Trends in Mathematics

Klaus Meer
Alexander Rabinovich
Elena Ravve
Andrés Villaveces
Editors

Model Theory, Computer Science, and Graph Polynomials

Festschrift in Honor of Johann A. Makowsky

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Trends in Mathematics

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This book is dedicated to our dear friend and colleague Johann Andreas Makowsky on occasion of his 75th birthday.

Preface

Johann (János) Andreas Makowsky was born on March 12, 1948, in Budapest, Hungary. In 1949 his family moved to Zurich in Switzerland, where he got his education, and in 1966 his citizenship. He majored in mathematics and physics at the Federal Institute of Technology (ETH) in Zurich, graduating with a master's degree in both fields in 1971. His MSc thesis was in Model Theory. He then spent part of 1972 and 1973 in Warsaw, first as an exchange student working under A. Mostowski, then as a participant in the Logic Year held at the Banach Center in 1973. Still enrolled at ETH, he completed his Ph.D. thesis under the supervision of H. Läuchli and E.P. Specker. Following the invitation by G. Kreisel, János worked as a visiting assistant professor at the philosophy department of Stanford University from fall 1973 to spring 1974. He returned to Zurich in spring 1974 and received the doctoral degree (Dr. math.sc.) for his thesis "Δ-Logics and Generalized Quantifiers." After a short post-doctoral research appointment at Simon Fraser University in Vancouver under A. Lachlan, in 1975 he was appointed a visiting professor at the mathematics department "Ulisse Dini" of Florence University. János held two regular positions: from 1976 to 1980 at FU Berlin as a junior faculty (H1), where he also got his Habilitation degree in 1980. Also in 1980 he joined the Faculty of Computer Science at the Technion in Haifa (Israel) as a senior research associate. He was given tenure in 1984 as associate professor, and became full professor in 2001 and professor emeritus in 2016. He still occasionally teaches advanced courses and supervises graduate students.

János held many visiting positions at various academic institutions: the Hebrew University and at Bar Ilan University in Israel; at the Simons Institute in Berkeley and at MIT in Cambridge, USA; at the Fields Institute in Toronto, Canada; at Lausanne University, Bern University, and ETH Zurich in Switzerland; at LaBRI of the University of Bordeaux and at Paris Diderot University in France; at Vienna Technical University in Austria; at Charles University in Prague, Czech Republic; the Stekhlov Institute in Moscow, Russian Federation; the Monash University, Melbourne, Australia; and at the Institute of Mathematical Sciences in Chennai, India.

János' research shows both an impressive broadness and scientific depth. The common thread of his research is Model Theory. Over the years János contributed significantly to areas ranging from Model Theory, Database Theory, Theory of Programming Languages and their Verification, to Data Modeling and Software Engineering, and more recently, to Algorithmic Graph Theory and Knot Theory, and a General Theory of Graph Polynomials.

János (co-)authored and edited 9 Books and Lecture Notes and published over 175 scientific papers. He gives a complete list and detailed comments in his contribution "My writing" in this volume. Without going more into detail here, the various contributions in the present volume convincingly testify to the diversity of his scientific interests and the breadth of the topics he has worked on.

Needless to say that his scientific career was flanked by numerous services to the community, such as long periods of being on the editorial board of various journals and a guest editor of special issues. He has also regularly served as a member of program and steering committees of scientific conferences worldwide. Among these multiple engagements let us highlight that he was one of the founding members of the European Association of Computer Science Logic (EACSL). During his presidency of the EACSL, in 2004 he initiated the now well-known "Ackermann Award" for outstanding dissertations in the field of Logic in Computer Science. As a teacher he developed numerous advanced courses and supervised 12 Ph.D. and 22 master's theses.

The present Festschrift gathers 24 research articles authored by scientific companions, friends, and colleagues. They cover a broad variety of areas to which János made significant contributions himself. This reflects impressively the significant impact of his work.

The volume is enriched by four essays shedding light on János as a personality, and with two contributions by the celebrant.

Last but not least, everyone who knows János as a private person or communicated with him during free time at conferences can affirm his formidable knowledge in so many areas besides science, whether it is literature, history, art, music, or other fields. For an impression of János' non-mathematical activities, especially creative and essay writing, one can access his homepage https://janos.cs.technion.ac.il/ and also read Claudia Gehrke's contribution in this volume. His own comments on his non-technical writing are also discussed in his contribution "My writing" in this volume. It is always a joy but also a challenge to discuss with János topics of his diverse interests.

We are happy to have you and your wife Masha as friends. We wish you many fruitful and healthy years full of activities!

Cottbus, Germany	Klaus Meer
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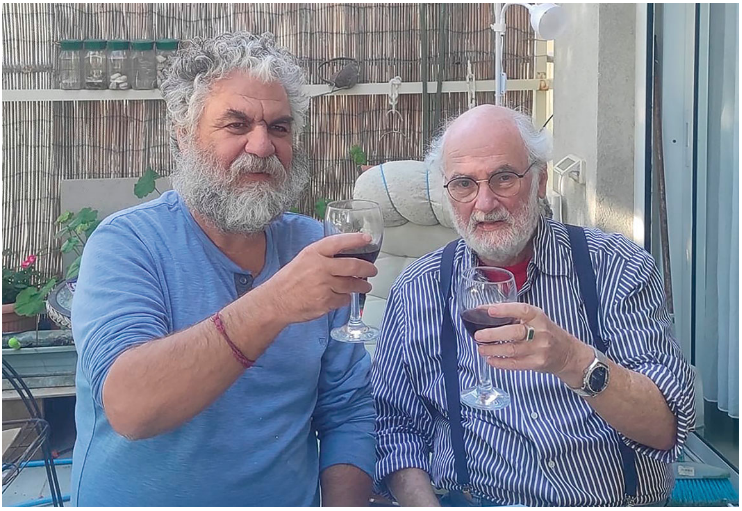
October 2024	

