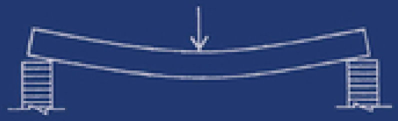

PARKER/AMBROSE SERIES OF SIMPLIFIED DESIGN GUIDES

Thirteenth Edition

SIMPLIFIED ENGINEERING
for ARCHITECTS AND BUILDERS

JAMES E. AMBROSE
PATRICK TRIPENY
SHARON S. BAUM KUSKA

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PREFACE TO THE THIRTEENTH EDITION

This book focuses on the design of structures for buildings. As with previous editions, the material in this book has been prepared for those without formal training in engineering. Mathematical work is limited mostly to simple algebra. It is particularly well suited for programs in architecture and building construction.

However, as most programs in engineering offer limited opportunity for study of the general fields of building planning and construction, this book can serve as a valuable supplement to engineering texts. It emphasizes the development of practical design, which typically involves minimal effort in structural investigation and significant consideration for circumstantial situations relating to the existence of the building structure. Readers should be better able to converse intelligently with the engineering professional and understand the reasoning behind suggestions being offered.

Changes that occur in reference sources and in design and construction practices make it necessary for periodic updates of the material in this book. This edition has been revised accordingly, although the reader should note that these changes are ongoing, so it is inevitable that some of the material presented here will be outdated within a short time. However, this work concentrates on fundamental concepts and processes of investigation and design; thus, the use of specific data is less critical to the learning of the fundamental material. For actual design work, current references, listed in the reference section of this book, should be consulted.

In addition to updating, each new edition affords an opportunity to reconsider the organization, presentation, and scope of the material contained in the book. This new edition features minor alterations to the basic content of previous editions, although just about everything contained in the previous edition has been retained. Some trimming has occurred, largely to add new material without significantly increasing the size of the book.

One notable change in this edition is a decrease in the amount of information provided for the load and resistance factor design (LRFD) method for wood structures to lower the confusion level of the reader. Because the allowable stress design (ASD) method is still the preferred method used by designers, it remains standard in this edition. Also, this edition provides information on topics such as the thermal effects in axially loaded members, wood decking and wood columns, nailed connections, and steel column base plates. Mass timber is introduced but is still regionally and manufacturer dependent.

To support instructors using this book as a course text, an accompanying website contains complete answers to all the additional problems presented in the text. This website also includes suggestions on teaching and learning strategies, along with potential course curriculum models.

There is also a student website that provides study material to assist in learning this complex material. It is available to anyone using this text, and we encourage all students to take advantage of this resource. Solutions to alternate problems are available for student usage.

For text demonstrations, as well as for the exercise problems, we include some data sources; I extend my gratitude to various industry organizations for their permission to use excerpts from these data sources, with proper acknowledgment provided.

As the author of this edition and as a representative of the academic and professional communities, I want to thank John Wiley & Sons for their continued publication of this widely utilized reference source. I am truly thankful for the support provided by the Wiley editors and production staff.

Finally, I would like to express my appreciation to my family. Writing, especially when added to an already full-time occupation, is very time-consuming. I am grateful for the patience, support, and encouragement of my spouse, children and grandchildren which has made this work possible.

Sharon S. Baum Kuska

PREFACE TO THE FIRST EDITION

(The following is an excerpt from Professor Parker's preface to the first edition.)

To the average young architectural draftsman or builder, the problem of selecting the proper structural member for given conditions appears to be a difficult task. Most of the numerous books on engineering which are available assume that the reader has previously acquired a knowledge of fundamental principles and, thus, are almost useless to the beginner. It is true that some engineering problems are exceedingly difficult, but it is also true that many of the problems that occur so frequently are surprisingly simple in their solution. With this in mind, and with a consciousness of the seeming difficulties in solving structural problems, this book has been written.

To understand the discussions of engineering problems, it is essential that the student have a thorough knowledge of the various terms which are employed. In addition, basic principles of forces in equilibrium must be understood. The first section of this book, "Principles of Mechanics," is presented for those who wish a brief review of the subject. Following this section are structural problems involving the most commonly used building materials, wood, steel, reinforced concrete, and roof trusses. A major portion of the book is devoted to numerous problems and their solution, the purpose of which is to explain practical procedure in the design of structural members. Similar examples are given to be solved by the student. Although handbooks published by the manufacturers are necessities to the more advanced student, a great number of appropriate tables are presented herewith so that sufficient data are directly at hand to those using this book.

Care has been taken to avoid the use of advanced mathematics, a knowledge of arithmetic and high school algebra being all that is required to follow the discussions presented. The usual formulas employed in the solution of structural problems are given with explanations of the terms involved and their application, but only the most elementary of these formulas are derived. These derivations are given to show how simple they are and how the underlying principle involved is used in building up a formula that has practical application.

No attempt has been made to introduce new methods of calculation, nor have all the various methods been included. It has been the desire of the author to present to those having little or no knowledge of the subject simple solutions of everyday problems. Whereas thorough technical training is to be desired, it is hoped that this presentation of fundamentals will provide valuable working knowledge and, perhaps, open the doors to more advanced study.

Harry Parker
Philadelphia, Pennsylvania
March, 1938

ABOUT THE COMPANION WEBSITE

This book is accompanied by a companion website:

www.wiley.com/go/SimplifiedEngineering13e

- The website features an instructor's manual and a student study guide.
- The instructor's manual contains examples of teaching materials relevant to the textbook content. Course models and scenarios of required technology courses on structures within a non-engineering curriculum are presented, highlighting teaching challenges and opportunities drawn from experiences.
- Additionally, a companion study guide for students serves as a resource for the topic of building structures, offering illustrated solutions to exercise problems to enhance their general understanding of the subject matter.

