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Polymer Extrusion

Pierre G. Lafleur and Bruno Vergnes

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Introduction

Extrusion is by far the most important and probably the oldest transformation and shaping process of thermoplastic polymers. After synthesis (polymerization) up until the formulation and production of the finished or semi-finished product, this process is almost entirely concerned with synthetic polymers as well as elastomers or food products. However, it is important to specify from the beginning that different applications and processes exist.

The development of the single-screw extrusion, which is directly derived from the Archimedean screw principle, began during the 1880s, initially with rubber and then with polymers around 1940. Nowadays, it is primarily used to make finished or semi-finished products that will undergo a second transformation (e.g. the extrusion of sheets that will then be thermoformed for use in packaging). This process essentially involves the melting and homogenization of the raw materials before pumping the molten polymer through a die (continuous process) or into a mold (in the case of injection, a cyclic process). Globally, more than 90 million tons of thermoplastics are transformed in this way each year.

Table I.1 provides an insight into the main processes involving a single-screw extruder, as well as examples of manufactured products. This process will be clarified and explained in more detail in Chapter 3.

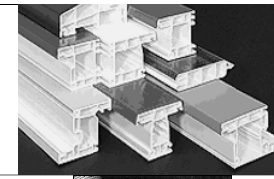
Process	Examples of products	
Profile extrusion	Tubes, pipes, window frames and exterior cladding	
Extrusion – calendering	Wall and floor coverings and tablecloths	
Film blowing	Films for packaging, bags, agricultural mulch, etc.	
Extrusion – blow molding (of hollow bodies)	Water and soda bottles, gasoline containers and tanks.	
Injection molding	Buckets, buttons, containers, syringes, electronic and car parts and pens.	

Table I.1. *Examples of manufactured products from injection and extrusion processes*

Twin-screw extrusion, as the name suggests, is a process involving two screws rotating inside a barrel whose cross section has a figure-eight shape. Also, counter-rotating systems are distinguished from co-rotating systems according to the direction in which the screws rotate with respect to each other.

Counter-rotating extruders are primarily used in polyvinyl chloride (PVC) extrusion, for the manufacture of pipes or profiles. They are not described in this work. A significant development has been seen in co-

rotating extruders, due to their great versatility and diverse uses. First and foremost, by being used as a mixing tool or as continuous chemical reactors, they allow specific materials to be produced. It is with this type of machine that polymer alloys, thermoplastic elastomers, filled polymers and nanocomposites are produced as well as all other materials made by reactive extrusion (chemical modifications, grafting, controlled degradation, etc.). This process, shown in Figure I.1, will be reviewed in Chapter 4.

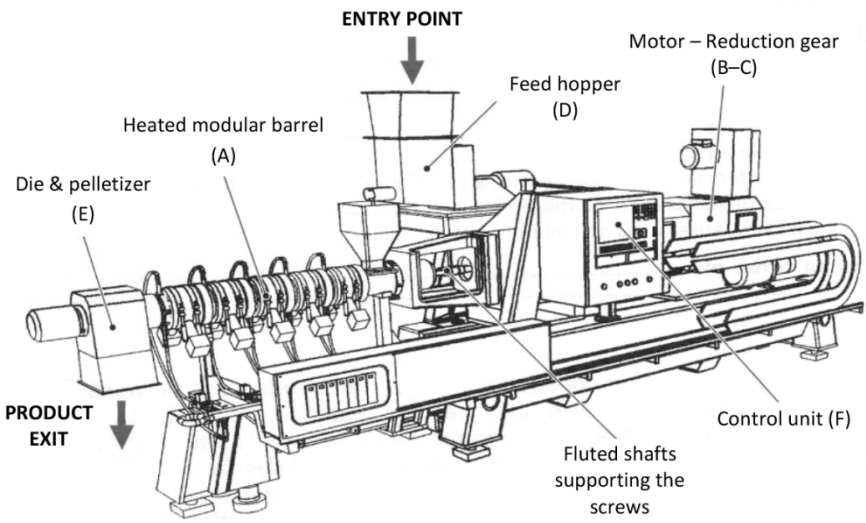


Figure I.1. *Diagram of a co-rotating twin screw extruder (Source: Clextral)*

Manufacturing of a finished product using extrusion is carried out by pushing the molten polymer through a tool called a die, which will give the final shape to the product.