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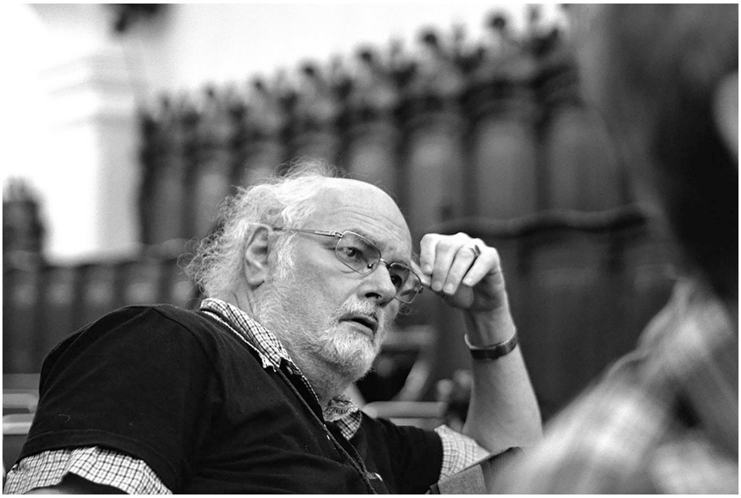

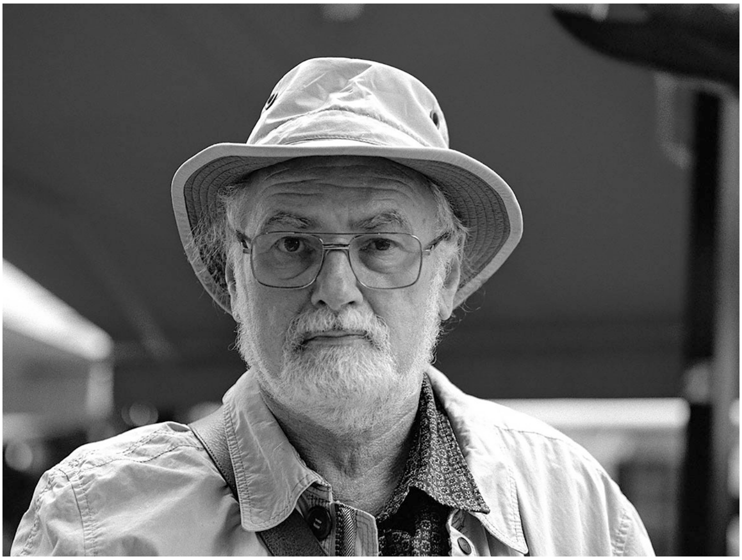

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Personal Notes

My Writing

Johann A. Makowsky

Abstract I first describe how I became a mathematician, discussing other possibilities and paths not taken. Next I discuss my scientific publications. Finally, I describe my non-scientific publications, essays and literary miniatures. All this is arranged by topics rather than chronologically. Finally there is a complete list of my publications, scientific and otherwise.

1 Mathematics as a Literary Genre

I knew very early that I wanted to be an academic, creative but rooted in some kind of reality. Writing would be part of it, but what kind of writing was not clear yet.

1.1 Paths Not Taken

1.1.1 Path Not Taken: Chemistry

My maternal grandfather was an industrialist. In Hungary he was the CEO of a big chemical and mining company. After emigration to Switzerland he founded a small chemical firm and traded in commodities, mostly mining products and chemical components. Before the war he had talked his son, my uncle, into studying chemistry. He was groomed to be the "crown prince" of the chemical empire, but it did not work out as planned. Grandfather was sure I should follow suit, as a future heir of his enterprises. When I was eleven, he passed away and his enterprise was acquired by his silent partner who bought our shares cheaply due to our lack of funds for badly needed further investments. In a way I was lucky: I was now free to

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realize my own career choices. A path not taken not as a choice but by force majeur, hence path not taken number zero.

1.1.2 Path Not Taken: Atomic Physics

In 1960, when I was twelve, I wanted to become an atomic physicist. It was the time when passionate discussions about the effect of the atomic bomb on the future of mankind were reaching the peak. In 1962 the play "The Physicists" by Friedrich Dürrenmatt had its premiere, and my mother took me to the memorable performance of the Schauspielhaus in Zurich. In 1964, I saw a performance of "In the Matter of J. Robert Oppenheimer", a play by Heinar Kipphardt, based on the transcripts of the Oppenheimer security hearings. I became aware of the ambiguity of the role of science in technical progress. Eager to learn more about this ambiguity I got hold of Karl Jaspers' "The future of mankind" published in 1958.[1] It became clear to me that my wish to become an atomic physicist would confront me with questions of ethics and personal responsibilities which I was not willing to face at the time. This was the first time a path was not taken consciously.

1.1.3 Path Not Taken: Politics and Feuilleton

It was also the time of the Eichmann trial.[2] Mother was watching excerpts of the publicly broadcasted trial every evening. Due to my insistent questions, she started telling me about her own and her family's dramatic war-time experiences. I read Sartre's "Réflexions sur la question juive".[3] Jaspers and Sartre led to my early interest in existentialist philosophy and to my general interest in literature. During the six and a half years of gymnasium I developed a passion for writing and languages. I was reading the classics in the original Latin and Greek, and philosophy and literature in German and French. My school essays became ambitious writing projects and ambitious literary experiments. I wrote plays and excelled in translating Greek and Latin poetry. I was curious to understand the "behind-the-scenes"

[1] Karls Jaspers, "Die Atombombe und die Zukunft des Menschen: Politisches Bewusstsein in unserer Zeit", Piper, München, 1958.

