WAVES SERIES

Acoustics of Fluid Media 1

Principles and Applications

Daniel Juvé
Marie-Annick Galland
Vincent Clair

iSTE

WILEY

Acoustics of Fluid Media 1

Series Editor
Frédérique de Fornel

Acoustics of Fluid Media 1

Principles and Applications

Daniel Juvé
Marie-Annick Galland
Vincent Clair

ISTE

WILEY

First published 2024 in Great Britain and the United States by ISTE Ltd and John Wiley & Sons, Inc.

ISTE Ltd
27-37 St George's Road
London SW19 4EU
UK

www.iste.co.uk

John Wiley & Sons, Inc.
111 River Street
Hoboken, NJ 07030
USA

www.wiley.com

Library of Congress Control Number: 2024940982

British Library Cataloguing-in-Publication Data
A CIP record for this book is available from the British Library
ISBN 978-1-78630-932-7

Contents

List of Abbreviations, Acronyms and Symbols

(ρ, φ, z)	cylindrical coordinates
(r, θ, φ)	spherical coordinates
(x_1, x_2, x_3)	Cartesian coordinates
α_m	random incidence absorption coefficient
α_r	energy reflection coefficient
α_t	energy transmission coefficient
β	thermal dilation coefficient
$\boldsymbol{I}$	(vector) acoustic intensity
$\boldsymbol{k}$	wave vector
M	Mach number
δ	Dirac distribution
γ	adiabatic index
κ_s	adiabatic compressibility coefficient
κ_T	isothermal compressibility coefficient
λ	wavelength
λ_{th}	thermal conductivity coefficient
μ	dynamic viscosity coefficient

μ_B volume viscosity coefficient

∇ nabla differential operator

ν kinematic viscosity coefficient

ω angular frequency

ϕ acoustic potential

ρ density

He Helmholtz number

Pr Prandtl number

Re Reynolds number

σ diffusion cross-section

$\zeta = Z/Z_0$ reduced acoustic impedance

c_0 speed of sound

c_f bending wave velocity

c_p constant pressure specific heat

c_v constant volume specific heat

E Young's modulus

E signal energy

e acoustic energy density

f frequency

f_{coin} coincidence frequency

f_{cr} critical frequency

G_0 Green's function of the Helmholtz equation in free space

g_0 Green's function of the free-space wave equation

H Heaviside function

h specific enthalpy

$H_n^{(2)}$ nth-order Hankel function of the second kind

J_n nth-order Bessel function of the first kind

k_0	acoustic wavenumber
k_f	bending wavenumber
L_I	acoustic intensity level (dB)
L_p	sound pressure level (dB)
L_W	sound power level (dB)
$p'(t)$	acoustic pressure
$p(f)$	complex amplitude of sound pressure
p_{ref}	reference sound pressure
p_{rms}	r.m.s. value of sound pressure fluctuations
R	ideal gas constant
r	gas constant
R_{pp}	autocorrelation of pressure fluctuations
s	specific entropy
S_{pp}	power spectral density of pressure fluctuations
T	absolute temperature
t	amplitude transmission coefficient
T_c	temperature in degrees Celsius
T_{60}	reverberation time
$u'(t)$	acoustic velocity fluctuation
$u(f)$	complex amplitude of acoustic velocity
U	flow velocity
v_g	group speed
v_φ	phase speed
Y_n	nth-order Bessel function of the second kind
Z	specific acoustic impedance
Z_0	characteristic impedance
Z_{ray}	radiation impedance
Zs	surface impedance

Preface

This book is based on courses taught by the authors at the École Centrale de Lyon and at the Université de Lyon, both at the undergraduate and graduate levels. It has also benefited from their interactions with the audience of professional training sessions, held in particular at the Collège de Polytechnique.

The book is intended for undergraduate students and engineering students, as well as graduate students and professionals in industry who are increasingly faced with the need to consider acoustic constraints when developing new products. It is limited to acoustics in fluids, with applications to atmospheric and underwater acoustics.