Figure I.2 shows the conventional setup of an extrusion line, with the extruder, the die and the cooling system required to generate an end product.

The die must be designed in a way that allows a product of controlled dimensions to be obtained at a uniform rate and temperature. This poses serious challenges that will be discussed in Chapters 5 (profiles and pipes), 6 (films and sheets) and 7 (cables).

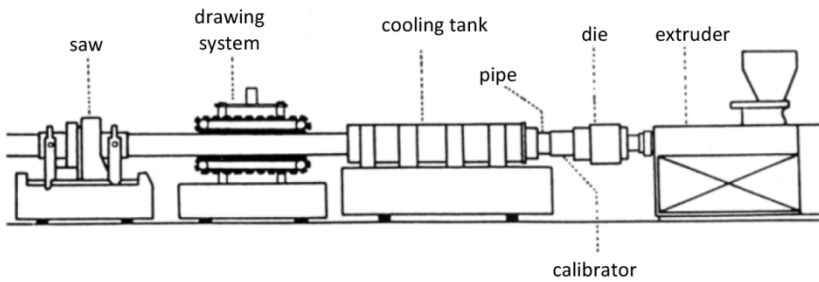


Figure I.2. *Example of a pipe extrusion line*

For each of the points mentioned above, this book first presents the physics of the mechanisms involved and then suggests some models of various sophistication to describe these mechanisms, interpret the results and gain an in-depth understanding of the flow conditions. To help readers with this, some essential reminders about continuum mechanics, rheology and heat transfer are presented in Chapter 1. Chapter 2 includes a short description of the most commonly used calculation methods.

Chapter 1

Continuum Mechanics, Rheology and Heat Transfer Overview

The aim of this chapter is to provide the readers with the basics of continuum mechanics, rheology and heat transfer, which will be of fundamental importance throughout the remainder of this book. We will keep this presentation as concise as possible by avoiding unnecessarily detailed mathematical manipulations, and referring interested readers to other pertinent references in the literature.

1.1. Continuum mechanics

1.1.1. *Strain*

Let us consider the deformation of a continuous medium defined by the displacement vector field of components: $U(x,y,z)$, $V(x,y,z)$, $W(x,y,z)$. The corresponding strain (provided that it is small) can be described by a symmetric tensor $\underline{\underline{\varepsilon}}$ as follows [SAL 88, AGA 14]:

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \dfrac{\partial U}{\partial x} & \dfrac{1}{2}\left(\dfrac{\partial U}{\partial y}+\dfrac{\partial V}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial U}{\partial z}+\dfrac{\partial W}{\partial x}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial U}{\partial y}+\dfrac{\partial V}{\partial x}\right) & \dfrac{\partial V}{\partial y} & \dfrac{1}{2}\left(\dfrac{\partial V}{\partial z}+\dfrac{\partial W}{\partial y}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial U}{\partial z}+\dfrac{\partial W}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial V}{\partial z}+\dfrac{\partial W}{\partial y}\right) & \dfrac{\partial W}{\partial z} \end{bmatrix} \quad [1.1]$$

Hereafter, a symmetric tensor will be considered as a square matrix, involving six independent terms:

– the diagonal terms $(\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz})$ correspond to uniaxial deformations of traction or compression;

– the symmetric terms $(\varepsilon_{xy} = \varepsilon_{yx}, \varepsilon_{yz} = \varepsilon_{zy}, \varepsilon_{xz} = \varepsilon_{zx})$ correspond to shear deformations.

