



Principles of
**Interferometric
and Polarimetric
Radiometry**

Ignasi Corbella

 **IEEE Press**

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Principles of Interferometric and Polarimetric Radiometry

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Principles of Interferometric and Polarimetric Radiometry

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To Roser, Pau and Oriol. And also to my parents Leopoldo and Isabel

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Foreword

To my knowledge, this is the first book ever published dedicated to the topic of aperture synthesis (or interferometry) for the remote sensing of the Earth. Much of its contents can, however, be applied to other types of microwave radiometers and applications. The author, Prof. Ignasi Corbella of the Polytechnic University of Catalunya (UPC), Barcelona (Spain), is, in my humble opinion, the engineer, among all I know, who best understands aperture synthesis for Earth Observation, a field that started in the 1980s. Readers of this book will have the privilege to learn directly from him.

In fact, I started learning microwave theory having Ignasi as teacher at UPC back in 1983, and he was also the supervisor of my master's thesis in 1986, and of my PhD in 1996. Therefore, it is an honor to write this foreword to his book some 41 years after I attended his lessons at UPC. From 1993 till 1999, he was involved in several studies and experiments to develop the Microwave Imaging Radiometer with Aperture Synthesis (MIRAS) within contracts with the European Space Agency (ESA) I was responsible for. Since 2000 till today, Prof. Corbella has been part of the team supporting, in many aspects, ESA's Soil Moisture and Ocean Salinity (SMOS) mission, which carries MIRAS as its only payload. It is the SMOS mission, in which I am involved from ESA side, which has allowed us to enjoy this very long professional link and friendship.

The development of MIRAS is a story truly worth telling. It all started with the work of David LeVine (NASA-GSFC) and Carl Swift (University of Massachusetts at Amherst) in the late 1980s on the Electronically Scanning Thinned-Array Radiometer (ESTAR). ESTAR was a one-dimensional interferometer presenting a new technological solution to deploy a very large L-band antenna in space, to make observations in the 1400–1427 MHz protected band, to provide global maps of soil moisture and ocean salinity. Inspired by this research, and following some work carried out by the French Space Agency (CNES), in 1992, ESA decided to initiate the development of a two-dimensional interferometer: MIRAS.

During the first 10 years of MIRAS, Prof. Corbella was involved in the theoretical aspects, the calibration, and the image processing of the instrument. In principle, although aperture synthesis had never been applied to remote sensing, it was a matter of translating the knowledge accumulated in radio-astronomy to the field of Earth Observation. Given this, we were all quite confident that the new type of radiometer should work very finely.

Radio-astronomy was (and is) based on a theorem that was first formulated by the Dutch physicist Pieter Hendrik van Cittert in 1934, and four years later proved in a simpler way by another Dutch physicist, Frits Zernike. Zernike was a prominent scientist who was awarded the Nobel Prize in 1953 for having invented the Phase-Contrast Microscope. The so-called Van Cittert-Zernike theorem, or in short, the VCZ theorem, was the theoretical basis of radio-astronomy, and in turn, of ESA's initiative to embark on two-dimensional aperture synthesis for remote sensing.

Nobody would have anticipated that such solid theoretical grounds were going to be shaken by the development of MIRAS, to the point of proving them completely inadequate for the application of interferometry to Earth Observation.

Based on the so-far successful breadboarding of MIRAS subsystems and the scientific need to monitor soil moisture and ocean salinity, ESA's Earth Science Advisory Committee recommended the implementation of the SMOS mission on April 27, 1999, as second Earth Explorer Opportunity Mission.

By 2002, the breadboarding activities (MIRAS Demonstrator Pilot Project – MDPP) were running in parallel with the Phase B of the SMOS project. The prototype built within the MDPP consisted of three linear arrays of four receivers each which could be mechanically moved along three horizontal tracks 120° equi-spaced from each other. The MDPP also included the optical harness, the correlator, and the internal calibration subsystems.

In December 2002, the MDPP demonstrator was brought inside the Electro Magnetic Compatibility (EMC) chamber of INTA (Instituto Nacional de Técnica Aeroespacial, Madrid) for the very first end-to-end test. The imaging algorithm, based on the VCZ theorem, was predicting high correlation values due to the high emissivity of the microwave absorber material (at room temperature) of the walls of the EMC chamber. To our surprise, the correlations were two orders of magnitude (!) smaller than those predicted by the VCZ theorem. After carefully assessing the breadboard, and realizing that the hardware was working well, we had to accept that the value of the correlations was real. As mentioned, SMOS was already in Phase B ...

