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Sung-Soo Kim

Fundamentals and Principles of Electromagnetic Wave Absorbers

From Theory, Design, and Materials to Measurement

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Fundamentals and Principles of Electromagnetic Wave Absorbers

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Sung-Soo Kim
Emeritus Professor
Chungbuk National University
Cheongju, Korea (Republic of)

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To my parents, Gui Taek and Young Sook Kim and to my families, Kyung Sook, Jung Hwan, Hee Jin, Jae Hyung, Hyo Jin, Ha Seong, Jung Woo, and Ga Yoon

Preface

Today, the electromagnetic (EM) radiation from various electronics of high-speed digital and analogue circuits in specific narrow frequency band or wide frequency spectrum is considered a significant challenge with regard to electromagnetic interference (EMI), electromagnetic compatibility (EMC), and reliability of electronic systems. Moreover, the rapid development of the information and communication industries is characterized by high speed of information processing (high frequency) and high integration by chip technology (miniaturization and mobility). High-performance EM wave absorbers are required for electromagnetic wave control to eliminate the interference between electronic devices and systems, particularly with a thin layer thickness at a specific frequency or broad bandwidth from radio frequency even up to a millimeter wave spectrum, depending on their applications. Another area of application is military stealth technology to cope with radar systems by reducing radar cross section (RCS). Radar absorption structures (RAS) and radar absorption materials (RAM) with light weight and wide absorption bandwidth will obviously be needed to achieve RCS reduction.

Entitled *Fundamentals and Principles of Electromagnetic Wave Absorbers: From Theory, Design, Materials to Measurement*, this book consists of three main parts: fundamental theory, design principles and methodology, and potential materials that can be applied to EM wave absorbers. The theoretical part provides the basis of electromagnetism, circuit and transmission line theory, EM wave propagation and reflection, and complex permittivity and permeability by electric polarization and magnetization of materials. The design part describes the design methods for various forms of EM wave absorbers based on equivalent circuit models and simulation techniques. Starting from the traditional resonant absorber, we review the latest metamaterial and frequency selection surface (FSS) absorbers with more advanced design techniques. Recent research results are also included associated with how to design the ultrawide-bandwidth absorbers through multilayering FSSs or shape control of lossy materials. The materials section examines a variety of lossy materials that can be used as EM wave absorbers, including conductive materials, magnetic materials, dielectric materials, core-shell materials, fiber-reinforced composites, metamaterials and metasurfaces. A literature review of electromagnetic properties and EM wave absorption performance is also provided. Finally, the methods and principles for measuring the high-frequency properties (complex permittivity and permeability) and EM wave absorption are described.

This book is suitable for scientists and engineers who are interested in EMI or EMC technology in related fields. By providing a physical and electrical engineering foundation in electromagnetism, it is possible to build up understanding and knowledge of the operating principles and applications of EM wave absorbers. In addition, the included high-frequency physical properties and EM wave absorption characteristics of various absorption materials will be helpful for them to design and select materials suitable for the target application. The book can also be used as a textbook for two semesters of undergraduate or graduate studies. Students will be able to study the physical basis of wave dynamics of EM waves and their interaction with materials. Circuit theory, including transmission lines, is included to model and interpret these physical phenomena as electrical circuits. By introducing high-frequency behavior and loss mechanisms based on electrical conduction, dielectric polarization, and magnetization of materials, the atomic or molecular properties of EM wave absorption materials can be understood. It is hoped that the readers will be able to acquire integrated knowledge and up-to-date information on EM absorber engineering, and this book can be used as a textbook or guidebook for industrial engineers and school students.