The book is divided into two volumes. The first is devoted to fundamental elements, the knowledge of which allows for a good mastery of acoustics in fluids. The second is an introduction to more advanced aspects, some of which are the subject of active research and whose status is sometimes still evolving (aeroacoustics, propagation in a moving medium, nonlinear acoustics). Some synthesis problems are also presented, focusing on noise control issues.

Volume 1 consists of 10 chapters plus an appendix of fluid mechanics reminders and a second one with some mathematical elements.

The first two chapters establish the equations of acoustics in homogeneous fluids and describe the properties of plane waves and spherical waves, as fundamental elements in the construction of more general solutions.

Chapter 3 is an interlude in the physical analysis offered throughout the book. It is devoted to elements of signal processing useful to the acoustician, to the definition of sound levels and decibel scales, and to notions of human sound perception and the characterization of the associated nuisances.

Chapter 4 describes the phenomena of reflection and transmission of plane and spherical waves at the interface between two fluids or between a fluid and a solid. In particular, the transmission of plane waves through a thin wall subjected to bending vibrations is discussed.

In Chapter 5, volume acoustic sources associated with mass, force or heat contributions within the fluid are introduced. The powerful method of Green's functions is then extensively discussed and used.

Integral methods, which complement the local formulations used so far, are introduced in Chapter 6, as well as their application to radiation from vibrating surfaces and diffraction by obstacles.

Chapter 7 describes these diffraction phenomena in more detail with their application to the characterization of the efficiency of sound barriers. This is followed by a description of wave scattering exerted by rigid obstacles, with emphasis on low-frequency (Rayleigh scattering) and high-frequency (geometric limit) behavior. The effect of fluid inclusions of low contrast relative to the surrounding medium is addressed within the framework of the Born approximation.

Chapters 8 and 9 deal with guided propagation in ducts, first in the general form using the notion of propagation modes, and then in the low-frequency version of one-dimensional networks. This simplified formulation is very useful for defining acoustic filters such as Helmholtz resonators and passive silencers for selective reduction of sound levels.

Chapter 10 is devoted to the acoustics of confined spaces and applications in room acoustics. The concept of diffuse field and the important notion of reverberation time are introduced, as well as elements for characterizing the acoustic quality of rooms from the point of view of human perception.

Each of these chapters is accompanied by a limited number of exercises, ranging from the simple application of definitions and formulas to problems requiring more advanced theoretical analyses or numerical solutions.

Throughout the book, we have striven to illustrate the theoretical results with many figures obtained from measurements and numerical simulations resulting from the evaluation of complex theoretical formulas or the use of a finite element solver. The purpose of these illustrations is to facilitate the physical interpretation of the phenomena involved by making our own Richard Hamming's aphorism, "The purpose of computing is insight, not numbers". They do not, of course, replace the theoretical developments that allow us to highlight the influence of the most influential parameters. However, theoretical formulations are all too often limited to highly simplified or asymptotic situations. Rather than resorting to too often unintuitive expressions using series of special functions, we have found it preferable, for example, to plot maps of acoustic levels that are much more meaningful.

This aspect distinguishes our work somewhat from the vast existing bibliography. The main works upon which we have relied while writing this book are listed in the references section, which is of course very far from exhaustive. In the text, we sometimes refer to some of these books or articles for additional elements or computational details that we felt it was unnecessary to develop.

We would like to warmly thank all our colleagues at the Acoustic Center of the LMFA at École Centrale de Lyon with whom we have had extensive interactions during our teaching and research activities.

July 2024

1

Equations of Linear Acoustics

In this chapter, we establish the equations that govern the propagation of small-amplitude acoustic waves in fluids under the simplest possible conditions. The fluid is considered as perfect, that is non-viscous and non-heat-conductive. Acoustic disturbances are regarded as small-amplitude perturbations of the ambient state of a time-independent homogeneous fluid at rest. External forces, such as gravity, are not taken into account.