1.1.2. *Strain rate*

We will now consider the velocity field $u(x,y,z)$, $v(x,y,z)$, $w(x,y,z)$ which is associated with the aforementioned strain. Just like the strain tensor, the strain rate tensor can be defined by:

$$\underline{\underline{\dot{\varepsilon}}} = \begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial u}{\partial z}+\dfrac{\partial w}{\partial x}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\right) & \dfrac{\partial v}{\partial y} & \dfrac{1}{2}\left(\dfrac{\partial v}{\partial z}+\dfrac{\partial w}{\partial y}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u}{\partial z}+\dfrac{\partial w}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial v}{\partial z}+\dfrac{\partial w}{\partial y}\right) & \dfrac{\partial w}{\partial z} \end{bmatrix} \quad [1.2]$$

Unlike the strain tensor, defined by relation [1.1] for small deformations, the strain rate tensor is defined in a general manner. It is, therefore, well adapted to the description of fluid flows, for which the deformations are always very large. Just like $\underline{\underline{\varepsilon}}$, the terms for the strain-rate tensor have a particular meaning:

– the diagonal terms are elongation rates, often referred to, below, as $\dot{\alpha}$;

– the symmetric terms are shear rates, often referred to, below, as $\dot{\gamma}$.

EXAMPLE 1.1.– Let us consider two elementary flows, which we will come across often in the remainder of this book. The first flow is a planar shear flow between two plates (see Figure 1.1.(a)). The bottom plate is immobile, whereas the top plate is mobile with a velocity V. The velocity field is, *a priori*, in the following form: $u = u(y)$, $v = 0$ and $w = 0$. The strain rate tensor is reduced to:

$$\underline{\underline{\dot{\varepsilon}}} = \begin{bmatrix} 0 & \frac{1}{2}\frac{du}{dy} & 0 \\ \frac{1}{2}\frac{du}{dy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad [1.3]$$

The second flow occurs between two immobile plates under a pressure drop (Figure 1.1(b)). This is called a Poiseuille flow. The velocity field has the same form as above, i.e. $u = u(y)$, $v = 0$ and $w = 0$.

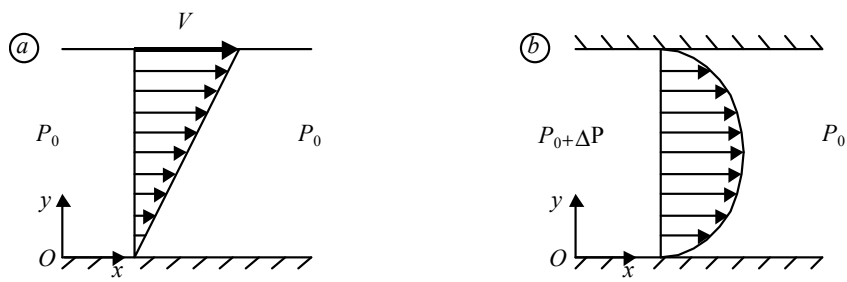


Figure 1.1. *Flow between parallel plates: a) simple shear flow and b) Poiseuille flow*

Consequently, the strain rate tensor will also have the same expression. All flows which give rise to this type of tensor (a single symmetric non-zero term) are called simple shear flows. In the corresponding Cartesian coordinate system, Ox is the shear direction, Oxy is the shear plane and du/dy is the shear rate.

1.1.3. *Stress*

Let us consider a small surface ds upon which a force dF is exerted (Figure 1.2). By definition, the stress vector $\underline{T}$ at point O of this surface is the limit dF / ds as ds tends toward zero. This vector is projected:

– on the normal $\underline{n}$ to the surface at point O: $\sigma_n = \underline{T}.\underline{n}$ is the normal stress, traction or compression;

– on the tangent plane: σ_t is the shear stress.

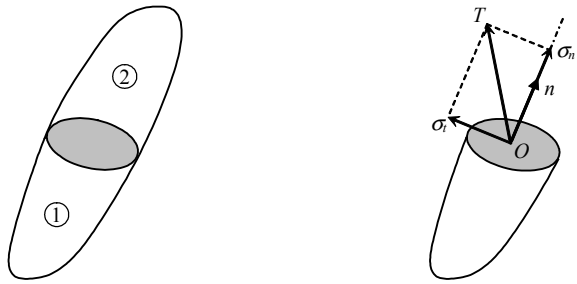


Figure 1.2. *Stress applied on a surface*

The stress vector cannot define a general state of stress, since it is associated with the orientation of the surface on which it is exerted. It is the stress tensor which will define this general state. As with the tensors mentioned above, it is a symmetric tensor written as:

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} \qquad [1.4]$$

As with the previous tensors, there are six terms involved:

– the diagonal terms, corresponding to normal stresses (along axes x, y and z), of traction or compression;

– the symmetric terms, corresponding to the shear stresses (in the planes xy, xz and yz).