INTRODUCTION

The principal purpose of this book is to develop the topic of *structural design*. However, to do the necessary work for design, use must be made of various methods of *structural investigation*. The work of investigation consists of the consideration of the tasks required of a structure and the evaluation of the responses of the structure in performing these tasks. Investigation may be performed in various ways, the principal ones being the use of modeling by either mathematics or the construction of physical models. For the designer, a major first step in any investigation is the visualization of the structure and the force actions to which it must respond. In this book, extensive use is made of graphic illustrations in order to encourage the reader in the development of the habit of first clearly *seeing* what is happening, before proceeding with the essentially abstract procedures of mathematical investigation. When working a problem within the book or in practice, the reader is encouraged to begin by drawing an illustration of the problem while identifying the key information that has been provided.

Structural Mechanics

The branch of physics called *mechanics* concerns the actions of forces on physical bodies. Most of engineering design and investigation is based on applications of the science of mechanics. *Statics* is the branch of mechanics that deals with bodies held in a state of unchanging motion by the balanced nature (called *static equilibrium*) of the forces acting on them. *Dynamics* is the branch of mechanics that concerns bodies in motion or in a process of change of shape due to actions of forces. A static condition is essentially unchanging with regard to time; a dynamic condition implies a time-dependent action and response.

When external forces act on a body, two things happen. First, internal forces that resist the actions of the external forces are set up in the body. These internal forces produce *stresses* in the material of the body. Second, the external forces produce *deformations*, or changes in shape, of the body. *Strength of materials*, or *mechanics of materials*, is the study of the properties of material bodies that enable them to resist the actions of external forces, of the stresses within the bodies, and of the deformations of bodies that result from external forces.

Taken together, the topics of applied mechanics and strength of materials are often given the overall designation of *structural mechanics* or *structural analysis*. This is the fundamental basis for structural investigation, which is essentially an analytical process. On the other hand, *design* is a progressive refining process in which a structure is first visualized; then it is investigated for required force responses and its performance is evaluated. Finally—possibly after several cycles of investigation and modification—an acceptable form is derived for the structure.

Units of Measurement

Early editions of this book used U.S. units (feet, inches, pounds, etc.) with equivalent SI (Standard International—aka metric) units in brackets for the basic presentation. In this edition, the basic work is developed with U.S. units only. While the building industry in the United States is now in the slow process of changing to SI units, our decision for the presentation here is a pragmatic one. Most of the references used for this book are still developed primarily in U.S. units and most readers educated in the United States use U.S. units as their first language, even if they now also use SI units.

Table I.1 lists the standard units of measurement in the U.S. system with the abbreviations used in this work and a description of common usage in structural design work. In similar form, Table I.2 gives the corresponding units in the SI system. Conversion factors to be used for shifting from one unit system to the other are given in Table I.3.

Direct use of the conversion factors will produce what is called a *hard conversion* of a reasonably precise form. Even though all of the work done in this book with be in U.S. units, the tables with SI units are given as a handy reference to readers who may be using reference books in SI units or using both systems.

Accuracy of Computations

Structures for buildings are seldom produced with a high degree of dimensional precision. Exact dimensions are difficult to achieve, even for the most diligent of workers and builders. Add this to considerations for the lack of precision in predicting loads for any structure, and the significance of highly precise structural computations becomes moot. This is not to be used for an argument to justify sloppy mathematical work, overly sloppy construction, or use of vague theories

TABLE I.1 Units of Measurement: U.S. System

Name of Unit	Abbreviation	Use in Building Design
Length		
Foot	ft	Large dimensions, building plans, beam spans
Inch	in.	Small dimensions, size of member cross sections
Area		
Square feet	ft^2	Large areas
Square inches	$in.^2$	Small areas, properties of cross sections
Volume		
Cubic yards	yd^3	Large volumes, of soil or concrete (commonly called simply "yards")
Cubic feet	ft^3	Quantities of materials
Cubic inches	$in.^3$	Small volumes
Force, Mass		
Pound	lb	Specific weight, force, load
Kip	kip, k	1000 pounds
Ton	ton	2000 pounds
Pounds per foot	lb/ft, plf	Linear load (as on a beam)
Kips per foot	kips/ft, klf	Linear load (as on a beam)
Pounds per square foot	lb/ft^2, psf	Distributed load on a surface, pressure
Kips per square foot	k/ft^2, ksf	Distributed load on a surface, pressure
Pounds per cubic foot	lb/ft^3	Relative density, unit weight
Moment		
Foot-pounds	ft-lb	Rotational or bending moment
Inch-pounds	in.-lb	Rotational or bending moment
Kip-feet	kip-ft	Rotational or bending moment
Kip-inches	kip-in.	Rotational or bending moment
Stress		
Pounds per square foot	lb/ft^2, psf	Soil pressure
Pounds per square inch	$lb/in.^2$, psi	Stresses in structures
Kips per square foot	$kips/ft^2$, ksf	Soil pressure
Kips per square inch	$kips/in.^2$, ksi	Stresses in structures
Temperature		
Degree Fahrenheit	°F	Temperature