These assumptions may seem very strong, and we will have to relax some of them in later chapters. However, they offer considerable advantages in terms of simplicity, while often remaining unrestrictive in practice. This simplicity makes it possible to emphasize the fundamental properties of acoustic waves, which are generally only slightly modified in more complex situations where, for example, the inhomogeneous nature of the medium will have to be taken into account. They also allow the construction of analytical solutions with very wide applications, which also serve as references for the numerical simulations that become necessary when the geometries of the problems under consideration become complex.

1.1. Validity of the assumptions of linear acoustics and a perfect fluid

It is important to form a first qualitative idea on the legitimacy of the assumptions of a perfect fluid and linearization of fluid dynamics equations by performing order-of-magnitude analyses in a simplified situation. We thus consider a wave of frequency f chosen within the range of audible sounds, for instance, $f = 1$ kHz, and propagating through the air in the x_1 direction only.

To evaluate the validity conditions of the linearization process, we consider the two components of acceleration in the Navier–Stokes equations (these equations are recalled in Appendix 1):

$$\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x_1}$$

where u_1 is the component in the x_1 direction of the velocity associated with the propagation of an acoustic wave. The partial derivative of velocity with respect to time $\partial u_1/\partial t$ is linear with respect to fluctuations, while its convective derivative $u_1 \partial u_1/\partial x_1$ exhibits a quadratic nonlinearity. The order of magnitude of the linear term is ωu_1, where $\omega = 2\pi f$ is the angular frequency of the wave. That of the nonlinear term is u_1^2/λ, where λ is the spatial scale of the wave, called the wavelength. The wavelength is related to the angular frequency by $\lambda = 2\pi c_0/\omega$, where c_0 denotes the speed of propagation of acoustic waves in the medium under consideration; in air, the speed of sound is about 340 m.s^{-1}. The ratio of the orders of magnitude of the nonlinear term to the linear term is therefore $u_1/c_0 = \mathrm{M}_a$; this ratio defines the *acoustic Mach number*. We will see in Chapter 3 that a sound wave with a 94 dB level, which is perceived by humans as very intense and painful, is only associated with a root-mean-square (rms) value of pressure fluctuations of only 1 Pa. For this wave, the acoustic Mach number is extremely low, being about 10^{-5}. In the most unfavorable situations, it will hardly exceed 10^{-3}, which fully justifies why terms composed of products of fluctuations in the equations of motion can be neglected, and thus validates the linearization process of the equations.

To evaluate the validity of the perfect fluid assumption, we simply analyze the influence of viscous terms, since the thermal effects are typically of the same order of magnitude as them. The viscous term in the Navier–Stokes equations:

$$\nu \frac{\partial^2 u_1}{\partial x_1^2}$$

has an order of magnitude $\nu u_1/\lambda^2$, where ν is the kinematic viscosity of the fluid. The ratio of the linear acceleration term to the viscous term is therefore of the order $c_0^2/\nu\omega$, or $c_0\lambda/\nu$, some sort of local acoustic Reynolds number. For air, $\nu \approx 15 \times 10^{-6}$ m^2.s^{-1} and for a 1 kHz frequency, this number is about 10^6. This very high value of the Reynolds number ensures that viscous effects are negligible for the usual range of frequencies.

It is important to note, however, that the above reasoning is qualitative and only has a *local* value, that is to say on the scale of a wavelength. Viscous, thermal and nonlinear effects are cumulative, and their consequences can be significant if the distances traveled by waves correspond to a large number of wavelengths. As a case in point, very high frequency acoustic waves (greater than a hundred kilohertz) propagating in the air are very quickly attenuated by visco-thermal effects over distances of only a few meters. It will therefore sometimes be necessary to take into account dissipative effects, often in an approximate way by an a posteriori correction of a calculation made for a perfect fluid. This is, for example, the case in room acoustics, as we shall see in Chapter 10.

1.2. Linearized equations of fluid dynamics

Since the fluid is now considered perfect and external forces are neglected, the equations of fluid dynamics are reduced to the system of Euler's equations (see Appendix A1.4):

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho U_i)}{\partial x_i} = 0 \quad [1.1]$$

$$\rho \frac{dU_i}{dt} = -\frac{\partial p}{\partial x_i} \quad [1.2]$$

$$\frac{ds}{dt} = 0 \quad [1.3]$$

$$p = p(\rho, s) \quad [1.4]$$

where p, ρ, s and U_i denote fluid pressure, density, entropy and velocity, respectively[1].

The propagation medium is assumed to be time independent, homogeneous and at rest. The variables describing the ambient state (i.e. when there is no acoustic disturbance) are therefore uniform; they will be expressed using a "0" subscript. Since the medium is at rest, the ambient velocity is zero, $\boldsymbol{U}_0 = \mathbf{0}$. Acoustic disturbances, which are time and space dependent, will be indicated by a "prime" symbol. During the propagation of an acoustic wave, the different variables will thus be decomposed according to:

$$p(\boldsymbol{x}, t) = p_0 + p'(\boldsymbol{x}, t) \quad [1.5]$$

$$\rho(\boldsymbol{x}, t) = \rho_0 + \rho'(\boldsymbol{x}, t) \quad [1.6]$$

$$U_i(\boldsymbol{x}, t) = 0 + u_i'(\boldsymbol{x}, t) \quad [1.7]$$

$$s(\boldsymbol{x}, t) = s_0 + s'(\boldsymbol{x}, t) \quad [1.8]$$

The disturbances are assumed to be small enough so that the products of fluctuations that will appear when the above decompositions are introduced into the equations of fluid dynamics can be neglected. This constitutes the fundamental assumption of linear acoustics.

1 The notations are specified in Appendix A1.1.

We now outline the linearization procedure on the conservation of mass equation (or "continuity equation"):

$$\frac{\partial}{\partial t}\left(\rho_0+\rho'\right)+\frac{\partial}{\partial x_i}\left(\rho_0+\rho'\right)\left(0+u_i'\right)=0 \qquad [1.9]$$

which reduces to:

$$\frac{\partial \rho'}{\partial t}+\rho_0\frac{\partial u_i'}{\partial x_i}+\frac{\partial}{\partial x_i}(\rho' u_i')=0 \qquad [1.10]$$

The last term of the left hand side of [1.10] is of second order as the divergence of a product of fluctuations. It is therefore neglected in the linear approximation, and the linearized continuity equation is finally written as:

$$\frac{\partial \rho'}{\partial t}+\rho_0\frac{\partial u_i'}{\partial x_i}=0 \qquad [1.11]$$

To linearize the equations in which the total time-derivative appears, it can be observed that the total derivative of a fluctuation can be reduced to its partial derivative with respect to time, because:

$$\frac{d()'}{dt}=\frac{\partial()'}{\partial t}+u_j'\frac{\partial()'}{\partial x_j}\approx\frac{\partial()'}{\partial t} \qquad [1.12]$$

by neglecting terms of order greater than or equal to 2. To obtain the linearized equations, thanks to the assumptions of homogeneity of the medium and the absence of ambient flow, any total derivative can therefore be replaced by a simple partial derivative with respect to time.

The system of linearized Euler's equations is thus written as:

$$\frac{\partial \rho'}{\partial t}+\rho_0\frac{\partial u_i'}{\partial x_i}=0 \qquad [1.13]$$

$$\rho_0\frac{\partial u_i'}{\partial t}+\frac{\partial p'}{\partial x_i}=0 \qquad [1.14]$$

$$\frac{\partial s'}{\partial t}=0 \qquad [1.15]$$

The last equation shows that the entropy fluctuation associated with an acoustic disturbance is identically zero, which leads to an important simplification. The pressure then depends only on the density and the equation of state takes the simple form $p = p(\rho, s_0) = p(\rho)$.

For the equation of state [1.4], the process of linearization must be carried out differently insofar as there is no explicit form for it, except for the special but important case of ideal gases to which we will return later. The idea is to consider that the presence of acoustic waves only slightly modifies the ambient state; the perturbed variables can then be obtained through a Taylor-series expansion around the ambient state that is limited to the first order. It thus follows that:

$$p(\rho_0+\rho', s_0) = p_0+p' = p(\rho_0, s_0)+\rho' \left(\frac{\partial p}{\partial \rho}\right)_{s,0} + \frac{\rho'^2}{2}\left(\frac{\partial^2 p}{\partial \rho^2}\right)_{s,0} + ... \quad [1.16]$$

Partial derivatives are taken at constant entropy (as recalled by the subscript "s") and evaluated at the ambient state. By definition, $p(\rho_0, s_0) = p_0$ and only considering the first order in the linearization assumption, a simple relation of proportionality between pressure and density fluctuations is therefore obtained.

$$p'(\boldsymbol{x}, t) = \left(\frac{\partial p}{\partial \rho}\right)_{s,0} \rho'(\boldsymbol{x}, t) \quad [1.17]$$

It is important to understand that the coefficient $(\partial p/\partial \rho)_{s,0}$ is a thermodynamic quantity characteristic of the medium in which acoustic waves propagate (but that it is independent of them, since it is evaluated at the ambient state). In thermodynamics, it is shown that this coefficient is strictly positive and it will later be seen that it is equal to the square of the propagation speed of acoustic waves (the speed of sound), denoted by c_0:

$$c_0^2 = \left(\frac{\partial p}{\partial \rho}\right)_{s,0} \quad ; \quad p' = c_0^2 \rho' \quad [1.18]$$

1.3. The wave equation

We therefore have a linear system of three first-order equations for three variables, p', ρ' and u_i'. These equations are generally rearranged to eliminate fluctuations in density and velocity to form a second-order equation for the pressure fluctuation alone, called the wave equation. To this end, the density fluctuation is eliminated using the equation of state [1.18], and then the time derivative of [1.13] is subtracted from the divergence of [1.14]. We obtain:

$$\frac{\partial^2 p'}{\partial x_i^2} - \frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} = \nabla^2 p' - \frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} = 0 \quad [1.19]$$

It can be verified that the density fluctuations ρ' and each of the components of the velocity vector fluctuations u_i' satisfy the same equation due to the linearity of the equations and the homogeneity of the medium.

When one variable is known, it is possible to evaluate the other two using the linearized equations [1.13]–[1.15] and [1.18]. The pressure field is often more directly available than other variables, either experimentally or numerically, and the linearized Euler's equation [1.14] can then be used to determine the acoustic velocity.

1.3.1. *The special case of ideal gases*

Many applications focus on acoustic propagation in air. Since air can be considered with an excellent approximation as an ideal gas under the usual pressure and temperature conditions, the following explicit form can then be used for the equation of state:

$$p = \rho r T \qquad [1.20]$$

where $r = R/M$ is the gas constant, ratio of the ideal gas constant $R = 8.314$ S.I. (J.K^{-1}.mol^{-1}) to the molar mass M of the gas under consideration.

Since the behavior is isentropic, as implied by equation [1.15], we also have the classical relation:

$$p\rho^{-\gamma} = \text{constant} = p_0\rho_0^{-\gamma} \qquad [1.21]$$

where $\gamma = c_p/c_v$ is the specific heat ratio, that is, the ratio of the specific heats at constant pressure and volume, respectively. Expanding this last relation to first order and using [1.18] yields:

$$p' = \frac{\gamma p_0}{\rho_0}\rho' = c_0^2\rho' \qquad [1.22]$$

Therefrom the following relations are derived for the speed of sound:

$$c_0 = \sqrt{\gamma\frac{p_0}{\rho_0}} = \sqrt{\gamma r T_0} \qquad [1.23]$$

For air at a temperature of 20°C (293.15 K), with $\gamma = 1.4$ and $M = 29$ g.mol^{-1}, the speed of sound is about 343 m.s^{-1}.

A simple and useful approximate formula, well adapted to the range of common temperatures, makes it possible to express this speed as a function of temperature in degrees Celsius T_c. By writing $T_0 = 273.15 + T_c$ and assuming that $T_c/T_0 \ll 1$, it follows that:

$$c_0 \approx 331 + 0.6T_c \ (\mathrm{m.s}^{-1}) \qquad [1.24]$$

This formula shows that there is an (approximately) linear relation between the temperature T_c and the speed of sound. This approximation is excellent in the usual temperature range, with the relative error not exceeding 0.3% over the range 0–40°C.

The presence of an acoustic disturbance also causes a temperature fluctuation that can be calculated using the equation of state and the linearized isentropic relation. One obtains:

$$\frac{T'}{T_0} = \frac{\gamma - 1}{\gamma}\frac{p'}{p_0} \qquad [1.25]$$

The ratio p'/p_0 rarely exceeds 10^{-5}, so the associated temperature variations will always be extremely small, about a few millikelvin. This formula also shows that there is a linear relationship between temperature fluctuations and pressure fluctuations. It is therefore also possible to write a wave equation similar to [1.19] for the temperature fluctuation.

1.3.2. *The velocity potential*

The linearized Euler's equation [1.14], written as:

$$\frac{\partial u_i'}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x_i} \qquad [1.26]$$

and in vector notation as:

$$\frac{\partial \boldsymbol{u}'}{\partial t} = -\frac{1}{\rho_0}\nabla p' \qquad [1.27]$$

shows that the acoustic velocity derives from a potential.

Since the curl of a gradient is zero, taking the curl of [1.27] leads to:

$$\frac{\partial}{\partial t}(\nabla \times \boldsymbol{u}') = 0 \qquad [1.28]$$

from which it is deduced that $\nabla \times \boldsymbol{u}' = 0$ if the medium is initially at rest.

Therefore, there exists an *acoustic velocity potential* ϕ defined by:

$$\boldsymbol{u}' = \nabla\phi \tag{1.29}$$

and then

$$\frac{\partial}{\partial x_i}\left(p' + \rho_0\frac{\partial\phi}{\partial t}\right) = 0 \tag{1.30}$$

from which one can deduce:

$$p' + \rho_0\frac{\partial\phi}{\partial t} = f(t) \tag{1.31}$$

and finally

$$p' = -\rho_0\frac{\partial\phi}{\partial t} \tag{1.32}$$

as it is possible to choose a velocity potential such that $f(t) = 0$ without restricting the generality.

The potential ϕ is a solution to the wave equation, just as all other acoustic variables. Although it has no physical meaning of its own, it is a convenient variable to use in a number of calculations. The physical variables acoustic pressure and velocity can then be determined by a simple time and spatial differentiation, respectively.

1.3.3. *Validity conditions for the linearization of equations*

Broadly speaking, the linearization of the equations of motion assumes that the second-order terms are very small with respect to the first-order terms in each equation. We saw in section 1.1 that an order-of-magnitude analysis of Euler's equations in which the time derivatives are assumed to be equivalent to a multiplication by f and the spatial derivatives to a multiplication by $1/\lambda = f/c_0$ leads to $u'/c_0 = \mathrm{M}_a \ll 1$. By use of the linearized continuity and momentum equations, it is verified that this imposes that $\rho'/\rho_0 = \mathrm{M}_a \ll 1$ and $p'/\rho_0 c_0^2 = \mathrm{M}_a \ll 1$. For an ideal gas, the latter condition is equivalent to $p'/p_0 \ll 1$, but this is not the case for liquids, in which the condition $p'/\rho_0 c_0^2 \ll 1$ is much less restrictive than $p'/p_0 \ll 1$. As an example for water, the product $\rho_0 c_0^2$ is about 2×10^9 Pa. An underwater acoustic wave of amplitude 1 atm (10^5 Pa) is therefore still a "small" signal for which the linearization process remains valid.

1.4. Acoustic energy, acoustic intensity and source power

Since the fluid is assumed to be perfect, there is no mechanism in a volume of fluid that can dissipate energy. Thus, in the absence of energy creation from volume or surface sources (which we will introduce later) and of absorbing materials covering surfaces, the total energy and that associated with fluctuations are conserved.

However, considering only linearized equations to construct an equation of conservation of energy, which involves second-order quantities, raises theoretical problems. Strictly speaking, a general equation for the total energy has to be written and then expanded up to second order (see [PIE 89], section I.1). In the following, we will however use the classic approach, based on a combination of the first-order equations, which yields the correct result.

1.4.1. *Definition of acoustic energy and acoustic intensity*

To construct an acoustic energy conservation equation, we form the product of the momentum equation by the acoustic velocity, that is symbolically the product of a force by a velocity (and therefore a power):

$$\rho_0 u_i' \frac{\partial u_i'}{\partial t} = -u_i' \frac{\partial p'}{\partial x_i} = -\frac{\partial (p' u_i')}{\partial x_i} + p' \frac{\partial u_i'}{\partial x_i} \qquad [1.33]$$

Using the linearized continuity equation and eliminating density fluctuations in favor of pressure fluctuations leads to:

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho_0 u_i'^2 + \frac{p'^2}{2\rho_0 c_0^2}\right) + \frac{\partial (p' u_i')}{\partial x_i} = 0 \qquad [1.34]$$

This equation has indeed the general form of a conservation equation that can be written in a condensed manner as:

$$\frac{\partial e}{\partial t} + \nabla \cdot \boldsymbol{I} = 0 \qquad [1.35]$$

with

$$e = \frac{1}{2}\rho_0 u_i'^2 + \frac{p'^2}{2\rho_0 c_0^2} \qquad [1.36]$$

$$\boldsymbol{I} = p' \boldsymbol{u}' \qquad [1.37]$$

The quantity e is called the acoustic energy density. The first term of its expression clearly corresponds to a kinetic energy density; the second can be interpreted as a potential energy density associated with the compression-expansion that the medium experiences when an acoustic wave passes through it. Its form is indeed similar to that of the potential energy of a spring $\frac{1}{2}kx^2$, if the following analogy is introduced: the pressure disturbance p' is identified with the deviation x from the equilibrium position of the spring and $1/\rho_0 c_0^2$ with the stiffness k of the spring. It should also be noted that $1/\rho_0 c_0^2$ is none other than the adiabatic compressibility κ_s of the medium:

$$\kappa_s = -\frac{1}{V^*}\left(\frac{\partial V^*}{\partial p}\right)_{s,0} = \frac{1}{\rho_0}\left(\frac{\partial \rho}{\partial p}\right)_{s,0} = \frac{1}{\rho_0 c_0^2} \qquad [1.38]$$

where V^* denotes the specific volume, the inverse of the density ρ.

$\boldsymbol{I}$ is called the acoustic intensity (vector[2]). This vector is associated with the flux of acoustic energy passing through a surface of unit area, as shown in the integral version of equation [1.35]. If this equation is integrated over a fixed control volume D, bounded by a surface Σ whose external unit normal is denoted $\boldsymbol{n}$ (Figure 1.1(a)), it follows that:

$$\int_D \left(\frac{\partial e}{\partial t} + \nabla \cdot \boldsymbol{I}\right) \mathrm{d}V = 0 \qquad [1.39]$$

and using the divergence theorem (or Gauss' theorem):

$$\frac{\partial}{\partial t}\int_D e \, \mathrm{d}V + \int_\Sigma \boldsymbol{I} \cdot \boldsymbol{n} \, \mathrm{d}\Sigma = 0 \qquad [1.40]$$

This integral form shows that the change over time in the total acoustic energy contained in the volume D is the opposite of the energy flux crossing its boundary Σ.

The acoustic intensity, whose normal component measures the energy transported per unit area and time, is a key quantity for analyzing energy transfers from one spatial region to another and for defining the notion of an acoustic source.

1.4.2. *Acoustic sources*

The acoustic energy conservation equation [1.40] shows that, on average, the flux of acoustic energy through a closed surface is zero.

2 The term *vector* will often be omitted in the following.