[2] "The Eichmann trial was the 1961 trial in Israel of major Holocaust perpetrator Adolf Eichmann who was kidnapped in Argentina by Israeli agents and brought to Israel to stand trial. Eichmann was a senior Nazi party member and served at the rank of Obersturmbannführer (Lieutenant-Colonel) in the SS, and was one of the people primarily responsible for the implementation of the Final Solution. He was responsible for the Nazis' train shipments from across Europe to the concentration camps, even managing the shipment from Hungary directly, where 564,000 Jews died". (Cited from https://en.wikipedia.org/wiki/Eichmann_trial).

[3] "Reflections on the Jewish Question" is an essay about antisemitism written by Jean-Paul Sartre shortly after the Liberation of Paris from the German occupation in 1944. The first part of the essay, "The Portrait of the Antisemite", was published in December 1945 in Les Temps modernes. The full text was then published in 1946.

and wrote term papers based on interviews I conducted with the brewmaster of "Wädenswiler Bier", with the stage master of the Zurich Opera, and with the head of the typesetting department of *Die Tat*, a now defunct but then important evening paper, for which I also worked as a delivery boy to earn some pocket money. When I was 14 I started writing essays about theater and music which were published in the quarterly magazine *Die Mittelschulzeitung* edited by students of the Zurich's gymnasia, the *Zürcher Mittelschulzeitung* (ZMZ). Later I joined the magazine as an editor. Once at the university, I continued to write for the bimonthly *Zürcher Student* and in 1969 became one of its editors. I had learned newspaper making in all its aspects: writing, editing, designing page layout, and acquisition of advertisements. I could have started a journalistic career. I even had my moment of investigative journalism and political activism: One was when ZMZ defended the freedom of cinematic expression and exposed an obstinate enemy of the avant-guardish movie club. Another one was when in 1968 I suggested to the Swiss Student Union to initiate a referendum against a misconceived university reform, which took place in 1969, and we won. The *Zürcher Student* played an importan role in the campaign while I was one of its editors.

Many of my fellow students involved over the years with the *Zürcher Student* became journalists, writers, public servants in cultural institutions, or politicians. One became a professor of philosophy at Zurich University, and one even served as a Federal Council. I retained my wish to write essays and prose and I did publish about theater, music, politics and society during all my career, see Sect. 2.5, but not for a living. Second path not taken.

1.1.4 Path Not Taken: The World as a Stage

In 1966 my mother was diagnosed cancer just before starting a new job. She remained unemployed and I earned my money working in the newly founded *Theater am Neumarkt* as an all-purpose hand: selling programs, controlling tickets and managing the cloakroom. I also fulfilled tasks backstage for special effects involving sound and lightening. I attended rehearsals and occasionally offered my comments. I did have a rather solid background in drama, as a theatergoer and as a reader of plays, and I was familiar with some of the theoretical literature, see again Sect. 2.5. Soon the actors and the director began to pay attention to my sharp comments and constructive suggestions. I started flirting with the idea of making a career as a director. At the time there was only one way to learn the trade: One had to become a director's assistant and apprentice. I kept working in the theater part-time from 1966 to 1970 while Felix Rellstab[4] was its artistic director and Reinhardt

[4] Felix Rellstab 1966–1971 first head and director at the *Theaters am Neumarkt* in Zürich. 1960–1991 head and director of the *Bühnenstudio*, Switzerland's prestigeous acting school. Under his leadership the *Bühnenstudio* was renamed *Schauspiel-Akademie Zürich*. in 1972/1973.

Spörri[5] was its dramaturg who was also directing. Two years into my studies at ETH, Spörri was desperately looking for an assistant for his newest production planned at a very short notice. There it was, my dream had come true. I was offered the job. However, it came too late. I had already started to work on my diploma thesis, an equivalent to an MSc thesis. I did ponder over the offer seriously, but I finally rejected it. Mathematics and logic had captured me and did not let go. Third path not taken.

1.2 From Philosophy and Epistemology to Logic and Mathematics

Shortly before my matriculation exams in 1966 I decided to study mathematics and philosophy. Originally I wanted to go to Paris and study mathematics and philosophy. Paris for me was Sartre and Foucault on the one hand, and Bourbaki on the other. My father, who lived in Paris at the time, bought me the first 6 volumes of Bourbaki as a gift for my matriculation diploma. One could enroll at the École Normal Supérieure (ENS) as a foreign student without passing the concours. However, my Greek teacher introduced me to a friend of his who was a philosophy professor well versed in mathematics and exact sciences. He convinced me that for undergraduate studies I would fare better in Zurich, because in Paris I was unlikely to even get close to my admired intellectual superstars. In 1967, I enrolled at the Swiss Institute of Technology (ETH) in the Department of Mathematics and Physics.

I got interested in mathematics through the philosophy of Plato and Leibniz. In the last years of the gymnasium I discovered the early work of Ernst Cassirer.[6] His monograph on Leibniz and his two volumes on epistemology in modern times left a deep impact on me. The latter is a "magisterial and deeply original contribution to both the history of philosophy and the history of science. It is the first work, in fact, to develop a detailed reading of the scientific revolution as a whole in terms of the 'Platonic' idea that the thoroughgoing application of mathematics to nature (the so-called mathematization of nature) is the central and overarching achievement of this revolution."[7] Through Cassirer I was naturally led to the Vienna Circle and to Wittgenstein as well as to Russel and Frege. I was

[5] Reinhart Spörri was 1968–1971 dramaturg and first director at the *Theater am Neumarkt Zurich* under Felix Rellstab.