From the stress tensor, the stress vector at any normal point $\underline{n}$ can be calculated using the following equation:

$$\underline{T} = \underline{\underline{\sigma}}.\underline{n} \qquad [1.5]$$

For any stress state, the hydrostatic pressure can be defined by the following equation:

$$p = -\frac{1}{3}\mathrm{tr}(\underline{\underline{\sigma}}) \qquad [1.6]$$

where $\mathrm{tr}(\underline{\underline{\sigma}})$ is the trace of the stress tensor, i.e. the sum of the diagonal terms: $\mathrm{tr}(\underline{\underline{\sigma}}) = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$.

This allows the stress tensor to be broken down into the sum of the hydrostatic pressure and the traceless tensor, called the deviator:

$$\underline{\underline{\sigma}} = -p\underline{\underline{I}} + \underline{\underline{s}} \qquad [1.7]$$

where $\underline{\underline{I}}$ is the identity tensor. It is the deviator $\underline{\underline{s}}$ that allows us, in section 1.2.2, to define the constitutive laws.

1.1.4. *General equations in continuum mechanics*

From the above concepts, we can now establish the basic equations that will be used to solve a flow problem.

1.1.4.1. *Continuity equation*

The continuity equation expresses the conservation of mass in a flow. It is generally written as:

$$\frac{d\rho}{dt} + \mathrm{div}(\rho\underline{u}) = 0 \qquad [1.8]$$

where ρ is the density, $\underline{u}$ is the velocity vector and "div" is the divergence operator.

In the case of an incompressible medium, as in the case of molten polymers, this equation is simplified to:

$$\mathrm{div}(\underline{u}) = 0 \qquad [1.9]$$

i.e. in Cartesian coordinates: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$.

Using the definition of the trace of a tensor, this can also be written as:

$$\mathrm{tr}(\underline{\underline{\dot{\varepsilon}}}) = 0 \qquad [1.10]$$

1.1.4.2. *Equilibrium equation*

The equilibrium equation expresses the equilibrium of forces being exerted upon an element of material. If F represents the forces of mass (e.g. gravity) and $\rho\gamma$ the forces of inertia, the equilibrium equation is written as:

$$\mathrm{div}(\underline{\underline{\sigma}}) + \underline{F} - \rho\underline{\gamma} = 0 \qquad [1.11]$$

The divergence of a tensor is a vector whose components are the divergence of each row of the tensor.

$$\mathrm{div}(\underline{\underline{\sigma}}) = \begin{vmatrix} \frac{\partial \sigma_{xx}}{\partial x} + & \frac{\partial \sigma_{xy}}{\partial y} + & \frac{\partial \sigma_{xz}}{\partial z} \\ \frac{\partial \sigma_{xy}}{\partial y} + & \frac{\partial \sigma_{yy}}{\partial y} + & \frac{\partial \sigma_{yz}}{\partial z} \\ \frac{\partial \sigma_{xz}}{\partial z} + & \frac{\partial \sigma_{yz}}{\partial z} + & \frac{\partial \sigma_{zz}}{\partial z} \end{vmatrix} \qquad [1.12]$$

In the case of an incompressible medium, the unknowns of the problem are the three components of the velocity vector and the six components of the stress tensor. The continuity equation [1.8] and the equilibrium equation [1.11] give us four equations. The five missing equations will be supplied by the constitutive equation, i.e. a relation between the stress tensor (or its deviator) and the strain rate tensor. The definition of these laws is the purpose of rheology, which is presented in the next section.

1.2. Rheology

The molten polymers discussed in this book are naturally viscoelastic. In fact, in the majority of flows encountered in extrusion, it is the viscous characteristic that will predominate and we will limit ourselves solely to the discussion of this behavior. We refer readers who are interested in viscoelasticity to more specialized sources [FER 70, BIR 77, DEA 90].

