winding 1 is

$$Z = \frac{\tilde{V}_1}{\tilde{I}_1} = r_1 + j(X_{l1} + X_{m1}) \tag{1A-4}$$

With $\tilde{V}_1$ as the reference phasor, $\tilde{V}_1 = 110\underline{/0^\circ}$, $\tilde{I}_1 = 1.05\underline{/-86.7^\circ}$. Thus,

$$r_1 + j(X_{l1} + X_{m1}) = \frac{110\underline{/0^\circ}}{1.05\underline{/-86.7^\circ}} = 6 + j104.6\ \Omega \tag{1A-5}$$

If we neglect core losses, then, from (1A-5), $r_1 = 6\ \Omega$. We also see from (1A-5) that $X_{l1} + X_{m1} = 104.6\ \Omega$. For the short-circuit test, we will assume that $\tilde{I}_1 = -\tilde{I}_2'$ since transformers are designed so that at rated frequency $X_{m1} >> |r_2' + jX_{l2}'|$. Hence, using (1A-1) again,

$$\phi_{pf} = \cos^{-1}\frac{44}{(30)(2)} = 42.8^\circ \tag{1A-6}$$

In this case, the input impedance is $Z = \left(r_1 + r_2'\right) + j\left(X_{l1} + X_{l2}'\right)$. This may be determined as

$$Z = \frac{30\underline{/0^o}}{2\underline{/-42.8^o}} = 11 + j10.2\ \Omega \tag{1A-7}$$

Hence, $r_2' = 11 - r_1 = 5\,\Omega$ and, since it is assumed that $X_{l1} = X_{l2}'$, both are $10.2/2 = 5.1\ \Omega$. Therefore, $X_{m1} = 104.6 - 5.1 = 99.5\ \Omega$. In summary, $r_1 = 6\ \Omega$, $L_{l1} = 13.5$ mH, $L_{m1} = 263.9$ mH, $r_2' = 5\,\Omega$, $L_{l2}' = 13.5$ mH. Make sure we converted from X's to L's correctly.

1.2.1 Nonlinear Magnetic System

Although the analysis of transformers and electric machines is often performed assuming a magnetically linear system, economics and physics dictate that in the practical design of many of these devices, some saturation occurs and that heating of the magnetic material exists due to hysteresis loss [2]. The magnetization characteristics of transformer or machine materials are typically given in the form of

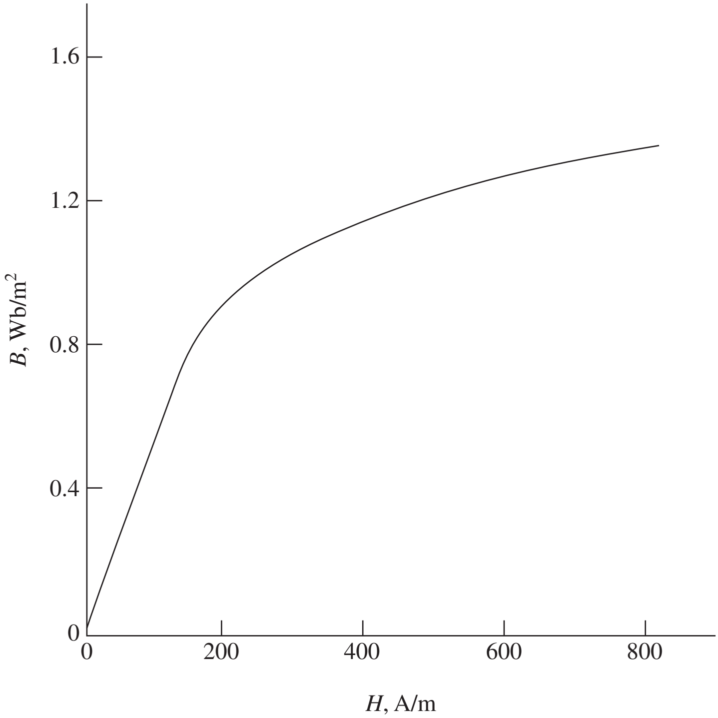

Figure 1.2-3 Typical *B–H* curve for silicon steel used in transformers.

the magnitude of flux density versus magnitude of field strength (*B–H* curve) as shown in Fig. 1.2-3.

If it is assumed that the magnetic flux is uniform through most of the core, then B is proportional to Φ and H is proportional to magnetomotive force (mmf). Hence, a plot of flux versus current is of the same shape as the *B–H* curve. A transformer is generally designed so that some saturation occurs during normal operation. During transients, saturation may occur resulting in large currents during startup transients. Electric machines are also designed similarly in that a machine generally operates slightly in the saturated region during normal, rated operating conditions. Since saturation causes coefficients of the differential equations describing the behavior of an electromagnetic device to be functions of the coil currents, transient analysis is difficult without the aid of a computer. Our purpose here is not to set forth methods of analyzing nonlinear magnetic systems. A method of incorporating the effects of saturation into a computer representation is of interest.

Formulating the voltage equations of stationary coupled windings appropriate for computer simulation is straightforward and yet this technique is fundamental to the computer simulation of ac machines. Therefore, it is to our advantage to consider this method here. For this purpose, let us first write (1.2-31) and (1.2-32) as

$$\lambda_1 = L_{l1} i_1 + \lambda_m \tag{1.2-39}$$

$$\lambda_2' = L_{l2}' i_2' + \lambda_m \tag{1.2-40}$$

where

$$\lambda_m = L_{m1} \left(i_1 + i_2' \right) \tag{1.2-41}$$

Solving (1.2-39) and (1.2-40) for the currents yields

$$i_1 = \frac{1}{L_{l1}} (\lambda_1 - \lambda_m) \tag{1.2-42}$$

$$i_2' = \frac{1}{L_{l2}'} \left(\lambda_2' - \lambda_m \right) \tag{1.2-43}$$

If (1.2-42) and (1.2-43) are substituted into (1.2-37), and if we solve the resulting equations for flux linkages, the following equations are obtained:

$$\lambda_1 = \int \left[v_1 + \frac{r_1}{L_{l1}} (\lambda_m - \lambda_1) \right] dt \tag{1.2-44}$$

$$\lambda_2' = \int \left[v_2' + \frac{r_2'}{L_{l2}'} \left(\lambda_m - \lambda_2' \right) \right] dt \tag{1.2-45}$$

Substituting (1.2-42) and (1.2-43) into (1.2-41) yields

$$\lambda_m = L_a \left(\frac{\lambda_1}{L_{l1}} + \frac{\lambda_2'}{L_{l2}'} \right) \tag{1.2-46}$$

where

$$L_a = \left(\frac{1}{L_{m1}} + \frac{1}{L_{l1}} + \frac{1}{L_{l2}'} \right)^{-1} \tag{1.2-47}$$

We now have the equations expressed with λ_1 and λ_2' as state variables. In the computer simulation, (1.2-44) and (1.2-45) are used to solve for λ_1 and λ_2' and (1.2-46) is used to solve for λ_m. The currents can then be obtained from (1.2-42) and (1.2-43).

If the magnetization characteristics (magnetization curve) of the coupled winding are known, the effects of saturation of the mutual flux path may be incorporated into the computer simulation. Generally, the magnetization curve can be adequately determined from a test wherein one of the windings is open-circuited (winding 2, for example) and the input impedance of the other winding (winding 1) is determined from measurements as the applied