[6] My first readings of Cassirer's work: Leibniz' System in seinen wissenschaftlichen Grundlagen. Marburg, 1902; (1906) Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit. Band 1 und 2. Berlin: Bruno Cassirer Verlag 1906/7. Kant und die moderne Mathematik. Kant-Studien 12, 1–40. 1907.

[7] Quoted from: Michael Friedman, "Ernst Cassirer", The Stanford Encyclopedia of Philosophy (Fall 2023 Edition), Edward N. Zalta & Uri Nodelman (eds.), URL = https://plato.stanford.edu/entries/cassirer.

intrigued by epistemological questions of mathematics. Was mathematics a branch of logic? Or was it a language to describe abstract configurations? Are the basic elements of the mathematical language innate, hardwired in the human brain? I read Hadamard's booklet "Psychology of Invention in the Mathematical Field", Poincaré's philosophical works, especially "Science and Method", and Piaget's "The Child's Conception of Number".

In the first semester of my university studies I also enrolled in the philosophy seminar of ETH dedicated to Wittgenstein's early masterpiece, the"Tractatus logico-philosophicus" given by E. Specker and G. Huber. I had read the "Tractatus" while still in the gymnasium. In my precocious but innocent enthusiam I volunteered to give the first talk in this seminar, unaware that among the over eighty participants of the seminar all the philosophical elite of Zurich and beyond would be present. Among them Paul Bernays, the eminent logician and the author of an important critique of Wittgenstein's later work[8] Had I known more about the audience of the seminar, I might have shied away from volunteering to prepare a talk at such a short notice. Surprisingly, my talk, given freely and only based on some scribelled notes, was a stunning success. Later, both E. Specker and P. Bernays played decisive roles in my further mathematical development and career. From the first semester and until after my PhD I attended regularly the logic seminar, founded by Bernays and Gonseth in the 1930s. In my time it was run by Ernst Specker and my future MSc advisor Hans Läuchli. Other logic-related seminars I attended whenever I was in Zurich were organized by Ernst Specker and Volker Strassen on the complexity of computations, and by Erwin Engeler on logic and its role in computer science.

1.3 Epilogue

The paths not taken were many. I did not become an atomic phycisist, I did not choose to make literary criticism as my career, nor did I become a scholar of Latin and Greek literature and culture. I did realize that classical philology had passed its zenith and was on the way of becoming marginal. My dabbling in journalism and politics did not satisfy my intellectual ambitions. Besides my interest in mathematics in all its aspects, my passion for writing remained strong. I was 28 when my father passed away and I resolved to write a historical novel based on his life. But whenever I tried to dedicate time to this (or similar) projects I stumbled over a mathematical discovery which gently forced me to write mathematics. Even after my formal retirement, mathematics did not let go. Although I did spend more time on my literary projects and progressed well, mathematics kept intruding and interrupting.

For me mathematics is a literary genre.

[8] Paul Bernays, "Comments on Ludwig Wittgenstein's Remarks on the Foundations of Mathematics", Ratio 2: 1–22. 1959.

2 Comments on Selected Papers

My mathematical research interests still include Model Theory (Classical Model Theory, Abstract Model Theory, Finite Model Theory), Semantics of Programming Languages (Database Theory, Program Verification, Logic Programming), Logic and Complexity. However, since 1995 my work has concentrated more but not exclusively on Applications of Logic to Graph Theory, Knot Theory and Combinatorics. Since 2005 my work concentrates on a model theoretic view of Graph Polynomials and Combinatorial Counting Functions.

I have written two mathematical papers which have an autobiographical touch: [135] recounts how Model Theory is a recurrent theme in most of my work, and [75] recounts how graph polynomials became a dominant theme in my more recent research. My recollections of my encounters with A. Mostowski were published as [162]. For Yuri Gurevich's 80th birthday I wrote about our 40 years of friendship in [171]. I also co-edited and contributed to the memorial volume for my most influential teacher E. Specker (1920–2012) [55].

In the following subsections I comment on my publications grouped by topic. A complete list of my papers follows in Sects. 3 and 4.

2.1 Model Theory

2.1.1 Classical Model Theory

Already in my first semester I attended the Läuchli-Specker logic seminar and remained a faithful regular. Although the seminar was meant also for undergraduates, we also studied very advanced topics like Matyasevich's work on Hilbert's 10th problem, Morley's landmark paper on categoricity, and Tseitlin's work on the complexity of the resolution algorithm for satisfiability of propositional logic. In summer 1969, I spent 4 months as a recruit in the Swiss Army. During my service I had a copy of Morley's paper with me and used much of my free time in the army to prepare myself for the seminar, where I was to lecture about it. I also tried to attack some of the open problems listed in the paper. In summer 1970, I attended W. Hodges' logic conference at Bedford College, London. There I met not only W. Hodges, but also G. Müller, A. MacIntyre and A. Lachlan. All of them decisively influenced my future career as a logician. A. Lachlan gave a preprint of his and J. Baldwin's paper on categoricity after I told him what I was working on. The paper became the basis of my early research.

My work of 1971–1974 in classical Model Theory is based on my diploma thesis from 1971. The results were published as [1] and [3]. Two of the famous problems from M. Morley's papers[9] were (partially) solved in [3]: There is no

[9] Michael Morley, "Categoricity in power". Trans. Amer. Math. Soc. 114 (2): 514–538. 1965.

finitely axiomatizable complete almost strongly minimal ω-categorical (hence ω_1-categorical) theory, and there is a finitely axiomatizable complete superstable theory such that all of its types are non-principal.