1.2.1. *Newtonian behavior*

Let us consider the flow between parallel plates presented in Figure 1.1(a). Viscosity is, by definition, the ratio of the stress exerted upon the top plate to the shear rate:

$$\eta = \frac{\tau}{\dot{\gamma}} \qquad [1.13]$$

This viscosity is constant and does not depend on pressure or temperature. The overall equation of Newtonian behavior can be written in terms of the stress and strain rate tensors:

$$\underline{\underline{\sigma}} = -p\underline{\underline{I}} + 2\eta\underline{\underline{\dot{\varepsilon}}} \qquad [1.14]$$

By applying this ratio to the case of shearing between plates, we obtain a shear stress equal to $\eta\dot{\gamma}$ and three identical normal stresses:

$$\underline{\underline{\sigma}} = \begin{bmatrix} -p & \eta\dot{\gamma} & 0 \\ \eta\dot{\gamma} & -p & 0 \\ 0 & 0 & -p \end{bmatrix} \qquad [1.15]$$

The latter is characteristic of inelastic behavior.

The average viscosity of a molten polymer is approximately 10^2 to 10^4 Pa.s; this is very significant compared to the viscosity of conventional fluids such as water (10^{-3} Pa.s) and oil (10^{-2} to 1 Pa.s).

This has many important consequences for flow, especially since, in the majority of cases, the mass and inertia forces are negligible in comparison to

viscous forces. Therefore, in extrusion, even at high rates, the flow will always be laminar, well below turbulent regimes. Mixing operations will certainly become much more difficult.

By introducing the equation for Newtonian behavior [1.14] into the equilibrium equation [1.11], and after deeming the body forces and inertia forces to be negligible, we obtain the Stokes equations:

$$\text{grad}\, p = \eta \, \Delta \underline{u} \qquad [1.16]$$

linking the pressure gradient to the Laplacian of the velocity field. For simple shear flows between plates, which have already been mentioned, this is reduced to:

$$\frac{\partial p}{\partial x} = \eta \frac{d^2 u}{dy^2}, \ \frac{\partial p}{\partial y} = 0, \ \frac{\partial p}{\partial z} = 0 \qquad [1.17]$$

From this, we deduce that the pressure gradient is constant. For the shear flow (Figure 1.1(a)), this means that the pressure between the two planes is uniform. From this, we can deduce that the velocity profile is linear and independent of viscosity:

$$u(y) = \frac{V}{h} y \qquad [1.18]$$

For the Poiseuille flow (Figure 1.1(b)), by considering the velocities to be zero on the upper and lower planes, we obtain:

$$u(y) = \frac{1}{2\eta} \frac{\Delta p}{L} (h - y) y \qquad [1.19]$$

where L is the flow length. The velocity profile is parabolic. The average velocity is obtained from equation [1.20]. It is proportional to the pressure gradient and inversely proportional to the viscosity.

$$\bar{V} = \frac{h^2}{12\eta} \frac{\Delta p}{L} \qquad [1.20]$$

1.2.2. *General viscous behavior*

It is well known that molten polymers are not Newtonian, and their viscosity depends on shear rate: $\eta = \eta(\dot{\gamma})$. For the most part, we are dealing with shear thinning behavior, which means that the viscosity decreases when the shear rate increases (Figure 1.3).

A certain number of semi-empirical laws have been proposed to describe this behavior. The laws most commonly used are:

– the power law (or Ostwald-de Waele law):

$$\eta = K\dot{\gamma}^{m-1} \qquad [1.21]$$

where K is the consistency of the material (expressed in Pa.s^m) and m is the flow index. The index m has a value of 1 for a Newtonian fluid (thus, we find that $K = \eta$ = constant) and 0 for a rigid plastic body. The index m is generally between 0.2 and 0.5 for the majority of conventional polymers.

– Carreau's law:

$$\eta - \eta_\infty = (\eta_0 - \eta_\infty)\left[\, 1 + (\lambda\dot{\gamma})^2 \,\right]^{\frac{m-1}{2}} \qquad [1.22]$$

where η_0 is the viscosity at low shear rates (Newtonian plateau), η_∞ is the viscosity at very high (or theoretically "infinite") shear rates and λ is the characteristic time of the material. In practice, for molten polymers, $\eta_\infty = 0$. For high shear rates, this law is equivalent to the power law. By contrast, it allows the Newtonian plateau observed at low shear rates to be introduced.