Cheongju, Korea (Republic of) Sung-Soo Kim

Acknowledgments

It has been a great blessing and pleasure for me to be able to write this book after retirement through collecting all the lecture materials and research results so far. The main part of this book arose from the research on *electromagnetic wave absorption materials* carried out by the author's research group at Chungbuk National University. I am grateful to all the students who graduated from our lab, and I would not have been able to write this book without their efforts and research products. During the period when I started my research at the Agency for Defense Development, Korea, my collaboration with Dr. Sung-Baek Jo, Dr. Kyung-Il Kwon, and Dr. Kil-Sung Churn became the seed for the publication of this book, and I am very grateful for that. Collaboration and discussions with Dr. Byung-Il Yoon have also been of great help in understanding the physical properties of radar absorption and structural materials. I would like to thank Prof. Young-Cheol Kim for his help in planning and proposing this book. I would like to thank Dr. Kyung-Seop Lee, Mr. Min-Sung Kim, and Ms. Joo-Hee Cho for their help in preparing the manuscript and art production. I would also like to thank the National Research Foundation of Korea for research funding over the long-term period. I am grateful to my parents and my wife for their heartful support. Looking forward to the day when my beloved Ha Seong, Jung Woo, and Ga Yoon will be able to read this booklet.

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Contents

Chapter 1
Fundamentals of Electromagnetics

This chapter describes the fundamentals of electromagnetism required for the design and application of electromagnetic wave absorbers. Electromagnetic spectrum is introduced, with frequency (wavelength) ranges and applications for each frequency band. This chapter also introduces the physical properties of materials in reaction to the external electric and magnetic fields and the associated material parameters (conductivity, permittivity, permeability) that establish the constitutive equations. SI and decibel units are introduced for representing the electromagnetic physical quantities and the gain or loss of power (or voltage) in electronic devices. Sinusoidal harmonic waves and their vector notation, phasor, are described together with their complex number representation. Vector algebra and vector calculus are included for understanding and mathematical representations of electromagnetic field theory and related phenomena. Maxwell's equations are introduced, including Faraday's law related to electromagnetic induction, Ampere's law to explain the generation of magnetic fields by conduction and displacement currents, and Gauss's law related to electric and magnetic fields. Maxwell's equations in integral and differential forms have been described, expressing the above four electromagnetic phenomena.

1.1 Electromagnetic Wave Spectrum

Electromagnetic waves are synchronized oscillating electric and magnetic fields, transmitting through free space with a speed of light. In general, the wave equations are given by the time-harmonic sine (or cosine) functions:

$$E = A\,\sin(\omega t - \beta z) \tag{1.1}$$

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where A is the amplitude, $\omega = 2\pi f$ (f is the frequency) is the angular frequency (in radians/second), and β is the phase constant or wave number (in radians/meter). In Eq. (1.1), $\omega t - \beta z$ is the phase (in radians) of the wave and it depends on both time t and space z. At any point, the sinusoidal variation with time is period, given by $T = 2\pi/\omega = 1/f$. At any time, the variation with distance is wavelength $\lambda = 2\pi/\beta$. In response to the variation in both time and location, the speed of the travelling wave is determined by the movement of a point of a constant phase, which is called the phase velocity v:

$$v = f\lambda \tag{1.2}$$

The speed of electromagnetic waves in free space v_0 is

$$v_0 = 2.997925 \times 10^8 (\mathrm{m/s}) \tag{1.3}$$

All electromagnetic waves travel at the speed of light in a vacuum, but have a wide range of frequencies and associated wavelengths, as shown in Fig. 1.1. The entire electromagnetic spectrum, from the lowest frequency to the highest frequency (from the longest to the shortest wavelength), is based on all radio waves (e.g., commercial radio and television, microwaves, radar), infrared, visible light, ultraviolet, X-rays, and gamma rays [1].

Important applications of electromagnetic waves are communications and radar. Applications on the communication side include telephones, TVs, mobile communications, wireless local area networks (WLANs), and satellite communications. Radar is an electronic system based on measuring the distance from the target and the speed of a moving object, and it has various applications such as aircraft takeoff and landing, missile tracking, vehicle collision avoidance, climate prediction, speed measurement, and remote sensing. Table 1.1 shows the frequency bands and their names (originated as code-names during World War II), frequency range, wavelength, and applications [2]. Frequency range is very broad from HF to mm wave, ranging from 3 MHz to 300 GHz. Correspondingly, the wavelength for the bands is reduced from 100 m to 1 mm.