Paper [3] is referenced in the two major monographs, one by C.C. Chang and J. Keisler[10] and one W. Hodges[11] and the more specialized monograph by B. Zil'ber[12] The book by M. G. Peretyat'kin[13] develops a rich theory of work done by its author between 1982 and 1993 which was influenced by my diploma thesis of 1971. In P. Rothmaler's Monograph[14] a whole chapter is devoted to my results of 1971.

Paper [3] also asks whether there is a finitely presented infinite group which has only finitely many conjugacy classes. This is still open and attributed to [3] in *Open problems in combinatorial group theory*.[15]

After my early success in classical Model Theory, I spent two periods of 6 months in Warsaw. I had realized that with the tools available to me at the time, I could not make any further progress in categoricity theory. Under the guidance of W. Marek and A. Mostowski, I turned to generalized quantifiers and Abstract Model Theory. I was fascinated by Lindström's characterization of First Order Logic using generalized quantifiers.[16]

2.1.2 Abstract Model Theory and Abstract Elementary Classes

My PhD thesis marked the beginning of my own work in Abstract Model Theory. It also marked the beginning of an intensive collaboration with S. Shelah.

The results of my PhD Thesis are published as [2, 4, 78]. When I lectured in S. Feferman's logic seminar at Stanford about my PhD, J. Stavi found a mistake in one of the theorems of PhD thesis. Jonathan had studied together with Saharon before joining Stanford's math department. He helped me to correct the mistake and we planned to publish the journal version together. However, J. Stavi insisted to add Saharon as a third author, although he was not personally involved in working on this paper at all. Stavi explained that all he needed to fix my mistake he had learned from Saharon while they both were in Jerusalem. Although I met Saharon already in 1971 at the European ASL Meeting in Cambridge, UK, personal collaboration with

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[10] C.C. Chang and Jerome Keisler, "Model Theory", 2nd edition and later, North Holland, 1973.

[11] Wilfrid Hodges, "Model Theory", Cambridge University Press, 1993.

[12] iBoris Zil'ber. "Uncountably categorical theories". Mathematical Monographs Vol. 117. American Mathematical Soc., 1993.

[13] Mikhail G. Peretyat'kin, Finitely Axiomatizable Theories. 1997. English translation by V. Morley, J. Symbolic Logic 64.1 (1999): 1828–1830.

[14] Philipp Rothmaler. "Introduction to Model Theory". Vol. 15. CRC Press, 2000.

[15] Alexei G. Myasnikov, Gilbert Baumslag and Vladimir Shpilrain. "Open problems in combinatorial group theory." Combinatorial and Geometric Group Theory: AMS Special Session, Combinatorial Group Theory, November 4–5, 2000, New York: AMS Special Session, Computational Group Theory, April 28–29, 2001, Hoboken, New Jersey 296 (2002): 1.

[16] Lindström, P. (1969) "On Extensions of Elementary Logic." Theoria, 35: 1–11.

him started only when I met him again in 1974 in Vancouver. After that he offered me a 1 year Lady Davis post-doc position at the Hebrew University 1976. However, on his suggestion, this was cut short to a 2 month visit during a semester break due to my accepting a 5 year position in West-Berlin. During my Berlin appointment I visited Israel regularly and continued to work with S. Shelah and J. Stavi. This collaboration resulted in papers [9, 10, 12].

At the time I also worked in topological Model Theory [6, 11, 79, 80]. My approach to topological Model Theory was based on monotone quantifiers, which were also investigated in [5, 7]. I returned again to topological Model Theory most recently with the paper [77], where we count the number of finite topologies subject to various restrictions definable in monadic second order logic.

Some of my papers in abstract Model Theory (1973–1984) are widely referenced. The multi-author monograph 'Model Theoretic Logics' (J. Barwise and S. Feferman eds., 1985) gives best testimony for this: Chapters 18 and 20, [153], [155] were written by me alone summarizing my work with S. Shelah's and my own contribution to the field. Chapter 19 [154] is co-authored with D. Munidici. The three chapters comprise 146 pages and basically conclude my work in abstract Model Theory. Many other chapters quote my work.

In chapter 20, [155], I gave the first published presentation of S. Shelah's then unpolished results on "Abstract embedding relations". Both chapters also contain relevant original contributions to the field I had obtained before 1982. In 1978 in Berlin I supervised the MSc thesis by Gerhard Herrgott on the model theory of L^{pos}, but the results were never published. I also initiated and supervised (until I left for Israel) the MSc thesis of Sakae Fuchino.[17] When S. Shelah resumed work on this topic (published in 1987) he called the concept 'Abstract Elementary Classes", and misquoted my work with him. Due to this, Shelah's and my pioneering work from before 1983 was often overlooked.

I stopped working in abstract model theory due to my employment at the Computer Science Department of the Technion. However, in my later work in finite Model Theory my earlier work on generalized quantifiers plays a crucial role, see Sect. 2.2.7 below.

2.2 Foundations of Computer Science

2.2.1 My Early Exposure to Computer Science

Although ETH in Zurich was a pioneer in early computing there was no Computer Science Department until 1981. Programming was viewed as part of Numeric Analysis using ALGOL-60 as the programming language. In 1968, the Institute of

[17] Sakae Fuchino, "On the categoricity theorem in $L_{\omega_1,\omega}$", Tsukuba Journal of Mathematics, Vol.10, No.1 (1986), 117–120.

Applied Mathematics was split and the Group of Computer Science was established with H. Rutishauser (one of the fathers of ALGOL-60) and N. Wirth (the father of PASCAL). Later, they were joined by E. Engeler responsible for Theoretical Computer Science. Programming was still taught as part of Numeric Analysis, but switched to PASCAL. Also in 1968, V. Strassen joined the Math Department of Zurich University. He and E. Specker started a joint seminar on Complexity Theory. During my studies I attended various lectures delivered by Engeler and became a regular in the Specker-Strassen seminar. Quite a few of the early participants of this seminar became leading computer scientists, among them J. Heintz, J. von zur Gathen, E. Zachos and M. Fürer.