The UPC team, led by Prof. Corbella, was responsible for processing the data from the INTA test. Therefore, we all packed the hardware and went on Christmas Holidays with the heavy weight of the insignificant correlation values on our shoulders...

It was during the Christmas of 2022 that Ignasi set himself into resolving the mystery. He found a paper on noise waves and passive linear multiports of 1991 which gave the expression of the correlation of the outgoing noise waves of a passive microwave circuit. Such problem resembled the configuration of the MDPP breadboard inside the EMC chamber. The article referred to a theorem stated by a Dutch engineer, Hendrik Bosma, in his PhD thesis of 1967. According to Bosma's theorem, the correlation of the noise waves coming out of a passive microwave circuit in thermal equilibrium with its terminations, as the MDPP breadboard inside the EMC chamber, had to null.

Based on these findings, Prof. Corbella carefully reformulated the basic measurement of interferometry, the correlation between the output signals of two receivers forming a baseline, considering the presence of the neighboring receivers as well as the physical temperature of the receivers and the target. By February 2003, he had come with a new equation, more general than the VCZ theorem, which could explain the correlations in the INTA chamber as well as those measured by radio-telescopes (for large antennas and spacings, in wavelength units, the new formulation reduced to the VCZ theorem). Later the same year, Prof. Corbella had derived the polarimetric version of his equation. The new equation, which I very deservedly coined as the *Corbella Equation*, was first published in TGARS, August 2004.

The fact that the VCZ theorem could not explain all interferometric experiments, like the test of the MDPP breadboard in the EMC chamber (by two orders of magnitude), was further evidenced when the opposite test was carried out: the imaging of the Cold Sky at the Dwingeloo radio-observatory facility in The Netherlands. In the Dwingeloo experiment, four receivers were manually moved along three arms to get an image of the Cold Sky. While the VCZ theorem was predicting now very low correlations based on the low brightness temperature of the Cosmic Microwave Background Radiation (CMBR), around 2.7 K, the expected values according to the Corbella equation were two orders of magnitude larger in this case. The measurements were perfectly in accordance with Corbella's predictions and, once again, about two orders of magnitude away from the VCZ theorem-predicted values.

The Corbella Equation was then adopted to process the data of the SMOS mission instead of the VCZ theorem. The calibration approach had to be adapted, and, fortunately, the Flat Target Response of the instrument could be obtained by using the CMBR and the subsequent Flat Target Transformation could be devised to remove the $-T_r$ term Corbella had introduced into the VCZ equation. With the data processing and calibration following his equation, Ignasi had brought the SMOS project back on track half way through Phase B.

It is worth noting that it took some time for some engineers to accept and understand the profound implications of the "Corbella Equation." After all, the VCZ theorem, established by renowned (Nobel Prize awarded) scientists, had

been working perfectly well in radio-astronomy for decades. It was only a matter of making new experiments in new conditions (as taking the MDDP breadboard inside the INTA EMC chamber), realizing of other scientists' research (as Bosma's PhD thesis), and having a clear and humble mind to put the pieces of the puzzle together. This ultimately culminated in a scenario where *David beat Goliath* with a new more general formulation of aperture synthesis.

This book starts with a tedious Chapter 1 on probability concepts and stochastic processes, Hilbert transform, and analytic signals. However, the usefulness of its contents will be much appreciated throughout the rest of the book, as they will be frequently called upon to make progress in the formulation of the different flavors of interferometers.

Chapter 2 introduces the basic concepts of radiometry. Prof. Corbella has made an effort to explain interferometry and all its equations in a way that collapses to the case of a conventional total power radiometer. In this respect, it makes interferometry easier to digest for engineers knowledgeable in conventional radiometry.

The Corbella Equation is derived in Chapter 3. This chapter can be considered central and including the nucleus of the aperture synthesis theory applied to remote sensing. An engineer confronted with the design of this type of radiometers should absorb these contents thoroughly. This chapter introduces also the tools necessary to derive the sensitivity of an interferometer, and the subtleties which occur when digitizing the output signals.

