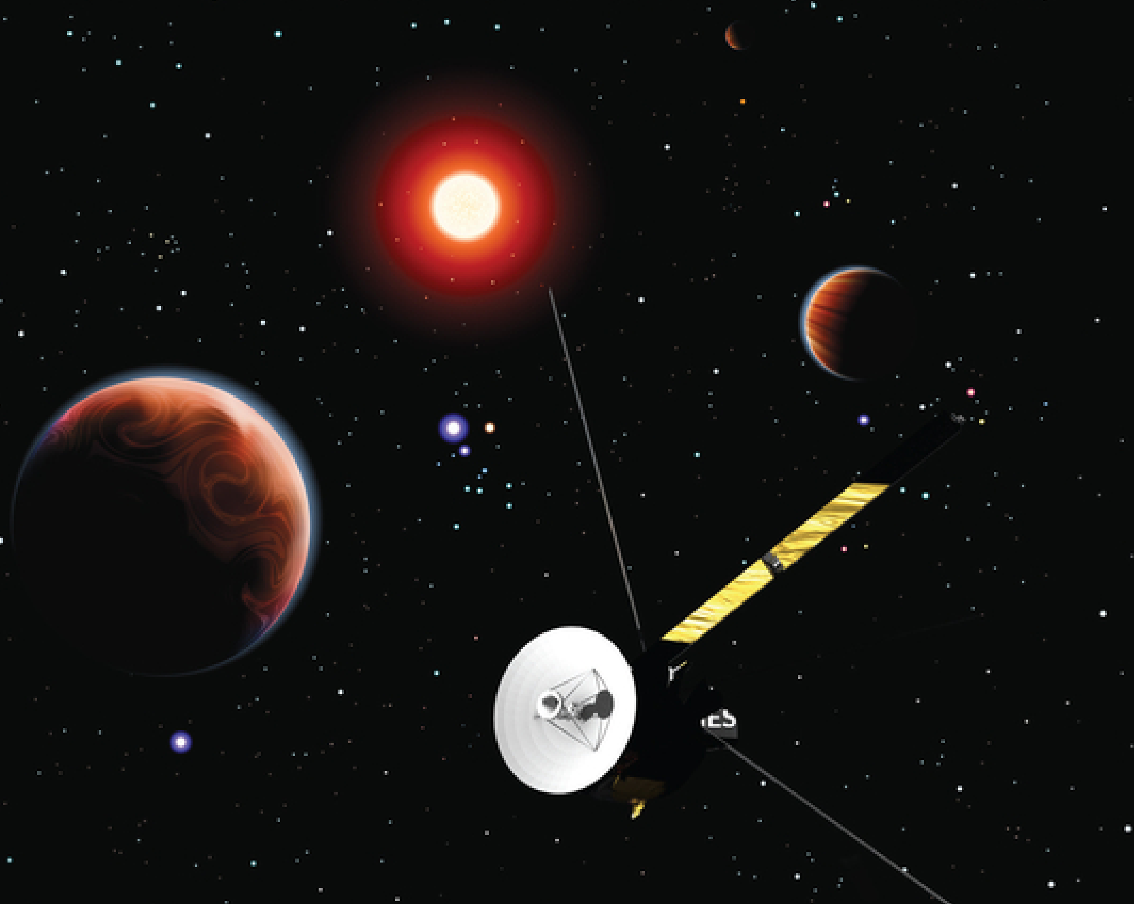
MAX CERF

SPACE TRAJECTORIES

BASIC AND ADVANCED TOPICS



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Basic and Advanced Topics

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To Mallow, who witnessed the completion of this book in March 2024.

Contents

About the Author

Max Cerf has been an engineer at ArianeGroup since 1990, specializing in optimization of space trajectories and spacecraft. He is also a university professor and author of a book on optimization techniques. He is graduate of Ecole Centrale de Paris (1989), holds a PhD (2012) and a "Habilitation à Diriger des Recherches" (2019), and was appointed "Chevalier de l'Ordre National du Mérite" (2015).

Foreword

The conquest of space is a major adventure in the history of mankind.

In the 17th century, Newton discovered the laws governing the motion of celestial bodies, thus founding classical mechanics, notably the theory of universal gravitation. Although mathematically simple, these laws conceal an immense wealth of possible dynamics. The Newtonian gravitational field possesses a host of remarkable properties, discovered over the centuries through the development of increasingly sophisticated mathematics, which are of great interest to space agencies. For example, the existence of periodic orbits, points of equilibrium, invariant manifolds, and other sometimes unexpected properties make possible space missions that are awe-inspiring.