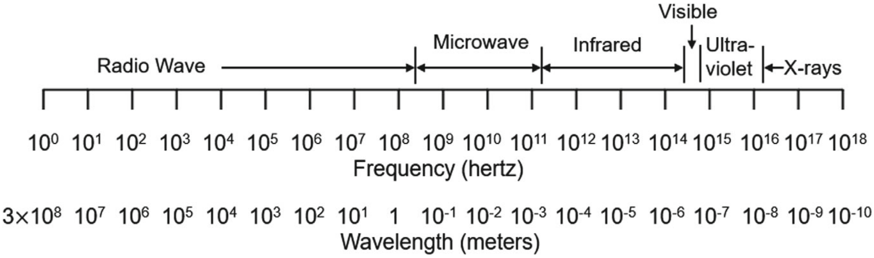

Fig. 1.1 Electromagnetic spectrum and frequency (wavelength)

Table 1.1 Frequency bands for electromagnetic waves and their applications

Frequency band	Band name	Frequency range	Wavelength	Applications
High frequency (HF)	HF	3–30 MHz	10–100 m	Telephone, telegraph, coastal radar systems
Very high frequency (VHF)	P	30–300 MHz	1–10 m	VHF TV, FM radio, very long-range radar
Ultra high frequency (UHF)	UHF	300–1000 MHz	0.3–1 m	UHF TV, satellite, cellular phones, long-range radar
	L	1–2 GHz	15–30 cm	
Super high frequency (SHF)	S	2–4 GHz	7.5–15 cm	Airplane radar, missile guidance, marine and weather radar, speed detector, mapping, airport surveillance
	C	4–8 GHz	3.75–7.5 cm	
	X	8–12 GHz	2.5–3.75 cm	
	Ku	12–18 GHz	1.67–2.5 cm	
	K	18–27 GHz	1.11–1.67 cm	
Extreme high frequency (EHF)	Ka	27–40 GHz	0.75–1.11 cm	Various radar applications, military communications, visual sensor for autonomous vehicles
	mm	40–300 GHz	1–7.5 mm	
	Q	40–60 GHz	5–7.5 mm	
	V	50–75 GHz	4–6 mm	
	W	75–110 GHz	2.7–4 mm	

1.2 Material Parameters and Constitutive Relation

When an electric field E (V/m) and a magnetic field H (A/m) are applied to a material or substance, it exhibits the inherent electromagnetic properties of the material. When an electric field is applied to metals and conductors, the phenomenon of electrical conduction occurs due to the movement of free electrons or mobile ions, which is expressed as the conduction current density J (A/m^2). When electric field is applied to an ionic bonding material such as ceramics, the phenomenon of electrical polarization appears due to the displacement of ions, which is expressed as the electric flux density or displacement D (C/m^2). When electric currents flow through a coil, magnetic field is generated. When the magnetic field is applied to a material, magnetic polarization (magnetization) occurs due to the orbital motion of electrons or the alignment of spins. This is denoted by magnetic flux density B (Wb/m^2). These are summarized in Table 1.2.

The conductivity is a material constant that indicates the electrical conduction, and is denoted by the Greek σ (sigma), and the unit is (S/m). Permittivity is a material constant that indicates the ability of a material to generate the electrical polarization, expressed in the Greek ε (epsilon), and its unit is (F/m). Permeability refers to the magnetization ability of a material to an external magnetic field, written in the Greek μ (mu), and the unit is (H/m).