The discussion on sensitivity continues more deeply in Chapter 4, which focuses on the image processing. The basic theory on how to retrieve the brightness temperature from the visibility function are explained in great detail. Key concepts such as the reciprocal grids, the apodization window, the G-matrix, the aliasing, and the noise floor are found here.

The implementation of the interferometry theory into practical instruments is covered by Chapter 5. The nomenclature of this chapter is dense, but precise and consistent with the previous chapters. The reader is invited to understand and carefully realize the subtle differences between the different instrument architectures, including the effects of digitalization of the signals and the different ways complex correlations can be obtained (whether using two or four multipliers, with or without digitalization).

Finally, Chapter 6 gives the reader an overview on the characterization and calibration of an interferometric radiometer, based on the Corbella Equation. The key parameters which have to be found to correct the raw visibilities into a calibrated set are described, together with the methods to retrieve their value.

For every new topic in engineering and science, there is always a "first reference book," a "bible." I believe this book is the "bible" on polarimetric aperture synthesis for remote sensing. This book will be tremendously useful for engineers

working with this type of instruments, as well as a valuable reference for a university course on interferometry for Earth Observation.

I can see a future for aperture synthesis in remote sensing, as demonstrated by the SMOS mission and, in great part, thanks to Prof. Corbella's contributions. Interferometry combines well with formation flying as a way to deploy extremely large apertures. When the techniques to master formation flying will be sufficiently developed, interferometry will spread further. I do hope graduated students as well as practicing engineers will find in this book a great support for decades to come.

Oegstgeest 26 June, 2024

(14 years, 7 months, and 24 days from
SMOS launch, still up and running)

Manuel Martín-Neira

Senior Microwave Radiometer Engineer
European Space Agency

About the Author

Ignasi Corbella was born in Barcelona, Spain, in 1955. He received the telecommunication engineering degree and doctoral degree in telecommunication engineering from the Universitat Politècnica de Catalunya (UPC), Barcelona, Spain, in 1977 and 1983, respectively.

In 1976, he was with the Microwave Laboratory, School of Telecommunication Engineering, UPC, as a research assistant, where he worked on passive microwave integrated-circuit design and characterization. In 1979, he was with Thomson-CSF, Paris, France, working on microwave oscillators' design. In 1982, he was Assistant Professor, in 1986, Associate Professor, and, in 1993, Full Professor with UPC, where he is currently teaching basic microwaves and antennas at the undergraduate level and graduate courses on nonlinear microwave circuits. Since 1993, he has been actively participating as a researcher with the European Space Agency (ESA) SMOS mission in the frame of several contracts, directly with ESA, with the payload prime contractor EADS-Casa Espacio (Spain) or with the operational processor prime contractor Deimos Engenharia (Portugal). His expertise includes, among others, fundamentals of interferometric aperture synthesis radiometry, radiometer calibration, image reconstruction, radiometer hardware specification, and payload characterization.

From 1993 to 1997, he was Academic Director of the School of Telecommunications Engineering. From 2001 to 2003, he was Director of the Department of Signal Theory and Communications, UPC. From 1998 to 1999, he was Guest Researcher at the NOAA/Environmental Technology Laboratory (Boulder-Colorado), developing methods for total-power radiometer calibration and data analysis. In 2015, he was a visiting scholar at the University of Colorado at Boulder, working on emission models from layered media. From 2004 to 2010, he was a member of

the SMOS Science Advisory Group (SAG) and since 2010 of the SMOS Quality Working Group (QWG).

Dr. Corbella was the General Chairman of the IEEE 2007 International Geoscience and Remote Sensing Symposium (IGARSS), Barcelona.

He also was the Scientific Coordinator of a Dictionary of Telecommunication terms in Catalan language, with more than 4000 entries, published in March 2007.

Preface

Invention, it must be humbly admitted, does not consist in creating out of void, but out of chaos. (Mary Shelley, 1831)

This book compiles the expertise the author has gathered during his 30-year-long work on Microwave Interferometric and Polarimetric Radiometry, particularly in relation to the “Soil Moisture and Ocean Salinity” (SMOS) mission of the European Space Agency. The SMOS satellite was put into orbit in November 2009 with the objective of acquiring global measurements of the two important geophysical parameters mentioned in its full name. The satellite includes a single payload named MIRAS (Microwave Imaging Radiometer with Aperture Synthesis) that is the first ever microwave interferometric radiometer designed for observing the Earth surface from space.