A more general form is the Carreau–Yasuda law:

$$\eta = \eta_0\left[\, 1 + (\lambda\dot{\gamma})^a \,\right]^{\frac{m-1}{a}} \qquad [1.23]$$

The parameter a allows the transition between the Newtonian plateau and the power-law zone to be accurately adjusted.

All of these one-dimensional (1D) laws can be generalized as a tensorial equation:

$$\underline{\underline{\sigma}} = -p\underline{\underline{I}} + 2f(\dot{\bar{\gamma}})\underline{\underline{\dot{\varepsilon}}} \quad [1.24]$$

where $f(\dot{\bar{\gamma}})$ is the 1D law, as defined above, and $\dot{\bar{\gamma}}$ is the generalized shear rate, defined by:

$$\dot{\bar{\gamma}} = \sqrt{2\sum_{i,j}\dot{\varepsilon}_{ij}^2} \quad [1.25]$$

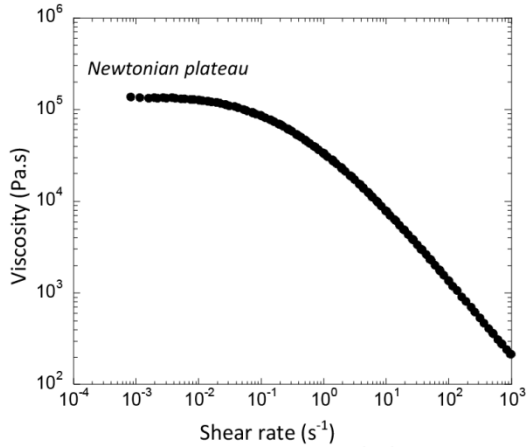


Figure 1.3. *Example of the viscosity curve of a molten polymer*

1.2.3. *Effects on pressure and temperature*

Temperature has a great impact on viscosity. Thermorheologically simple polymers follow the time–temperature superposition principle [FER 70], i.e. we can express the viscosity at temperature T as a function of both its value at a reference temperature T_0 and the shift factor a_T:

$$\eta(\dot{\gamma}, T) = a_T\,\eta(\dot{\gamma}a_T, T_0) \quad [1.26]$$

The latter is often expressed by the Arrhenius equation:

$$a_T = \exp\left[\frac{E_a}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right] \quad [1.27]$$

where E_a is the activation energy and R is the ideal gas constant. E_a varies between 20 and 100 kJ/mole for the majority of polymers, which means that a change of 10°C will cause the viscosity to change by 10–70%.

The viscosity also depends on pressure and is expressed as:

$$\eta(p) = \eta_0 \exp(\chi p) \qquad [1.28]$$

The compressibility coefficient χ is approximately 10^{-8} Pa^{-1}, which means that the effect of pressure becomes apparent beyond several dozen MPa only. This effect will often, therefore, be negligible in extrusion.

1.3. Heat transfer [CAR 59, BIR 60, AGA 14]

1.3.1. *The thermal balance equation*

The variation in temperature over time is controlled by the thermal balance equation, which – in the simplest thermodynamic example, used frequently hereafter – is written as follows:

$$\rho C_p \frac{dT}{dt} = -\text{div}\underline{q} + \dot{W} \qquad [1.29]$$

where:

– C_p is the specific heat capacity;

– $\underline{q}$ is the heat flux;

– $\dot{W}$ is the power dissipated per unit volume;

– dT / dt is the material derivative, which is expressed as:

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \underline{u}\,\text{grad}\,T \qquad [1.30]$$

where $\partial T / \partial t$ is the variation in temperature over time at a fixed point and $\underline{u}\,\text{grad}\,T$ is the amount of heat "induced" by movement, i.e. convection. Obviously, this term is zero if the material is at rest.

The heat flux $\underline{q}$ is most commonly described by Fourier's law:

$$\underline{q} = -k \operatorname{grad} T \qquad [1.31]$$

where k is the thermal conductivity. Therefore, conduction is written as:

$$-\operatorname{div} \underline{q} = k \Delta T \qquad [1.32]$$

where Δ is the Laplace operator.