In this remarkable book, in his inimitably clear style, Max Cerf reveals all the secrets of space dynamics and mission design. Senior engineer at ArianeGroup, expert in mission analysis and optimization, Max Cerf is a master of mission design and space vehicle control. In addition to being an accomplished and renowned engineer and researcher, he is also an exceptional teacher, as the reader will appreciate from the pleasant and fascinating reading of this book. From rocket launch and orbiting, through problems of atmospheric reentry, orbital transfer, and space debris clean-up, to interplanetary missions, Max Cerf provides all the keys you need to quickly master all aspects and implementation.

This book will appeal to students, researchers, and engineers looking for a global view of trajectory design problems in astrodynamics, and to acquire the main techniques, particularly in the fields of optimization and optimal control. After reading this book, all you want to do is take part in this amazing adventure!

Emmanuel Trélat
Sorbonne Université, Paris

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Introduction

Space trajectories is a field at the crossroads of mechanics and mathematics. Its development can be divided into two periods.

The first period followed the discovery of Kepler's laws and their demonstration by Newton who laid the foundations of classical mechanics. During this period spanning the 18th and the 19th centuries, research focused on celestial mechanics with the goal of predicting the motion of planets as precisely as possible.

The second period began with the invention of rocket propulsion at the beginning of the 20th century. The possibility of sending maneuvering vehicles in space stimulated the development of optimal control theory, which was originally mainly oriented toward space applications.

This book is aimed at students, researchers, and engineers interested in space applications and who wish to gain an overview of the field. It recalls the essential concepts of orbital mechanics and also presents advanced space applications involving trajectory optimization.

The content is divided into three parts, each of which composed of five chapters.

- The first part deals with free orbital motion with the following topics: Keplerian motion, perturbed motion, three-body problem, orbit determination, and collision risks in orbit.
- The second part deals with controlled orbital motion with the following topics: impulsive transfer, orbital rendezvous, thrust level optimization, low-thrust transfer, and space debris cleaning.
- The third part deals with ascent and reentry with the following topics: launch into orbit, launcher staging, analytical solutions in flat Earth, interplanetary mission, and atmospheric reentry.

These topics have been selected for their practical relevance to space applications. Each chapter covers the necessary concepts, so that it can be read independently of the rest of the book.

Part I

Free Orbital Motion

1

Two-Body Problem

1.1 Introduction

The two-body problem is the fundamental model of orbital mechanics. It describes the motion of two bodies in pure gravitational interaction to the exclusion of all other forces. In the restricted two-body problem, the mass of one body is assumed to be negligible, while the other is considered as point-like. This modeling is appropriate to the motion of artificial satellites around a celestial body.

The two-body problem has an analytical solution called Keplerian motion. The trajectories are conics, satisfying properties of conservation of energy and angular momentum. The conic nature (circle, ellipse, parabola, or hyperbola) depends only on the initial conditions of position and velocity and the motion time law is determined by Kepler's equation. The Keplerian orbit of a satellite around the attracting body is represented geometrically by its orbital parameters, which are analytically related to the position and velocity.

The first section presents the dynamic model and derives the main prime integrals of the Keplerian motion, namely those of angular momentum, eccentricity vector, and energy. The conic shape is then determined from the initial conditions of position and velocity.

The second section deals with the time law of the motion. Position and time are linked by Kepler's equation, which is transcendental. Kepler's equation takes different forms depending on the conic nature. It can also be expressed in a universal form valid for any kind of orbit. The solution of Kepler's equation requires a numerical iterative method.

The third section defines the orbital parameters locating the orbit in space and establishes their relation to position and velocity. The classical orbital parameters have singularities for circular or equatorial orbits, which motivates the definition of the equinoctial parameters, devoid of such singularities. An overview of Earth's orbits is finally presented, with a focus on Earth's coordinate systems and on specific properties such as geo-synchronism or sun-synchronism.

1.2 Keplerian Motion

The two-body problem, or Kepler's problem, concerns the relative motion of two particles exerting between them the force of gravitational attraction. The resulting dynamics, called Keplerian motion, is presented in this section.

Space Trajectories: Basic and Advanced Topics, First Edition. Max Cerf.

1.2.1 Dynamic Model

The two-body problem is formulated in a Galilean (or inertial) reference frame.

Both bodies are considered as material points of masses m_1 and m_2 and positions $\vec{r}_1$ and $\vec{r}_2$, respectively. The gravitational force exerted by the second body on the first body is

$$\vec{F}_{2\to1} = G\frac{m_1 m_2}{r^3}\vec{r} \tag{1.1}$$

The constant G is the universal gravitational constant and $\vec{r} = \vec{r}_2 - \vec{r}_1$ denotes the position of body 2 relative to body 1. Newton's third law (action and reaction) implies that body 1 exerts an opposite force on body 2: $\vec{F}_{1\to2} = -\vec{F}_{2\to1}$.