Except in some very few cases where text previously written by the author is reused, practically all writing is completely new, not extracted from any existing source. Many concepts and methods were indeed published in open literature or internal reports, often in collaboration with other authors (see acknowledgments) but, for the present work, the author has made an important effort to rewrite all the material from scratch in order to present the information with consistent terminology and complemented with detailed procedures and concepts not sufficiently and/or rigorously explained in the original papers. In consequence, even for the more established themes, for example in Chapters 1 and 2, the reader may find some explanations or points of view different from those found in published treatises. This feature will be probably useful to complement different visions of similar concepts.

The book starts with a review of the fundamental concepts that are used throughout the rest of the volume. They range from random variables and stochastic processes to microwave circuits and antennas. This first chapter is

essential to set the terminology and symbols and is frequently referenced in all others. Even though the reader may be familiar with these themes in general, the author recommends to start exploring this chapter, at least in a “fast read” mode, before going to others more specific.

The second chapter is dedicated to the quite established topic of Microwave Radiometry, focused especially on instrumental aspects. It is included as a kind of introduction of the main subject of the book, interferometric and polarimetric radiometry, in order to present the fundamental concepts, formulate the needed hypotheses, and derive the theorems that will be extended later to more complex systems. It should be helpful in facilitating the readers the study of the rest of the book, even if they have a sound knowledge of microwave radiometric systems in general.

Interferometric and polarimetric radiometry is presented in the third chapter. This is perhaps the core of the book as it presents the fundamentals of the interferometric technique in the microwave region of the spectrum. Optical systems are simply not covered, although they may have important similitudes with what is here presented. Unavoidably, the formulation is biased to its application in Earth observation, as this is the background of the author. Nevertheless, most of the results are perfectly applicable to Radioastronomy since the fundamental concept is exactly the same. It may happen that experts in this field find some material in this book interesting for their own research.

Combination of multiple baselines to form an image is the subject of the fourth chapter. Here, the content is even more specific than that of the previous one in terms of application to large instruments dedicated to Earth observation from space or aircraft. As a matter of fact, much of the material relies on algorithms or procedures actually implemented in the MIRAS instrument. This chapter should be useful to people designing similar systems for related applications.

Chapter 5 deals with specific instrument techniques. Consistent with a top-down vision, the general theory developed in Chapters 2–4 is here materialized into dedicated technologies. Many of the items included are already implemented in existing instruments or intended for their use in future ones. Digital processing, essential in today’s systems, has been intentionally relegated to this chapter as a convenient way to treat measured data. Up to this point the world in this book is analog.

Finally, calibration and characterization are covered in Chapter 6. These are essential aspects of any radiometer, either total power or interferometric, maybe less conceptual, more oriented to techniques and procedures, and mathematically simpler. Commonly used methods are here described and particularly those utilized in the MIRAS instrument. This chapter has been written with the aim of helping engineers designing an instrument of the same or similar kind.

An appendix is also included with some specific and supposedly well-known concepts and definitions, such as Fourier transform (analog and discrete), special functions, spherical coordinates, solid angles, and others. Surely, in many cases the advanced reader will not need to consult it, although it can serve as a quick reference guide.

The author hopes that this book will help engineers and scientists facing the design of any kind of Microwave Interferometric and/or Polarimetric Radiometer, especially for application in Earth observation.

Barcelona
September 12, 2024

Ignasi Corbella

Acknowledgments

Above all, I would like to express my most sincere gratitude to Dr. Manuel Martin-Neira, Senior engineer at the European Space Agency, for his careful revision of the manuscript. His opportune comments always hit in some weakness of the original writing and, once endorsed, enhanced the quality of the final version. I am also thankful to him for having written the excellent Foreword and for the generous words he expresses in it about my work and person.

A substantial amount of information in this book can be found scattered in articles and conference proceedings published by myself, always in collaboration with other colleagues. Three persons deserve to be mentioned in this regard: the already mentioned Dr. Manuel Martin-Neira, Dr. Francesc Torres and Dr. Adriano Camps, both professors at the “Universitat Politècnica de Catalunya.” This book would not have been possible without the exceptional job they have carried out during many years of fruitful cooperation. I am indebted to all three.