The power dissipated by deformation is expressed by:

$$\dot{W} = \underline{\underline{\sigma}} \, \underline{\underline{\dot{\varepsilon}}} = \sum_{i,j} \sigma_{ij} \, \dot{\varepsilon}_{ij} \qquad [1.33]$$

From the constitutive law used, we can deduce different expressions, for example:

– $\dot{W} = \eta \dot{\bar{\gamma}}^2$ for a Newtonian fluid;

– $\dot{W} = K \dot{\bar{\gamma}}^{m+1}$ for a power-law fluid.

Thus, for a stationary flow of a Newtonian fluid (see Figure 1.1(a)), the thermal balance equation is written as:

$$\rho C_p u \frac{\partial T}{\partial x} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \eta \left(\frac{V}{h} \right)^2 \qquad [1.34]$$

To calculate the temperature field, it is necessary to add the boundary conditions to this equation, both at the entry and at the wall. At the entry, we generally use an isothermal condition: $T(0) = T_0$. The boundary conditions can be:

– either an imposed temperature: $T(0) = T(h) = T_p$;

– or an imposed heat flux: $\underline{q} = -k \dfrac{\partial T}{\partial \underline{n}} = h_T (T_p - T_\infty)$.

where T_p is the wall temperature, h_T is the heat transfer coefficient and T_∞ is an infinite temperature of the surrounding environment.

In the remainder of this book, heat transfer is mainly considered in two cases:

– the case of a flow inside an extruder or a die;

– the case where a material is cooled at the die exit.

1.3.2. *Heat transfer during flow*

When a molten polymer flows through a piece of equipment, its temperature increases according to the power dissipated by deformation and heat transfer by conduction through the walls. Depending on the respective weights of these different terms, different thermal situations can be observed, which we will review using Poiseuille's flow between parallel plates (Figure 1.1(b)). In most cases, for a simple shear flow, the thermal balance equation in a stationary regime for a Newtonian fluid will be written as follows:

$$\rho C_p u \frac{\partial T}{\partial x} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \eta\left(\frac{du}{dy}\right)^2 \qquad [1.35]$$

We can show that in the x-direction, the conduction (in terms of $k\partial^2 T / \partial x^2$) is negligible compared to the convection (in terms of $\rho C_p u \partial T / \partial x$). Equation [1.35] can thus be simplified into:

$$\rho C_p u \frac{\partial T}{\partial x} = k\frac{\partial^2 T}{\partial y^2} + \eta\left(\frac{du}{dy}\right)^2 \qquad [1.36]$$

1.3.2.1. *The equilibrium regime*

Temperature is, *a priori,* a function of both x and y: there is a temperature profile along the thickness of the flow which will evolve with the latter. After a certain amount of time, equilibrium should be reached, which then does not alter throughout the rest of the flow. An ideal scenario of thermal equilibrium would be when the dissipated power is entirely compensated for by heat transfer through the walls. Using the velocity field equation of this flow (equation [1.19]), we can, therefore, write:

$$k\frac{\partial^2 T}{\partial y^2} = -\eta\frac{36\overline{V}^2}{h^4}(h-2y)^2 \qquad [1.37]$$

By supposing that the temperature imposed on the walls is equal to T_p, the integration of equation [1.37] gives:

$$T(y) = T_p + \frac{3}{4}\frac{\eta\overline{V}^2}{k}\left[1-\left(1-\frac{2y}{h}\right)^4\right] \qquad [1.38]$$

We obtain a quadratic temperature profile, with a maximum at the center of the flow, characterized by:

$$T_{max} = T_p + \frac{3}{4}\frac{\eta\overline{V}^2}{k} \qquad [1.39]$$

In practice, this equilibrium state is hardly ever found in molten polymer flows. In fact, given the weak thermal diffusivity of these products, the time or distance needed to achieve equilibrium would be very high compared to actual processing times or distances.