TABLE I.2 Units of Measurement: SI System

Name of Unit	Abbreviation	Use in Building Design
Length		
Meter	m	Large dimensions, building plans, beam spans
Millimeter	mm	Small dimensions, size of member cross sections
Area		
Square meters	m^2	Large areas
Square millimeters	mm^2	Small areas, properties of member cross sections
Volume		
Cubic meters	m^3	Large volumes
Cubic millimeters	mm^3	Small volumes
Mass		
Kilogram	kg	Mass of material (equivalent to weight in U.S. units)
Kilograms per cubic meter	kg/m^3	Density (unit weight)
Force, Load		
Newton	N	Force or load on structure
Kilonewton	kN	1000 newtons
Stress		
Pascal	Pa	Stress or pressure (1 pascal = 1 N/m^2)
Kilopascal	kPa	1000 pascals
Megapascal	MPa	1,000,000 pascals
Gigapascal	GPa	1,000,000,000 pascals
Temperature		
Degree Celsius	°C	Temperature

TABLE I.3 Factors for Conversion of Units

To Convert from U.S. Units to SI Units, Multiply by:	U.S. Unit	SI Unit	To Convert from SI Units to U.S. Units, Multiply by:
25.4	in.	mm	0.03937
0.3048	ft	m	3.281
645.2	$in.^2$	mm^2	1.550×10^{-3}
16.39×10^3	$in.^3$	mm^3	61.02×10^{-6}
416.2×10^3	$in.^4$	mm^4	2.403×10^{-6}
0.09290	ft^2	m^2	10.76
0.02832	ft^3	m^3	35.31
0.4536	lb (mass)	kg	2.205
4.448	lb (force)	N	0.2248
4.448	kip (force)	kN	0.2248
1.356	ft-lb (moment)	N-m	0.7376
1.356	kip-ft (moment)	kN-m	0.7376
16.0185	lb/ft^3 (density)	kg/m^3	0.06243
14.59	lb/ft (load)	N/m	0.06853
14.59	kips/ft (load)	kN/m	0.06853
6.895	psi (stress)	kPa	0.1450
6.895	ksi (stress)	MPa	0.1450
0.04788	psf (load or pressure)	kPa	20.93
47.88	ksf (load or pressure)	kPa	0.02093
$0.566 \times (°F - 32)$	°F	°C	$(1.8 \times °C) + 32$

of investigation of behaviors. Nevertheless, it makes a case for not being highly concerned with any numbers beyond three significant digits.

While most professional design work these days is likely to be done with computer support, most of the work illustrated here is quite simple and was actually performed with a hand calculator (the 8-digit, scientific type is adequate). Rounding off of these computations is done with no apologies. However, sometimes the “three significant digit” practice appears inadequate and more accuracy is justified—its all dependent on the situation.

With the use of the computer, accuracy of computational work is a somewhat different matter. Still, it is the designer (a person) who makes judgments based on the computations and who knows how good the input to the computer was and what the real significance of the degree of accuracy of an answer is.

Symbols

The following shorthand symbols are frequently used.

Symbol	Reading
$>$	Is greater than
$<$	Is less than
$\geq$	Equal to or greater than
$\leq$	Equal to or less than
$\sum$	The sum of
ΔL	Change in L

Standard Notation

Notation used in this book complies generally with that used in the building design field. A general attempt has been made to conform to usage in the reference standards commonly used by structural designers. The following list includes all of the notation used in this book that is general and is related to the topic of the book. Specialized notation is used by various groups, especially as related to individual materials: wood, steel, masonry, concrete, and so on. The reader is referred to basic references for notation in special fields. Some of this notation is explained in later parts of this book.

Building codes use special notation that is usually carefully defined by the code, and the reader is referred to the source for interpretation of these definitions. When used in demonstrations of computations, such notation is explained in the text of this book.

A = unit area
A_g = gross area of a section, defined by the outer dimensions
A_n = net area
C = compressive force
e = eccentricity of a nonaxial load, from point of application of the load to the centroid of the section
E = modulus of elasticity
f = actual computed stress
F = allowable stress
h = effective height (usually meaning unbraced height) of a wall or column
I = moment of inertia
L = length (usually of a span); uppercase L indicates units of ft, lowercase l is units of in.
M = bending moment
P = concentrated load
s = spacing, center to center
S = elastic section modulus
T = tension force
W = (1) total gravity load; (2) weight, or dead load of an object; (3) total wind load force; (4) total of a uniformly distributed load or pressure due to gravity
w = unit of weight or other uniformly distributed load per unit length of member
W = unit of weight or other total uniformly distributed load (note: usually $W = wL$ or wl)
Z = plastic section modulus

I

FUNDAMENTAL FUNCTIONS OF STRUCTURES

This Part I presents various considerations regarding the general nature and performance of structures for buildings. A major part of this material consists of basic concepts and applications from the field of applied mechanics as they have evolved in the process of investigation of the behavior of structures. The purpose of studying this material is twofold: first, the general need for a comprehensive understanding of what structures do and how they do it; second, the need for some factual, quantified basis for the exercise of judgment in the process of structural design. If it is accepted that the understanding of a problem is the necessary first step in its solution, this analytical study should be seen as the cornerstone of any successful, informed design process.

A second major concern is for the sources of the tasks that structures must undertake, that is, for what structures are basically needed. These tasks are defined in terms of the loads that are applied to structures. Considerations in this regard include the load sources, the manner of their application, the combinations in which they occur, and the quantification of their specific values.

Finally, consideration must be given to the possible forms and materials of structures. These concerns affect the determination of the construction of the structures, but they also relate to the general design development of the buildings.

This part provides an introduction to the basic components of structural analysis and design; the remaining parts of the book present amplifications of the issues raised in this part.

Simplified Engineering for Architects and Builders, Thirteenth Edition. James E. Ambrose, Patrick Tripeny, and Sharon S. Baum Kuska.