While I was employed at the Free University in West-Berlin I attended seminars at the Technical University: D. Siefkes' seminar on complexity theory and H. Ehrig's seminar on specification of abstract data types. There I started my collaboration with B. Mahr who would visit me later at the Technion in Haifa and with whom I wrote [88, 89, 13]. These papers became the basis of my [18] see also Sect. 2.2.5 below. I also supervised the MSc thesis of Michael Mötz on the complexity of resolution, but our results where superseded by Z. Galil's PhD thesis from 1977.[18]

Between 1978–1980, while visiting the Hebrew University on a German-Israeli Minerva Grant it became clear to me that for family reasons a permanent position in Israel would be beneficial for me. I was told by Eli Shamir that I would have good chances if I changed my research interest to Logic in Computer Science. This was 10 years before Logic in Computer Science became an established branch of theoretical computer science with its conferences LiCS (in the US) and CSL (in Europe).

Eli introduced me to Catriel Beeri. He knew that Catriel was working on databases, he was likely to be willing to discuss his work with a logician.

Consequently, I started working with Catriel on the foundations of database theory. Eli also suggested I should attend the 1979 Annual Symposium on Theory of Computing in Atlanta which I did. I made contact with V. Pratt, with whom I had a chance to discuss dynamic logic. He invited me for a 2 month stay at MIT starting in February 1980 in order to continue our emerging collaboration. I also learned about computable database queries from the talk by A. Chandra and D. Harel. Many other talks stressed the importance of logic for computer science.

Eli's suggestion opened my mind and helped me in choosing my first research topics in Computer Science. When I joined the Technion's Computer Science Department, I was well prepared to find ways to use my expertise in model theory for foundational problems in Theoretical Computer Science.

[18] Zvi Galil, On the complexity of regular resolution and the Davis-Putnam procedure, Theoretical Computer Science 4 (1), 1977, 23–46.

2.2.2 How to Use Model Theory in Computer Science

During 1980–82 I had various intensive discussions with Y. Gurevich and also with J. Stavi on the role model theoretic methods could play in Computer Science. In 1982 I was an invited speaker at the Logic Colloquium'82 in Florence, where I chose to speak about this, [87]. There were no technical results, but many suggestions and examples. At the time I missed what Y. Gurevich later pointed out:[19] Finite Model Theory is inherently different from model theory, in as much as most preservation theorems of classical Model Theory do not hold in the finite. My approach, however, was to use classical preservation theorems to explain how they can (still) be used when infinite models are allowed. I had two remarkable successes in this approach: I could explain why Horn formulas matter in Computer Science, [18], and find many new uses of the Feferman-Vaught Theorem and its variants for graph algorithms [43]. The first one is discussed further on pages in Sect. 2.2.5, the second in Sect. 2.2.8. I published two more papers similar in spirit to [87]. In 1992 [156] and in 1994 [99]. Finally, in 2011, I published a version of my retirement address as president of EACSL as [135] reflecting on how Model Theory remained a recurrent theme in my work over the years.

2.2.3 Dynamic Logic and Program Verification

My first published paper in computer science dealt with the expressive power of dynamic logic [82] and was presented at ICALP'80 in Noordweijkerhout, the Netherlands. There I also met S. Even who at the time was the head of the CS department of the Technion. Having heard that I was looking for an academic appointment in Israel, he recruited me to join his department. Already in my first semester at the Technion Michail Tiomkin, a very recent immigrant from the USSR became my PhD student. Michail was a graduate of Moscow State University. He had studied with Albert G. Dragalin who, together with A.N. Kolmogorov, had established a compulsory logic course for all mathematics majors at the university. He was also chosen to represent the USSR at the International Mathematics Olympiad, but, after he refused to join the KOMSOMOL (the Young Communist League), was dropped from the team. Michail's PhD thesis was published in two joint papers as [14, 23]. The first one dealt with propositional dynamic logic with local assignments, and the second with the decidability of finite probabilistic propositional dynamic logic. Michail left academia and joined the research laboratory of IBM, But he continued working with M. Kaminski, another immigrant from the same period, whom he knew from his Moscow times, and who joined our faculty after completing his PhD with M. Rabin in Jerusalem. They mostly worked together on non-monotonic

[19] Gurevich, Yuri. Logic and the challenge of computer science. 1985. (preprint), published in 1988 as: Gurevich, Yuri, Logic and the challenge of computer science, as: Chapter 1 of Trends in Theoretical Computer Science, E. Börger ed., Computer Science Press, 1988.

7 Related Topics and Open Problems

We have shown that local graph operations used in reductions and recursions are powerful tools for computing graph polynomials. In some cases, it may be useful to work with edge- or vertex-weighted graphs to reduce the number of terms of a recursion or to obtain new reductions.

Problem 1 The list of local graph operations presented in Sect. 2 is not complete. What other local graph operations are useful for graph polynomials? What do we gain if we allow to modify the graph within a certain distance from a given vertex?

Problem 2 Sometimes relations between graph polynomials are "hidden." For example, we can derive a polynomial of a given graph from a graph polynomial of its complement or its line graph. What other relations between known graph polynomials can we discover in this way?