Ignasi Corbella

1

Signals, Receivers, and Antennas

A radiometer, as a measuring system, processes input information in the form of *signals* and converts them to usable magnitudes proportional to observational parameters. Since all signals encountered in radiometry are of random nature, it is of paramount importance to establish a solid background of knowledge on stochastic processes. With the objective in mind of establishing notation and in order to have concepts and definitions at hand, the most relevant of them are collected in this chapter. Section 1.1 is an introduction to random variables, a fundamental tool on which random signals, developed in the section 1.2, are based. The reader is assumed to be familiar with basic random variables and processes, so only the definitions and methods needed for this book are included. More complete treatments can be found in classical books on statistics and its application to signal theory [1–5].

Receivers and *antennas* are the two basic building blocks of any radiometer, so mastering the main concepts about these two components is fundamental to perceive the details of radiometric operation. Both of them are also used in Telecommunications or Radar and are subjects of basic university courses. Consequently, there is a considerable amount of literature about them. This chapter presents their usage in microwave radiometry, aimed at gathering all important definitions and relations used later in this book. To know more, the interested reader may wish to complement the information with basic textbooks, as for example [6–10] among others. In any case, some familiarity with microwave circuits, including noise, and antenna theory is assumed.

1.1 Random Variables, Real and Complex

A real random variable is rigorously defined in, for example, [1]. It is fundamentally described by the “probability density function,” which is the probability that the variable takes a given value within a differential interval. For two random

variables, the “joint density function” is the same concept applied to a differential area in a cartesian plane defined by all possible values of both. Two real random variables are said to be *independent* if they describe completely unrelated events. In this case, their joint density function is equal to the product of both individual density functions. Not surprisingly, if two variables are independent, arbitrary functions of each one of them are also independent.

A complex random variable is defined by the joint density function of its real and imaginary parts. Most of the definitions and equations provided in this section apply to complex variables, although they are also valid for real ones just by setting the imaginary part equal to zero. To avoid confusion, *real variables are represented by upright roman typeface* ($x, y, z \dots$) *while complex variables use italic shape* ($x, y, z \dots$). Complex conjugation is denoted by a superscript asterisk.

1.1.1 Definitions

The “expected value” or “mean” of a real random variable x is

$$\bar{x} \triangleq \int_{-\infty}^{\infty} xf(x)dx \quad (1.1)$$

where $f(x)$ is the probability density function. In [1], it is demonstrated that this operation is linear so, for any two random variables x and y , $\overline{\alpha x + \beta y} = \alpha \bar{x} + \beta \bar{y}$ where α and β are arbitrary constants.

Given a *complex* random variable $z = x + jy$, the following definitions apply:

$$\begin{aligned} \text{Mean: } \bar{z} &= \bar{x} + j\bar{y} \\ \text{Variance: } \sigma_z^2 &= \overline{|z - \bar{z}|^2} = \overline{|z|^2} - |\bar{z}|^2 \\ &\sigma_z^2 = \sigma_x^2 + \sigma_y^2 \\ \text{Standard deviation: } \sigma_z &\quad (\text{square root of variance}) \\ \text{Autocorrelation: } \overline{|z|^2} &= \overline{zz^*} \end{aligned} \quad (1.2)$$

The standard deviation is *always* real and positive and it has the same units as the random variable itself. Conjugating a random variable does not change the standard deviation. The following property is immediate from the definition:

$$\text{if } z = \alpha x + \delta \text{ then } \sigma_z = |\alpha| \sigma_x \quad (1.3)$$

where α and δ are complex constants, and x is a complex random variable, not to be confused with x , the real part of z . The additive constant δ does not change the standard deviation.

The generalization of variance and autocorrelation for two arbitrary complex random variables z and w is

$$\begin{aligned} \text{Covariance: } C_{zw} &= \overline{(z - \bar{z})(w - \bar{w})^*} = \overline{zw^*} - \bar{z}\bar{w}^* \\ \text{Cross-correlation: } \overline{zw^*} & \end{aligned} \quad (1.4)$$