Universal Gravitational Constant

The accurate knowledge of the value of G is of considerable importance, both for the computation of space trajectories and for the confirmation of theories such as general relativity.

The first reliable measurement of G was made in 1798 by Henry Cavendish, using a torsion pendulum. The possibility of on-board experiments carried out by satellites has greatly improved the accuracy. The reference value adopted in the international system is: $G \approx 6.674\,08 \cdot 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$.

Newton's second law (also called fundamental relation of dynamics) applied to each body in the inertial frame yields the two differential equations

$$\begin{cases} \dfrac{d^2\vec{r}_1}{dt^2} = \dfrac{\vec{F}_{2\to1}}{m_1} = G\dfrac{m_2}{r^3}\vec{r} \\[2ex] \dfrac{d^2\vec{r}_2}{dt^2} = \dfrac{\vec{F}_{1\to2}}{m_2} = -G\dfrac{m_1}{r^3}\vec{r} \end{cases} \tag{1.2}$$

After subtracting the first equality from the second, we obtain the differential equation for the motion of body 2 relative to body 1.

$$\frac{d^2\vec{r}}{dt^2} = -\frac{\mu}{r^3}\vec{r} \tag{1.3}$$

where $\mu = G(m_1 + m_2)$ is called the gravitational constant of the two-body system.

In the restricted two-body problem, the mass of body 2 is negligible compared with that of body 1 (main attracting body). The constant $\mu = Gm_1$ is then called the gravitational constant of the main attracting body. This model is well suited to practical applications studying the motion of artificial satellites.

The values of μ for Earth, Moon, and Sun are (in m^3/s^2):

$$\mu_{Earth} \approx 3.986\,005 \cdot 10^{14}, \quad \mu_{Moon} \approx 4.902\,835 \cdot 10^{12}, \quad \mu_{Sun} \approx 1.327\,23 \cdot 10^{20}$$

The solution to the two-body problem is called **Keplerian motion**, after Johann Kepler, who first set out the laws of motion.

Kepler's Laws

The laws of planetary motion around the Sun were set out empirically by Johann Kepler (1571–1630) in *Astronomia Nova*, published in 1609.

The three laws of Kepler are as follows:

- the planets describe ellipses with the Sun as one of their foci;
- the radius vector sweeps equal areas in equal times;
- the square of the period of revolution is proportional to the cube of the semi-major axis.

Kepler had guessed these three laws from the collection of observations made by the Danish astronomer Tycho Brahé (1456–1601) on the motion of the planets.

Their demonstration came much later owing to Isaac Newton (1642–1727).

In *Philosophiae Naturalis Principia Mathematica*, published in 1687, Newton postulated the laws of motion (inertia, force, and reaction) and the law of gravitation, from which he rigorously established the solution to the two-body problem.

These fundamental results form the basis of orbital and celestial mechanics since the 18th century.

1.2.2 Prime Integrals

A prime integral of a dynamic system is a quantity that remains constant over time. Knowledge of prime integrals helps to delimit the system evolution, even when no analytical solution exists. The two-body problem admits several prime integrals, such as those of angular momentum, energy, and eccentricity vector.

1.2.2.1 Angular Momentum

The angular momentum per unit mass is $\vec{h} = \vec{r} \wedge \vec{v}$, where $\vec{v} = \frac{d\vec{r}}{dt}$ is the velocity of body 2 relative to body 1. Calculating the derivative of $\vec{h}$ with the assumption of a central acceleration given by (1.3) yields

$$\frac{d\vec{h}}{dt} = \frac{d\vec{r}}{dt} \wedge \vec{v} + \vec{r} \wedge \frac{d\vec{v}}{dt} = \vec{v} \wedge \vec{v} + \vec{r} \wedge \left(-\frac{\mu}{r^3}\vec{r}\right) = \vec{0} \tag{1.4}$$

The angular momentum vector $\vec{h}$ is therefore constant throughout the motion.

If $\vec{r}$ and $\vec{v}$ are collinear, then $\vec{h} = \vec{0}$ and the motion takes place along a straight line. Such a trajectory is said to be degenerate. Unless otherwise specified, we assume in the sequel that $\vec{h} \neq \vec{0}$.

As the position and the velocity are constantly perpendicular to $\vec{h}$, the motion takes place in a plane called the orbital plane.

Taking body 1 as origin O and a reference axis Ox in this plane (Figure 1.1):

- the position of body 2 is defined by the polar angle θ and the radius vector r;
- the velocity of body 2 has radial and orthoradial components $\dot{r}$ and $r\dot{\theta}$ (where $\dot{x}$ denotes the time derivative of x).

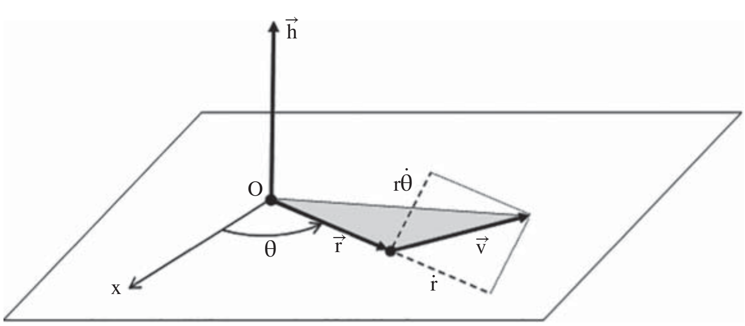

Figure 1.1 Orbital plane.

The modulus of the angular momentum is

$$h = \|\vec{r} \wedge \vec{v}\| = r^2\dot{\theta} \tag{1.5}$$

In an infinitesimal time dt, the radius vector sweeps the area dS of the triangle with sides $\vec{r}$ and $\vec{v}dt$. This area is

$$dS = \frac{1}{2}\|\vec{r} \wedge \vec{v}dt\| = \frac{1}{2}hdt \tag{1.6}$$

The areal velocity (area swept per unit time) is therefore constant, just as the angular momentum. This establishes **Kepler's second law** or **law of areas**.

1.2.2.2 Energy

The mechanical energy per unit mass noted w is the sum of the kinetic energy $\frac{v^2}{2}$ and the potential energy $-\frac{\mu}{r}$, from which the gravitational force $-\frac{\mu}{r^3}\vec{r}$ derives.

$$w = \frac{v^2}{2} - \frac{\mu}{r} \tag{1.7}$$

Taking the scalar product of each member of (1.3) with $\vec{v}$ and using $\vec{r} \cdot \vec{v} = r\dot{r}$ (Figure 1.1), we obtain

$$\frac{d\vec{v}}{dt} \cdot \vec{v} = -\frac{\mu}{r^3}\vec{r} \cdot \vec{v} = -\frac{\mu}{r^3}r\dot{r} = -\mu\frac{\dot{r}}{r^2} \quad \Rightarrow \quad \frac{d}{dt}\left(\frac{v^2}{2}\right) = \frac{d}{dt}\left(\frac{\mu}{r}\right) \tag{1.8}$$

Instantaneous variations in kinetic and potential energy cancel each other out. This statement is in fact true for any force deriving from a potential.

The mechanical energy $w = \frac{v^2}{2} - \frac{\mu}{r}$ is therefore constant throughout the motion.

The accessible domain depends on the energy sign. To escape the main attracting body, the energy must be zero or positive (kinetic energy ≥ potential energy).

In this case, the escape velocity v_{esc} associated with zero energy (w = 0) and the residual velocity at infinity v_∞ (for $r \to \infty$) are respectively given by

$$\begin{aligned} w &= \frac{v^2}{2} - \frac{\mu}{r} = 0 \quad \text{(for escape)} \quad \Rightarrow \quad v_{esc} = \sqrt{\frac{2\mu}{r}} \\ w &= \frac{v^2}{2} - \frac{\mu}{r} = \frac{v_\infty^2}{2} \quad (\text{for } r \to \infty) \quad \Rightarrow \quad v_\infty = \sqrt{2w} \end{aligned} \tag{1.9}$$

1.2.2.3 Eccentricity Vector

The eccentricity vector $\vec{e}$ (also called Laplace vector) is defined as:

$$\vec{e} = \frac{\vec{v} \wedge \vec{h}}{\mu} - \frac{\vec{r}}{r} \tag{1.10}$$

To calculate the derivative of $\vec{e}$, we observe that the radial unit vector $\vec{u}_r = \frac{\vec{r}}{r}$, which lies in the orbital plane, rotates around the fixed axis $\vec{u}_h = \frac{\vec{h}}{h}$ with an angular velocity $\dot{\theta}$ (see Figure 1.1).

Its derivative is therefore orthoradial and it can be expressed as:

$$\frac{d\vec{u}_r}{dt} = \dot{\theta}\,\vec{u}_h \wedge \vec{u}_r \tag{1.11}$$

Using (1.5) to substitute $\dot{\theta}$, then (1.3) to make $\vec{v}$ appear, we obtain

$$\frac{d\vec{u}_r}{dt} = \frac{h}{r^2}\vec{u}_h \wedge \vec{u}_r = \vec{h} \wedge \frac{\vec{r}}{r^3} = -\frac{1}{\mu}\vec{h} \wedge \frac{d^2\vec{r}}{dt^2} = -\frac{1}{\mu}\vec{h} \wedge \frac{d\vec{v}}{dt} \tag{1.12}$$