1.3.2.2. *The adiabatic flow*

Let us now consider the case where conduction is negligible compared to the dissipated power. The thermal balance equation is, therefore, reduced to:

$$\rho C_p u\frac{dT}{dx} = \eta\frac{36\overline{V}^2}{h^4}(h-2y)^2 \qquad [1.40]$$

In this case, the temperature profile will evolve with the flow, without any limitations, since heat exchange with the exterior is excluded. Equation [1.40] can no longer be solved analytically. We can, however, propose an approximate solution in the form of an average temperature. For this, we will consider a slab with thickness dx, whose average temperature is defined by:

$$\overline{T}(x) = \frac{1}{h\overline{V}}\int_0^h u(y)T(y)\,dy \qquad [1.41]$$

The thermal balance equation in terms of average temperature is therefore written as:

$$\rho C_p h \bar{V} \frac{d\bar{T}}{dx} = 36\eta \frac{\bar{V}^2}{h^4} \int_0^h (h-2y)^2 dy \qquad [1.42]$$

that is: $$\frac{d\bar{T}}{dx} = 12\eta \frac{\bar{V}}{\rho C_p h^2} \qquad [1.43]$$

Supposing that the viscosity remains constant and using the average velocity [1.20], by integrating equation [1.43] we obtain:

$$\bar{T}(x) = T_0 + \frac{\Delta p}{\rho C_p} \frac{x}{L} \qquad [1.44]$$

where T_0 is the initial temperature and L is the flow length. We found that the average temperature increases linearly. After a given length L, the increase in temperature is:

$$\Delta T = \bar{T}(L) - T_0 = \frac{\Delta p}{\rho C_p} \qquad [1.45]$$

In this form, we found that the increase in temperature in an adiabatic system is directly proportional to the pressure drop and is independent of viscosity. This value, which is easy to calculate, is overestimated, since heat exchange limits the increase in temperature. We can show that equation [1.45] is valid for any Poiseuille flow, independent of the rheological behavior and the geometry.

1.3.2.3. *Transitory regime*

In fact, in most cases, the terms “conduction” and “dissipation” will be taken into account. We are, therefore, left with the most general form of the thermal balance equation [1.36]. Also, with regard to the numerical solutions of this equation, it is often useful to propose an approximate solution for the average temperature. As in the previous section, by taking into account a slab of material with a thickness of *dx*, we can write:

$$\rho C_p h \bar{V} \frac{dT}{dx} = -2q + 36\eta \frac{\bar{V}^2}{h^4} \int_0^h (h-2y)^2 dy \qquad [1.46]$$

where q is the heat flux exchanged via the surface. This is generally expressed using heat transfer coefficient:

$$q = h_T(\overline{T} - T_p) \quad [1.47]$$

h_T can also be defined as an internal Nusselt number:

$$h_T = \frac{k\,Nu}{h} \quad [1.48]$$

The thermal balance equation for average temperature is written as follows:

$$\frac{d\overline{T}}{dx} = 2\frac{h_T}{\rho C_p \overline{V}}\frac{T_p - \overline{T}}{h} + 12\eta\frac{\overline{V}}{\rho C_p h^2} \quad [1.49]$$

which allows the local evolution of temperature in the section of material dx to be calculated. Later on, we will see many examples where these calculation methods are used to estimate temperature fields.

1.3.3. *Cooling temperature*

Let us consider a film with a thickness of $2e$ and a uniform velocity u exiting a flat die (Figure 1.4).

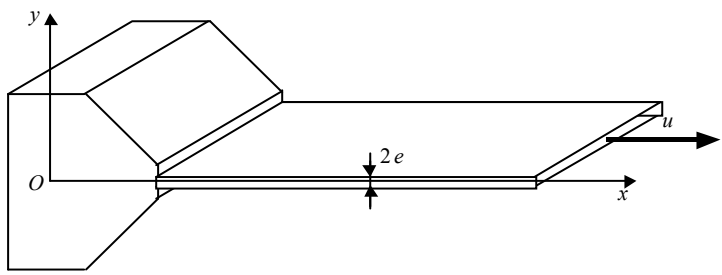


Figure 1.4. *Cooling of a film exiting the die*

If we assume that it is not subjected to stretching, the power dissipated by deformation is negligible. The thermal balance equation is written as: