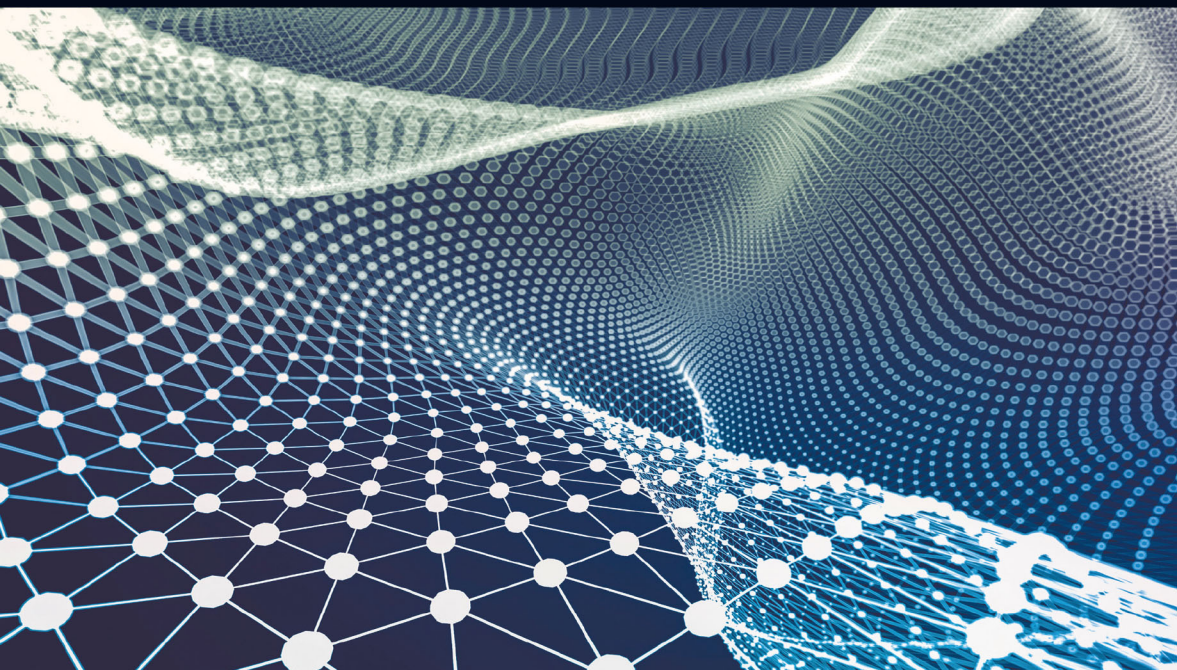


# **Rheology, Physical and Mechanical Behavior of Materials 4**

*Rigidity and Resistance of Composite  
Materials, Sizings of Laminate*

**Maurice Leroy**



**ISTE**

**WILEY**





Rheology, Physical and Mechanical  
Behavior of Materials 4



*Series Editor*  
*Noël Challamel*

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John Wiley & Sons, Inc.  
111 River Street  
Hoboken, NJ 07030  
USA

[www.wiley.com](http://www.wiley.com)

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Library of Congress Control Number: 2024951146

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British Library Cataloguing-in-Publication Data  
A CIP record for this book is available from the British Library  
ISBN 978-1-78630-973-0

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## Preface

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In the case of relatively low loads, the deformation mechanisms for materials, parts and structures are reversible, and the elastic deformations are proportional to the stresses (Hooke's law with  $E$ , Young's modulus of elasticity).

In the case of complex loads, Hooke's law is generalized into a three-dimensional relationship, and the linear nature of this law results in the following superposition principle: the stresses or deformations produced by the sum of several loading states on an elastic solid are equal to the sum of the stresses or deformations generated by each of the load states applied in isolation to the solid.

If the stress exceeds a certain value  $\sigma_e$  (or  $R_e$ ,  $\sigma_0$ ,  $Y$ ), known as the elasticity limit stress, the phenomenon ceases to be reversible and linear, and the theory of elasticity can no longer be applied.

For three-dimensional loads, different sets of criteria for yield strength will define the corresponding domain in the stress space. These include the Tresca and Von Mises criteria, while Hill's criteria are suitable for composites, and are often used in the calculations to determine the scale of parts and structures.

In many cases, it is sufficient to use the theory of elasticity, with the dimension criteria used to address safety concerns for the determination of the maximum permissible stress and/or maximum deformation.

NOTE.— The Tresca, Von Mises and Hill criteria are described in Leroy (2024), with special attention paid to the Hill criterion (Chapter 2, section 2.2) and its applications to composites.

Maurice LEROY  
November 2024

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## List of Symbols

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$[A]$	Rigidity matrix of a membrane for a symmetrical laminate (Pa)
$\mathcal{A}$	Bending moment diagram area $\mathcal{A} = MI/EI$ (dimensionless) of a symmetrical laminate ( $\text{Pa}^{-1}$ )
$[a]$	Flexibility matrix of a membrane for a symmetrical laminate ( $\text{Pa}^{-1}$ )
$[A^*], [a^*]$	Matrices $[A]$ , $[a]$ normalized, $[A^*] = [A]/h$ , $[a^*] = h[a]$
$b$	Width of a beam and of a plate (m)
$C$	Carbon
$\tilde{c}(\omega)$	Complex module
$[D]$	Rigidity bending matrix for a symmetrical laminate (Nm)
$[D^*]$	Normalized matrix $[D]$ , $[D^*] = 12[D]/h^3$ ( $\text{N/m}^2$ , Pa)
$[d]$	Flexibility bending matrix for a symmetrical laminate (Nm)
$[d^*]$	Normalized matrix $[d]$ , $[d^*] = h^3[d]/12$
dB	Decibel, one-tenth of a bel, which is commonly used to express the level of sound intensity

$d^*$	Normalized distance $d$ , $d^* = d/h$
$e$	Deformation value measured by extensometry gauges (in microdeformations $\mu D$ or $1\mu/m$ )
$E$	Young's modulus (GPa)
$E_i$	Young's modulus in the direction $i$
$E_s$	Shear modulus (GPa)
$\bar{E}$	$E/(1 - \nu \ell t \nu t \ell)$
$\tilde{E}$	Complex Young's modulus
$f$	Bend, or fiber (this symbol is generally used in subscript)
$f$	Frequency (hertz, Hz), $f = \frac{1}{T}$ , $1 \text{ Hz} = 1 \text{ s}^{-1}$
$f_n$	Frequency of order $n$
$E_s, G$	Shear modulus, or prefix for giga ( $10^9$ )
Hz	Hertz, for frequency ( $s^{-1}$ )
$h$	Total thickness of a laminate, in a sandwich or a construction with thin walls (m)
$h_o$	Thickness of the unit layer (m)
$I$	Quadratic moment ( $m^4$ )
$I^*$	Normalized quadratic moment per unit width ( $m^3$ ), $I^*$ : $I/b$ , where $b$ = width
$i$	Index, imaginary number, layer index
$\{K\}$	Curvature ( $m^{-1}$ )
$\{K^*\}$	Normalized curvature, $\{K^*\} = h \{K\}/2$



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$l$	Length (m)
$L$	Longitudinal (lengthwise) index
$L_p$	Sound pressure level (dimensionless), $L_p = 20 \lg (W/I_n)$ with $P_o = 2 \times 10^{-5} \text{ Pa}$
$L_w$	Acoustic power level (dimensionless), $L_w = 10 \lg (P/P_o)$ with $P_o = 10^{-12} \text{ W}$
$\{M\}$	Bending moment or load (Nm), and per unit of plate width (N)
$\{M^*\}$	Normalized moments, $\{M^*\} = 6 \{M\}/h^2$ for $b = 1 \text{ m}$
$m$	Mechanical effects, matrix (this symbol is generally used as an index), number of groups of layers; with $m = \cos \theta$
$\{N\}$	Membrane loading
$\{N^*\}$	Normalized membrane loading, $\{N^*\} = \{N\}/h$
$n$	Number of layers in a laminate; with $n = \sin \theta$
$P$	Slope
$\text{Pa}$	Pascal ( $\text{N/m}^2$ )
$[Q]$	Rigidity matrix of plane stresses (Pa)
$Q_{ij}$	Rigidity
$R$	Stiffness $EI$ ( $\text{Jm}$ )
$[S]$	Flexibility matrix ( $\text{Pa}^{-1}$ )
$s$	Shear component in the $xy^-$ or $l2$ plane, used in general as an index
$T$	Temperature, also indicates the crosswise (transverse) direction

UD	Unidirectional composite layer
U <sub>i</sub>	Linear combinations of the [Q] values, $i = 1, 2, 3, 4, 5$
V	Volume ( $\text{m}^3$ )
V <sub>f</sub>	Volume fraction of fibers (dimensionless)
V <sub>m</sub>	Volume fraction of matrix
x	Lengthwise axis of an orthotropic layer, usually the direction of the fibers in a unidirectional layer
y	Crosswise axis of an orthotropic layer, usually the crosswise direction to the fibers in a unidirectional layer
Yorel	Carbon violin prototype
z	Axis normal to the plane of a laminate
z(i)	Side or position of Layer i
$\alpha$	Expansion coefficient
( $\alpha_i$ )	Expansion coefficient in the $i$ th direction of a layer
{ $\beta_i$ }	Hygrometric expansion coefficient in the $i$ th direction
$\gamma$	Angle due to shearing
{ $\epsilon$ }	Components of the strain tensor
$\eta$	Damping coefficient
$\theta$	Angle
$\mu\text{m}$	Micrometer ( $10^{-6}\text{m}$ )
$\nu$	Poisson's coefficient (dimensionless)
$\nu_{ij}$	Poisson's coefficient and shear coupling coefficient
$\rho$	Density ( $\text{kg}/\text{m}^3$ )

$\Sigma$	Sum
$\{\sigma\}$	Component of the stress tensor (Pa)
$\sigma$	Stresses (Pa)
$\tau$	Shear stress (Pa)





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## List of Abbreviations and Definitions

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### Abbreviations

ABS	Acrylonitrile-butadiene-styrene
AP	Automate programmable
APV	Polyvinyl alcohol
BMC	Bulk molding compound
CAD	Computer-aided design
CADM	Computer-aided design and manufacturing
CAM	Computer-aided manufacturing (NC = numerical control; PA = programmable automaton)
CAPM	Computer-aided production management
CCC	Ceramic-ceramic composite
CVD	Gas-phase chemical deposition
DC	Digital control
DMC	Dough molding compound

EP	Epoxy
EPDM	Ethylene propylene diene monomer
EPS	Expanded polystyrene
EVA	Ethylene vinyl acetate
EVOH	Ethylene-polyvinyl alcohol copolymer
HDP	High-density polyethylene
HEL	High elastic limit
HM	High modulus
HP	High performance
HS	High strength
IMC	In-mold coating
LCP	Liquid crystal polymer
LDP	Low-density polyethylene
LMP	Lost mold process
MF	Melamine formalin
MMC	Metal-metal composite
MP	Melamine phenol
OC	Organic composite (BD = broad diffusion; HP = high performance)
PA	Polyamides
PAA	Polyarylamide
PAES	Polyaryl ether sulfone

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PAI	Polyamide-imide
PAN	Polyacrylonitrile = precursor of carbon fiber
PAR	Polyarylate
PBT	Polybutadiene terephthalate
PC	Polycarbonate
PE	Polyethylene
PE-BA	Polyether block amide
PEEK	Polyether-ether-ketone
PEI	Polyetherimide
PEK	Polyetherketone
PES	Polyethersulfone
PET	Polyethylene terephthalate
PF	Phenol-formol
PI	Polyimide
PMM	Polymethyl methacrylate
POM	Polyoxymethylene (or polyacetal or polyformaldehyde)
PP	Polypropylene
PPO	Phenylene polyoxide
PPS	Phenylene polysulfide
PS	Polystyrene
PSU	Polysulfone
PTFE	Polytetrafluoroethylene
PU	Polyurethane

PVC	Polyvinyl chloride
PVD	Physical deposit in the gas phase
PVDC	Polyvinylidene chloride
RIM	Reaction molding
R-RIM	Reinforced-reaction injection molding
RTM	Resin transfer molding
RTP	Reinforced thermoplastic
SAN	Styrene acrylonitrile
SBS	Styrene butadiene block copolymer
SG	Spheroidal graphite cast iron
SI	Silicone
SMC	Sheet molding compound
SRT	Stampable reinforced thermoplastic
TH	Thermohardening
TMC	Thick prepreg
TP	Thermoplastic
TPE	Thermoplastic elastomer
UF	Urea formol
UP	Unsaturated thermosetting polyester
XMC	Prepreg with oriented reinforcements
ZMC	Injection of reinforced TD (TD = thermohardening)
ZMC	Specific premix for injection

**Definitions**

Amorphous	Constituted by disordered molecules grouped into clumps
Anisotropy	Variable properties depending on the direction under consideration
Complex	Material made by combining films or sheets of different properties (plastic directions)
Composite	Material comprising a reinforcement in the form of a filament
Crystalline	Constituted by organized, aligned molecules
Epitaxy	Formation in the gas phase of high purity crystals
Hyperbaric	Under very high pressure ( $> 1,000$ bar)
Isostatic	Under uniform pressure in all directions
Isotropy	The same properties in all directions
Leaching	Preparation in the form of solvents in order to extract the constituents
Plasma	Gas brought to a very high temperature (ionized)
Pyrolysis	Chemical decomposition caused by heat
Slurry	Diluted paste used for pouring
Tribology	Study of the effects of friction
Trichitis	Monocrystals in the form of very pure filaments