Companion Website: www.wiley.com/go/SimplifiedEngineering13e

1

INVESTIGATION OF FORCES, FORCE SYSTEMS, LOADING, AND REACTIONS

Structural systems are designed to safely respond to the forces to which they are subjected. The forces are determined by the magnitude of the loads and the way they are applied to the structure. These loads include gravity loads based on the weight of building materials, the use or occupancy of the structure, and environmental loads. The purpose of the structure involves the transmission of the load forces to the supports for the structure. The external loads and support forces produce resistance from the structure in terms of internal forces that resist changes in the shape of the structure. This chapter treats the basic properties and actions of forces.

1.1 PROPERTIES OF FORCES

Force is a fundamental concept of mechanics but does not yield to simple definition. An accepted concept is that a *force* is an effort that tends to change the form or the state of motion of a physical object. Mechanical force was defined by Isaac Newton as being a product of mass and acceleration; that is, $F = ma$. Gravitational attraction is a form of acceleration, and thus weight—a force we experience—is defined as $W = mg$ with g being the acceleration of gravity. Physical objects have weight, but more precisely, they have mass and will thus have *different* weights when they experience different gravitational effects, for example, on the surface of Earth or on the surface of the moon.

In the United States, aka imperial, unit of gravity force is quantified as the weight of the body. Gravity forces are thus measured in pounds (lb) or in some other unit such as tons (T) or kips (1 kilopound, or 1000 pounds). In the SI (International System, aka metric) system, force is measured in a more scientific manner related to the mass of objects, the mass of an object being a constant, whereas weight is proportional to the precise value of the acceleration of gravity, which varies from place to place. Force in metric units is measured in newtons (N), kilonewtons (kN), or meganewtons (mN), whereas weight is measured in grams (g) or kilograms (kg).

In structural engineering work, forces are described as *loads*. Loads derive from various sources, including gravity, and are dealt with in terms of their application to a given structure. Thus, the gravity load on a beam begins with the weight of the beam itself and goes on to include the weight of everything else supported by the beam.

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Vectors

A quantity that involves magnitude, line of action (e.g., vertical), and sense (up, down, etc.) is a vector quantity, whereas a scalar quantity involves only magnitude and sense. Force, velocity, and acceleration are vector quantities, while energy, time, and temperature are scalar quantities. A vector can be represented by a straight line, leading to the possibility of constructed graphical solutions in some cases, a situation that will be demonstrated later. Mathematically, a scalar quantity can be represented completely as +50 or −50, while a vector must somehow have its line of action represented as well (50 vertical, horizontal, etc.).

Properties of Forces

In order to completely identify a force, it is necessary to establish the following:

Magnitude, or the amount of the force, measured in weight units such as pounds or kips.

Line of Action of the force, refers to the orientation of its path, usually described by the angle that the line of action makes with some reference, such as the horizontal.

Sense of the force, refers to the manner in which it acts along its line of action (e.g., up or down, right or left). Sense is usually expressed algebraically in terms of the sign of the force, either plus or minus.

Forces can be represented graphically in terms of these three properties by the use of an arrow, as shown in Figure 1.1*a*. Drawn to some scale, the length of the arrow represents the magnitude of the force.

The angle of inclination of the arrow represents the direction of the force. The location of the arrowhead represents the sense of the force. This form of representation can be more than merely symbolic, since actual mathematical

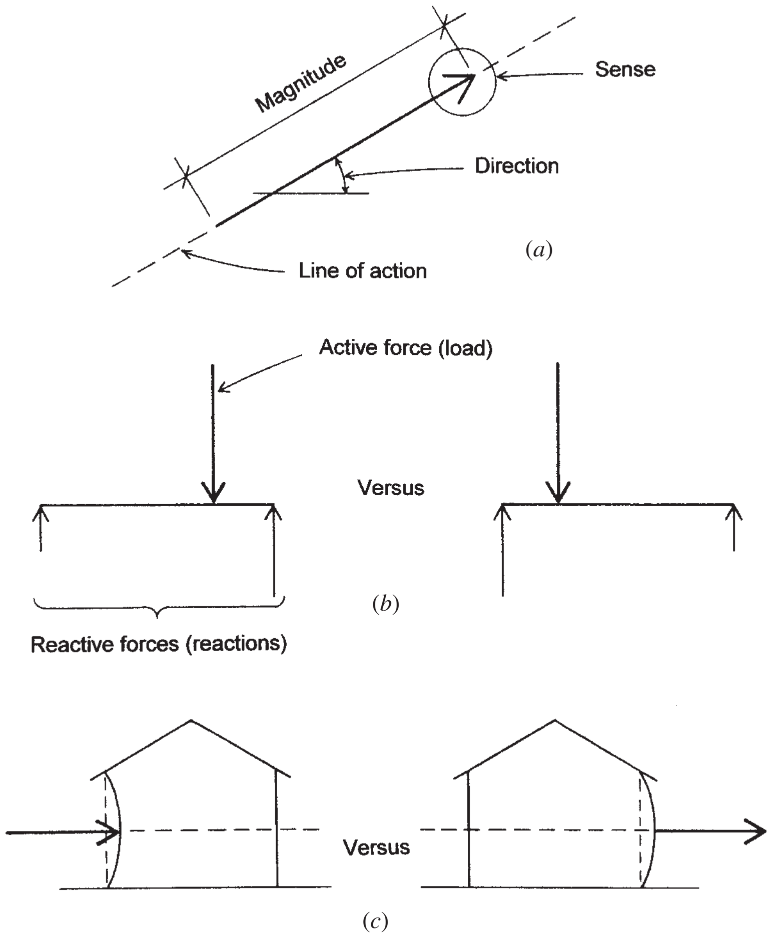


Figure 1.1 Representation of forces and force actions.

manipulations may be performed using the vector representation that the force arrows constitute. In the work in this book, arrows are used in a symbolic way for visual reference when performing algebraic computations and in a truly representative way when performing graphical analyses.In addition to the basic properties of magnitude, line of action, and sense, some other concerns that may be significant for certain investigations are:

The *position of the line of action* of the force with respect to the lines of action of other forces or to some object on which the force operates, as shown in Figure 1.1*b*. For the beam, shifting the location of the load (active force) affects changes in the forces at the supports (reactions).

The *point of application* of the force along its line of action may be of concern in analyzing the specific effect of the force on a structure, as shown in Figure 1.1*c*.

When forces are not resisted, they tend to produce motion. An inherent aspect of static forces is that they exist in a state of *static equilibrium*, that is, with no motion occurring. In order for static equilibrium to exist, it is necessary to have a balanced system of forces. An important consideration in the analysis of static forces is the nature of the geometric arrangement of forces in a given set of forces that constitute a single system. The usual technique for classifying force systems involves consideration of whether the forces in the system are:

Coplanar. All acting in a single plane, such as the plane of a vertical wall.

Parallel. All having the same direction though not along the same line of action.

Concurrent. All having their lines of action intersect at a common point.

Using these three considerations, the possible variations are given in Table 1.1 and illustrated in Figure 1.2.

It is necessary to qualify a set of forces in the manner just illustrated before proceeding with any analysis, whether it is to be performed algebraically or graphically.

TABLE 1.1 Classification of Force Systems[a]

		Qualifications	
System Variation	Coplanar	Parallel	Concurrent
1	Yes	Yes	Yes
2	Yes	Yes	No
3	Yes	No	Yes
4	Yes	No	No
5	No	Yes	No
6	No	No	Yes
7	No	No	No

[a]See Figure 1.2.

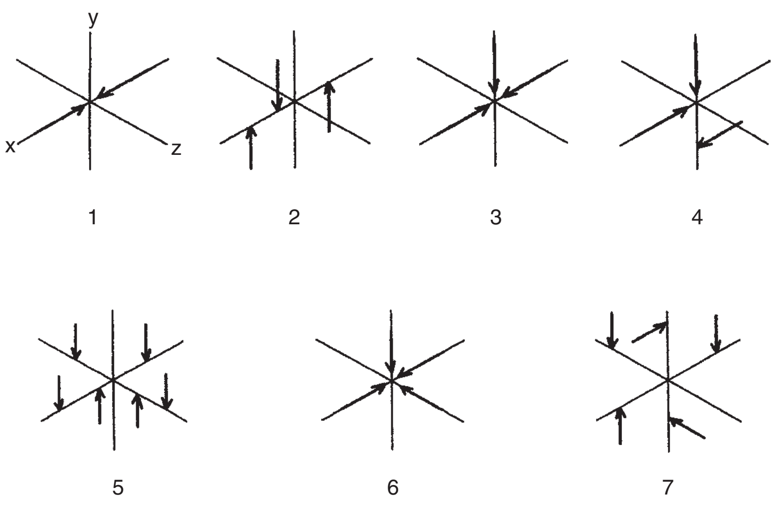


Figure 1.2 Types of force systems.

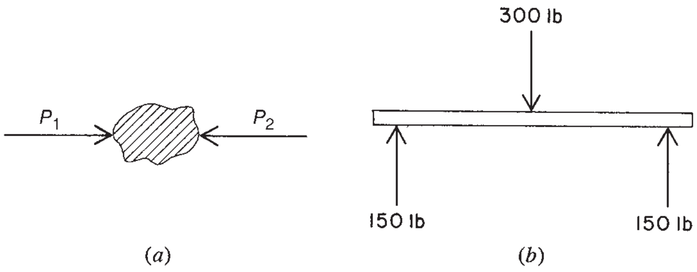


Figure 1.3 Equilibrium of forces.

1.2 STATIC EQUILIBRIUM

As stated previously, an object is in *equilibrium* when it either is at rest or has uniform motion. When a system of forces acting on an object produces no motion, the system of forces is said to be in *static equilibrium*.

A simple example of equilibrium is illustrated in Figure 1.3*a*. Two equal, opposite, and parallel forces, having the same line of action, P_1 and P_2, act on a body. If the two forces balance each other, the body does not move and the system of forces is in equilibrium. These two forces are *concurrent*. If the lines of action of a system of forces have a point in common, the forces are concurrent.

Another example of forces in equilibrium is illustrated in Figure 1.3*b*. A vertical downward force of 300 lb acts at the midpoint in the length of a beam. The two upward vertical forces of 150 lb each (the reactions) act at the ends of the beam. The system of three forces is in equilibrium. The forces are parallel and, not having a point in common, are *nonconcurrent*.

1.3 FORCE COMPONENTS AND COMBINATIONS

Individual forces may interact and be combined with other forces in various situations. Conversely, a single force may have more than one effect on an object, such as a vertical action and a horizontal action simultaneously. This section considers both of these issues: adding up of forces (combination) and breaking down of single forces into components (resolution).

Resultant of Forces

The *resultant* of a system of forces is the simplest system (usually a single force) that has the same effect as the various forces in the system acting simultaneously. The lines of action of any system of two coplanar nonparallel forces must have a point in common, and the resultant of the two forces will pass through this common point. The resultant of two coplanar, nonparallel forces may be found graphically by constructing a *parallelogram of forces*.

To construct a parallelogram of two forces, the forces are drawn at any scale (so many pounds to the inch), with both forces pointing toward or both forces pointing away from the point of intersection of their lines of action. A parallelogram is then produced with the two forces as adjacent sides. The diagonal of the parallelogram passing through the common point is the resultant in magnitude, line of action, and sense, the direction of the resultant being similar to that of the given forces, toward or away from the point in common. In Figure 1.4*a*, P_1 and P_2 represent two nonparallel forces whose lines of action intersect at point O. The parallelogram is drawn, and the diagonal R is the resultant of the given system. In this illustration, note that the two forces point *away* from the point in common, and hence, the resultant also has its sense away from point O. It is a force upward to the right. Notice that the resultant of forces P_1 and P_2 shown in Figure 1.4*b* is R; its sense is toward the point in common.

Forces may be considered to act at any point on their lines of action. In Figure 1.4*c*, the lines of action of the two forces P_1 and P_2 are extended until they meet at point O. At this point, the parallelogram of forces is constructed, and R, the diagonal, is the resultant of forces P_1 and P_2. In determining the magnitude of the resultant, the scale used is the same scale used in drawing the given system of forces.

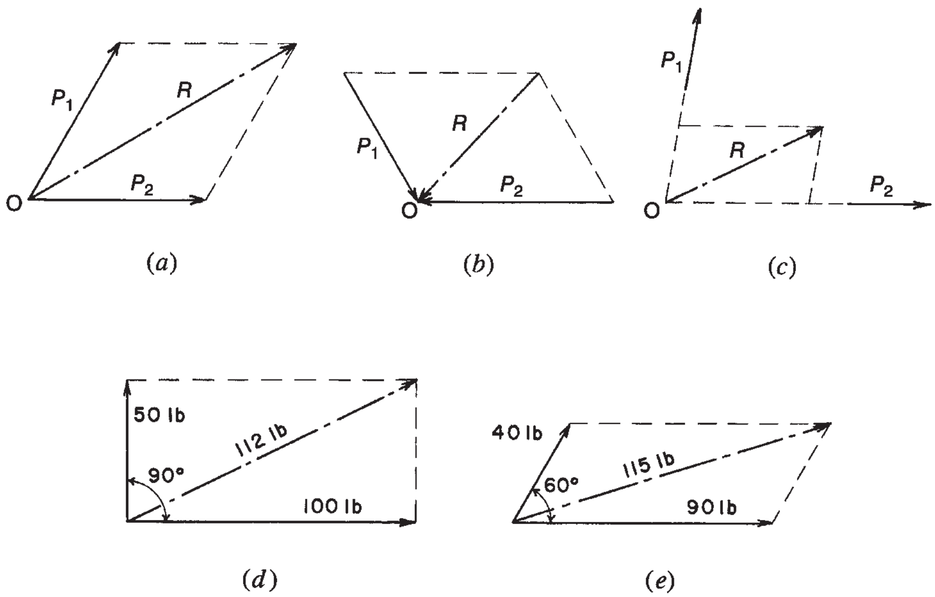


Figure 1.4 Consideration of the resultant of a set of forces.

Example 1. A vertical force of 50 lb and a horizontal force of 100 lb, as shown in Figure 1.4*d*, have an angle of 90° between their lines of action. Determine the resultant.

Solution: The two forces are laid off at a convenient scale from their point of intersection, the parallelogram is drawn, and the diagonal is the resultant. Its magnitude scales approximately 112 lb, its sense is upward to the right, and its line of action passes through the point of intersection of the lines of action of the two given forces. By use of a protractor it is found that the angle between the resultant and the force of 100 lb is approximately 26.5°.

Example 2. The angle between two forces of 40 and 90 lb, as shown in Figure 1.4*e*, is 60°. Determine the resultant.

Solution: The forces are laid off to scale from their point of intersection, the parallelogram of forces is constructed, and the resultant is found to be a force of approximately 115 lb. Its sense is upward to the right, and its line of action passes through the common point of the two given forces. The angle between the resultant and the force of 90 lb is approximately 17.5°.

Attention is called to the fact that these two problems have been solved graphically by the construction of diagrams. Mathematics might have been employed. For many practical problems, carefully constructed graphical solutions give sufficiently accurate answers and frequently require far less time. Do not make diagrams too small as greater accuracy is obtained by using larger parallelograms of forces.

Problems 1.3.A–F By constructing the parallelogram of forces, determine the resultants for the pairs of forces shown in Figures 1.5*a–f*.

Components of a Force

In addition to combining forces to obtain their resultant, it is often helpful to replace a single force with its *components*. The components of a force are the two or more forces that, acting together, have the same effect as the given force. In Figure 1.4*d*, if we are given the force of 112 lb, its vertical component is 50 lb and its horizontal component is 100 lb. That is, the 112 lb force has been *resolved* into its vertical and horizontal components. Any force may be considered as the resultant of its components.

Combined Resultants

The resultant of more than two nonparallel forces may be obtained by finding the resultants of pairs of forces and finally the resultant of the resultants.

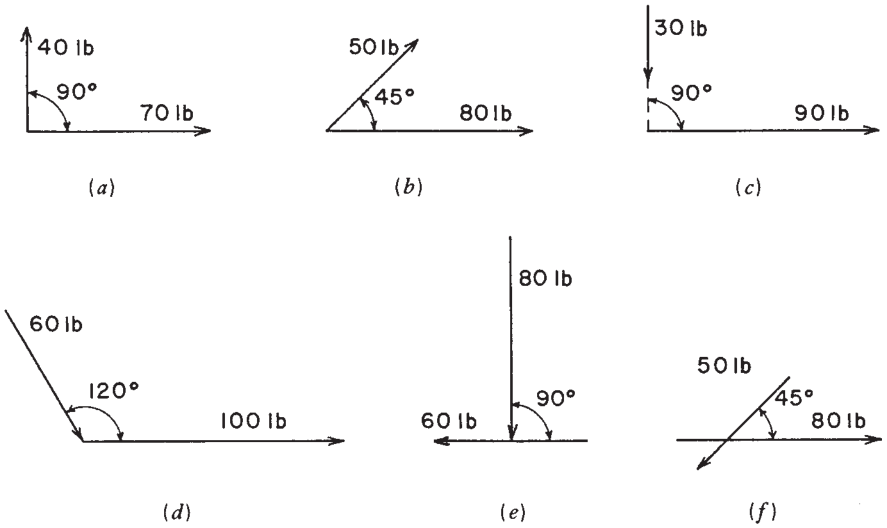


Figure 1.5 Reference for Problems 1.3.A–F, Part 1.

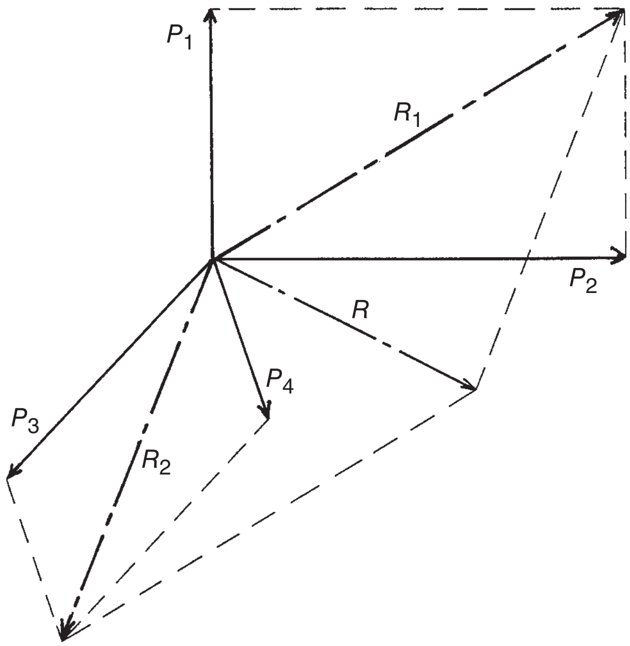


Figure 1.6 Finding a resultant by successive pairs.

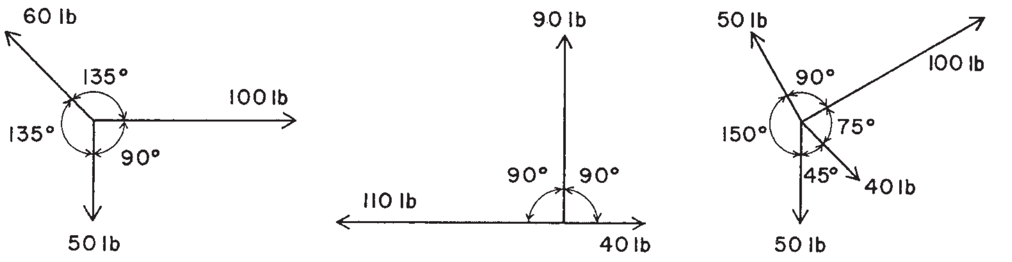
Example 3. Find the resultant of the concurrent forces P_1, P_2, P_3, and P_4 shown in Figure 1.6.

Solution: By constructing a parallelogram of forces, the resultant of P_1 and P_2 is found to be R_1. Similarly, the resultant of P_3 and P_4 is R_2. Finally, the resultant of R_1 and R_2 is R, the resultant of the four given forces.

Problems 1.3.G–I Using graphical methods, find the resultant of the systems of concurrent forces shown in Figure 1.7.

Equilibrant

The force, not a part of the system of forces, required to maintain a system of forces in equilibrium is called the *equilibrant* of the system. Suppose that we are required to investigate the system of two forces, P_1 and P_2, as shown in Figure 1.8. The parallelogram of forces is constructed, and the resultant is found to be R. The system is not in equilibrium. The force



(*a*) (*b*) (*c*)

Figure 1.7 Reference for Problems 1.3.G–I, Part 2.

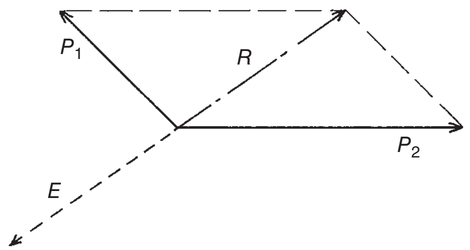


Figure 1.8 Finding an equilibrant.

required to maintain equilibrium is force E, shown by the dashed line. The equilibrant, E, is the same as the resultant in magnitude and line of action but is opposite in sense. Simply put, the equilibrant is a force equal and opposite of the resultant. The three forces, P_1, P_2, and E, constitute a system in equilibrium.

If two forces are in equilibrium, they must be equal in magnitude and opposite in sense and have the same direction and line of action. Either of the two forces may be said to be the equilibrant of the other. The resultant of a system of forces in equilibrium is zero.

1.4 GRAPHICAL ANALYSIS OF CONCURRENT FORCE SYSTEMS

Force Polygon

The resultant of a system of concurrent forces may be found by constructing a *force polygon*. To draw the force polygon, begin with a point and lay off, at a convenient scale, a line parallel to one of the forces, with its length equal to the force in magnitude, and having the same sense. From the termination of this line draw similarly another line corresponding to one of the remaining forces and continue in the same manner until all the forces in the given system are accounted for. If the polygon does not close, the system of forces is not in equilibrium, and the line required to close the polygon *drawn from the starting point* is the resultant in magnitude and direction. If the forces in the given system are concurrent, the line of action of the resultant passes through the point they have in common.

If the force polygon for a system of concurrent forces closes, the system is in equilibrium and the resultant is zero.

Example 4. Let it be required to find the resultant of the four concurrent forces P_1, P_2, P_3, and P_4 shown in Figure 1.9*a*. This diagram is called the *space diagram*; it shows the relative positions of the forces in a given system.

Solution: Beginning with some point such as O, shown in Figure 1.9*b*, draw the upward force P_1. At the upper extremity of the line representing P_1, draw P_2, continuing in a like manner with P_3 and P_4. The polygon does not close; therefore, the system is not in equilibrium. The resultant R, shown by the dot-and-dash line, is the resultant of the given system. Note that its sense is *from* the starting point O, downward to the right. The line of action of the resultant of the given system shown in Figure 1.9*a* has its line of action passing through the point they have in common; its magnitude and direction having been found in the force polygon.

In drawing the force polygon, the forces may be taken in any sequence. In Figure 1.9*c*, a different sequence is taken, but the resultant R is found to have the same magnitude and direction as previously found in Figure 1.9*b*.

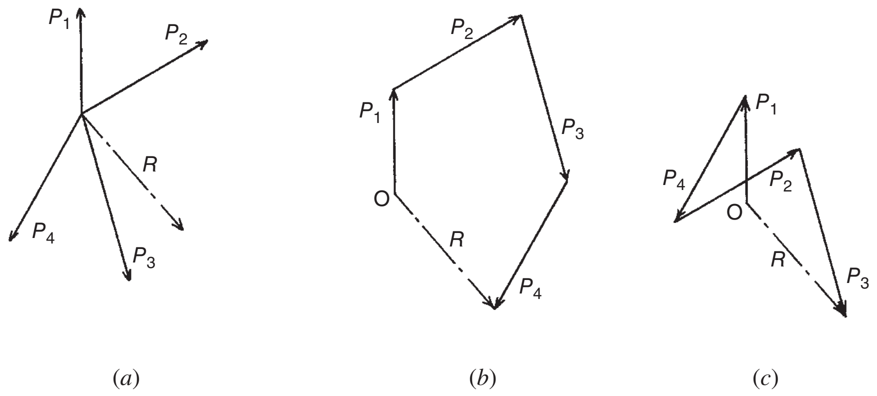


Figure 1.9 Finding a resultant by continuous vector addition of forces.

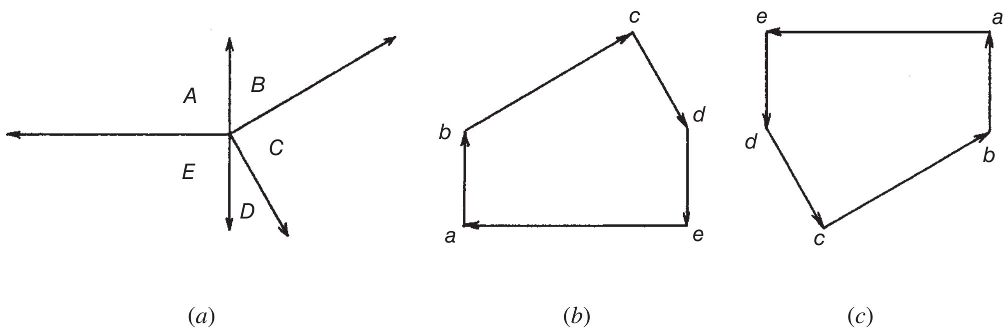


Figure 1.10 Construction of a force polygon.

Bow's Notation

Thus far, forces have been identified by the symbols P_1, P_2, and so on. A system of identifying forces, known as Bow's notation, offers many advantages. In this system, letters are placed in the space diagram on each side of a force and a force is identified by two letters. The sequence in which the letters are read is important. Figure 1.10*a* shows the space diagram of five concurrent forces. Reading about the point in common *in a clockwise manner* the forces are *AB, BC, CD, DE*, and *EA*. When a force in the force polygon is represented by a line, a letter is placed at each end of the line. As an example, the vertical upward force in Figure 1.10*a* is read *AB* (note that this is read clockwise about the common point); in the force polygon (Figure 1.10*b*), the letter *a* is placed at the bottom of the line representing the force *AB* and the letter *b* is at the top. Use capital letters to identify the forces in the space diagrams and lowercase letters in the force polygon. From point *b* in the force polygon, draw force *bc*, then *cd*, and continue with *de* and *ea*. Since the force polygon closes, the five concurrent forces are in equilibrium.

In reading forces, a clockwise manner is used in all the following discussions. It is important that this method of identifying forces be thoroughly understood. To make this clear, suppose that a force polygon is drawn for the five forces shown in Figure 1.10*a*, reading the forces in sequence in a counterclockwise manner. This will produce the force polygon shown in Figure 1.10*c*. Either method may be used, but for consistency, the method of reading clockwise is used here.

Use of the Force Polygon

Two ropes are attached to a ceiling and their lower ends are connected to a ring, making the arrangement shown in Figure 1.11*a*. A weight of 100 lb is suspended from the ring. Obviously, the force in rope *AB* is 100 lb, but the magnitudes of the forces in ropes *BC* and *CA* are unknown.

The forces in the ropes *AB*, *BC*, and *CA* constitute a concurrent force system in equilibrium (Figure 1.11*b*). The magnitude of only one of the forces is known—it is 100 lb in rope *AB*. Since the three concurrent forces are in equilibrium, their force polygon must close, and this fact makes it possible to find their magnitudes. Now, at a convenient scale, draw the line *ab* (Figure 1.11*c*) representing the downward force *AB*, 100 lb. The line *ab* is one side of the force polygon. From point *b*, draw a line parallel to rope *BC*; point *c* will be at some location on this line. Next, draw a line through point *a*