Universality of Graph Polynomials There are some graph polynomials which have the nice property of being a *universal polynomial* of graphs. A graph polynomial p is universal if any other graph polynomial that satisfies the same recursive relation as p (and perhaps some additional conditions) can be obtained from p by variable substitution and multiplication by some factor. The spanning subgraph (Tutte) polynomial, the covered components (edge elimination) polynomial, the subgraph component polynomial, and the extended cut polynomial are universal [9, 12, 25, 28, 55, 56]. An open question is to find a general strategy for obtaining a list of properties that guarantees universality, such as satisfying an edge deletion-contraction relation and being multiplicative with respect to components.

Problem 3 What other local graph operations lead to universal graph polynomials?

Recursive Relations and Complexity Since the computation of each graph polynomial presented in this paper is an NP-hard problem (often #P-complete, [36]), the algorithmic application of recurrence relations for the computation of the polynomial cannot provide an efficient way to its computation. However, often we can get *better exponential-time algorithms* by a proper selection of the edges and/or vertices used for local graph operations, together with a suitable set of initial graphs (terminal conditions for the recursion). In this way, we obtain for the chromatic or reliability polynomial an algorithm that needs $t(G)$ (number of spanning trees) instead of 2^m steps. For the independence polynomial, we can find an algorithm with time complexity $O(1.4^n)$ instead of $O(2^n)$ [54]. Reducing complexity becomes a real challenge when many different local graph operations, reductions and other methods are combined.

Problem 4 How can we find better exponential-time algorithms for graph polynomials?

Universal Versus Special Reductions The local graph operations introduced in Sect. 2 and applied in Sect. 3 are *universal* in the sense that they can be used for *any* vertex or edge of the graph. For vertices or edges with special properties, additional reductions may be possible. Consider, as an example, a *dominating vertex* (a vertex that is adjacent to any other vertex). It is not a hard exercise to prove that for the domination, independence, and chromatic polynomial the following relations are valid for any dominating vertex v:

$$D(G,x) = x(1+x)^{n-1} + D(G-v,x),$$
$$I(G,x) = x + I(G-v,x),$$
$$P(G,x) = xP(G-v,x-1).$$

Further reductions can be obtained for vertices of small degree, simplicial vertices, bridges or loops. Reductions for special vertices and/or edges often result in polynomial-time algorithms for graph polynomials of special graph classes like threshold graphs, co-graphs, sp-graphs or chordal graphs.

Problem 5 An interesting open question is what other local graph operations can be used to derive reductions for graph polynomials? In which case and for which class of graphs can we obtain a polynomial-time algorithm?

Problem 6 When we use several different local graph operations in an algorithm, the computational complexity can depend on the order of the operations. How can we find the optimum order of the local graph operations?

Sum Formulae and Recursive Definitions All above listed graph polynomials can be defined by a sum ranging over vertex subsets, edge subsets, or partitions of the vertex set. Alternatively, we can define each graph polynomial by a recurrence relation together with initial conditions. It seems natural that we can use the recursive definition to compute the graph polynomial by a *state distinction* or by a *decomposition tree*, so that we end up in a summation process that yields a sum formula (not necessarily over edge or vertex subsets, but more general *states*). For more details, see [32].

Starting from a definition of a graph polynomial as a generating function (a sum formula), it seems to be in general a more difficult task to split the sum in way that allows an interpretation of the resulting polynomials by some local graph operations. In some cases, we can at least show that a certain type of linear recurrence relation *does not exist*. For an example, see [43].

Problem 7 Is there a general way to find suitable local graph operations and a recursive relation for a graph polynomial with a given subset representation?

Toplogical Properties of Graphs Extracting of topological properties of graphs, like planarity, linkless embeddability, and genus, from a graph polynomial seems to be a widely open problem. Most attempts to deal with topological properties focus on generalizing graphs to embedded graphs, ribbon graphs, delta matroids or other more complex objects [18, 24, 29]. However, there seems to be no fundamental obstacle to a polynomial defined on undirected graphs containing the information necessary to derive all the topological properties of interest.

Problem 8 Can we find local graph operations that allow a topological interpretation? Can we find a graph polynomial with a fixed number of variables that provides interesting topological information? Is there a way to extract more topological properties of graphs from existing graph polynomials?

Problem 9 This problem is related to the previous problem. Let $\mathcal{H}$ be a given finite set of (small) graphs. Can we define a graph polynomial $f(G)$ that gives for each $H \in \mathcal{H}$ the number of occurrences of H as a minor of G?

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References

1. Aigner, M.: A Course in Enumeration. Springer-Verlag, Berlin (2007)
2. Aigner, M., Van der Holst, H.: Interlace polynomials. Linear Algebra and its Applications, vol. 377, pp. 11–30. Elsevier (2004)
3. Akbari, S., Oboudi, M.R.: On the edge cover polynomial of a graph. Eur. J. Comb. **34**(2), 297–321 (2013)
4. Akbari, S., Alikhani, S., Oboudi, M.R., Peng, Y.-H.: On the zeros of domination polynomial of a graph. Comb. Graphs **531**, 109–115 (2010)
5. Alikhani, S., Peng, Y.-H.: Introduction to domination polynomial of a graph. Preprint (2009). arXiv:0905.2251
6. Arocha, J.L., Llano, B.: Mean value for the matching and dominating polynomials. Discussiones Mathematicae Graph Theory **20**, 57–69 (2000)
7. Arratia, R., Bollobás, B., Sorkin, G.B.: The Interlace Polynomial: A New Graph Polynomial. IBM Thomas J. Watson Research Division, New York (2000)
8. Arratia, R., Bollobás, B., Sorkin, G.B.: A two-variable interlace polynomial. Combinatorica **24**, 567–584 (2004)
9. Averbouch, I.: Completeness and Universality Properties of Graph Invariants and Graph Polynomials. Ph.D. Thesis. Technion - Israel Institute of Technology, Haifa, Israel, 2011
10. Averbouch, I., Godlin, B., Makowsky, J.A.: A most general edge elimination polynomial. International Workshop on Graph-Theoretic Concepts in Computer Science, pp. 31–42. Springer, Berlin (2008)
11. Averbouch, I., Godlin, B., Makowsky, J.A.: An extension of the bivariate chromatic polynomial. Eur. J. Comb. **31**(1), 1–17 (2010)
12. Averbouch, I., Kotek, T., Makowsky, J.A., Ravve, E.: The universal edge elimination polynomial and the dichromatic polynomial. Electronic Notes Discrete Math. **38**, 77–82 (2011)
13. Barton, C., Brown, J.I., Pike, D.A.: Acyclic polynomials of graphs. Aust. J. Comb. **82**(2), 146–181 (2022)
14. Biggs, N.: Algebraic Graph Theory, vol. 67. Cambridge University Press, Cambridge (1993)
15. Birkhoff, G.D.: A determinant formula for the number of ways of coloring a map. Ann. Math. **14**(1/4), 42–46 (1912)
16. Bollobás, B.: Modern Graph Theory, vol. 184. Springer, Berlin (2013)

17. Bollobás, B., Riordan, O.: Polychromatic polynomials. Discrete Math. **219**(1–3), 1–7 (2000)
18. Bollobás, B., Riordan, O.: A polynomial invariant of graphs on orientable surfaces. Proc. Lond. Math. Soc. **83**(3), 513–531 (2001)
19. Bondy, J.A., Murty, U.S.R.: Graph Theory. Springer, Berlin (2008)
20. Bouchet, A.: Graph polynomials derived from Tutte–Martin polynomials. Discrete Math. **302**(1–3), 32–38 (2005)
21. Brylawski, T.H.: A combinatorial model for series-parallel networks. Trans. Am. Math. Soc. **154**, 1–22 (1971)
22. Chen, X.: Digraph Polynomials for Counting Cycles and Paths. Preprint (2017). arXiv:1712.00686
23. Chen, X.: Polynomials for Counting of Cycles and Paths. Bachelor Thesis. Mittweida University of Applied Sciences, 2017
24. Chun, C., Moffatt, I., Noble, S.D., Rueckriemen, R.: Matroids, delta-matroids and embedded graphs. J. Comb. Theory Series A **167**, 7–59 (2019)
25. Courcelle, B.: A multivariate interlace polynomial and its computation for graphs of bounded clique-width. Electron. J. Comb. R69–R69 (2008)
26. Doyle, P.G., Laurie Snell, J.: Random Walks and Electric Networks, vol. 22. American Mathematical Society, Providence (1984)
27. Duffin, R.J.: Topology of series-parallel networks. J. Math. Anal. Appl. **10**(2), 303–318 (1965)
28. Ellis-Monaghan, J.A., Merino, C.: Graph polynomials and their applications I: The Tutte polynomial. In: Structural Analysis of Complex Networks, pp. 219–255. Springer, Berlin (2011)
29. Ellis-Monaghan, J.A., Moffatt, I.: Graphs on Surfaces: Dualities, Polynomials, and Knots, vol. 84. Springer, Berlin (2013)
30. Ellis-Monaghan, J.A., Moffatt, I.: Handbook of the Tutte Polynomial and Related Topics. CRC Press, Boca Raton (2022)
31. Farrell, E.J.: An introduction to matching polynomials. J. Comb. Theory Series B **27**(1), 75–86 (1979)
32. Fischer, E., Makowsky, J.A.: Linear recurrence relations for graph polynomials. Pillars of Computer Science: Essays Dedicated to Boris (Boaz) Trakhtenbrot on the Occasion of His 85th Birthday, pp. 266–279 (2008)
33. Fortuin, C.M., Kasteleyn, P.W.: On the random-cluster model: I. Introduction and relation to other models. Physica **57**(4), 536–564 (1972)
34. Godlin, B., Katz, E., Makowsky, J.A.: Graph polynomials: From recursive definitions to subset expansion formulas. J. Logic Comput **22**(2), 237–265 (2012)
35. Godsil, C., Royle, G.F.: Algebraic Graph Theory, vol. 207. Springer Science & Business Media, Berlin (2013)
36. Goodall, A., Hermann, M., Kotek, T., Makowsky, J.A., Noble, S.D.: On the complexity of generalized chromatic polynomials. Adv. Appl. Math. **94**, 71–102 (2018)
37. Grimmett, G.: The Random-cluster Model, vol. 333. Springer, Berlin (2006)
38. Gutman, I., Harary, F.: Generalizations of the matching polynomial. Utilitas Math **24**(1), 97–106 (1983)
39. Heinrich, I., Tittmann, P.: Neighborhood and domination polynomials of graphs. Graphs Comb. **34**(6), 1203–1216 (2018)
40. Hoede, C., Li, X.: Clique polynomials and independent set polynomials of graphs. Discrete Math. **125**(1–3), 219–228 (1994)
41. Kikuno, T., Yoshida, N., Kakuda, Y.: A linear algorithm for the domination number of a series-parallel graph. Discrete Appl. Math. **5**(3), 299–311 (1983)
42. Kotek, T., Makowsky, J.A., Rave, E.V.: On sequences of polynomials arising from graph invariants. Eur. J. Comb. **67**, 181–198 (2018)
43. Kotek, T., Preen, J., Simon, F., Tittmann, P., Trinks, M.: Recurrence relations and splitting formulas for the domination polynomial. Electron. J. Comb. **19**(3), P47 (2012)
44. Kotek, T., Preen, J., Tittmann, P.: Subset-sum representations of domination polynomials. Graphs Comb. **30**(3), 647–660 (2014)