In a linear medium, the following constitutive relations are established between the applied field and the physical quantities of electromagnetic response that is specific

Table 1.2 Classification of electromagnetic properties of materials in response to applied fields

Properties	Applied field	Electromagnetic response	Material constants
Electric conduction	Electric field	Migration of Coulomb charges (electrons, ions)	Conductivity σ
Electric polarization	Electric field	Displacement of Coulomb charges (electrons, ions)	Permittivity ε
Magnetization	Magnetic field	Alignment of spins or orbital magnetic moment	Permeability μ

to a material or substance:

$$D = \varepsilon E \tag{1.4}$$

$$B = \mu H \tag{1.5}$$

$$J = \sigma E \tag{1.6}$$

If the constitutive parameters ε, μ, σ are different in position, the medium is said to be inhomogeneous. If different in applied strength of electric and magnetic field, the medium is non-linear. If different in direction, the medium is anisotropic. If different in frequency, the parameter is said to be dispersive.

For free space, the permittivity and permeability, denoted by ε_0 and μ_0, have the following values:

$$\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9} \simeq 8.854 \times 10^{-12}(\mathrm{F/m}) \tag{1.7}$$

$$\mu_0 = 4\pi \times 10^{-7}(\mathrm{H/m}) \tag{1.8}$$

The permittivity and permeability of the medium are usually expressed as relative values to free space, $\varepsilon_r = \varepsilon/\varepsilon_0$ and $\mu_r = \mu/\mu_0$, which are called the relative permittivity and relative permeability, respectively.

1.3 Units and Decibels

1.3.1 SI Units

SI unit is an acronym that comes from the French word '*Le Systeme International d'Unites*' and refers to the 'International System of Units' that are now officially adopted and used in most countries of the world. The basic units of SI are MKS

Table 1.3 SI derived units with special names and symbols used in electromagnetics [3]

Quantity	Name	Symbol	in SI units
Phase angle	radian	rad	m/m
Frequency	hertz	Hz	s^{-1}
Force	newton	N	$kg \cdot m/s^2$
Energy, work, heat	joule	J	N·m
Power	watt	W	J/s
Electric charge	coulomb	C	A·s
Voltage	volt	V	J/C, W/A
Capacitance	farad	F	C/V
Resistance, impedance	ohm	Ω	V/A
Electrical conductance	siemens	S	A/V
Magnetic flux	weber	Wb	V·S
Magnetic flux density	tesla	T	Wb/m^2
Inductance	henry	H	Wb/A

units based on meter (m, length), kilogram (kg, mass), and second (s, time), and with the use of ampere (A, electric current), the MKSA unit is formed. To this were added kelvin (K, thermodynamic temperature), candela (cd, luminous intensity), and mole (mol, amount of substance), making it the current seven base units of SI. The fundamental derived units in electromagnetics can be expressed in the SI base units and represented in Table 1.3 [3]. The SI system uses a mixture of basic units and various prefixes. These prefixes represent multiples or submultiples of original unit (for example, THz = 10^{12} Hz and fF = 10^{-15} F, etc.). A list of commonly-used prefixes for the SI units and their abbreviations and etymologies is presented in Table 1.4 [4].

1.3.2 Decibels

An electric system has inputs and outputs, and their ratio is used as one of the indexes that indicate the performance of the system. For the convenience of comparing the relative ratios, the unit decibel (dB) is often used [5]. Decibel is originated from the words of Deci and Bel (in short, B) and means 1/10 of Bel, in which Bel is named after Alexander Graham Bell, the inventor of telephone. dB is a unit that expresses the relative ratio of two power values (e.g., electric power and sound energy) on a logarithmic scale for magnitude comparison. Particularly, dB is useful in electrical and electronic engineering, because the handling range of basic quantities of power, voltage, and current is very wide (e.g., 1 μV to 200 V for voltages).

Taking the amplification circuit as an example (Fig. 1.2a), when the power of the input signal is referred to as P_{in} and the output power P_{out}, the power gain G_p is defined as

Table 1.4 Prefixes for the international system of units (SI)

Base 10	Symbol	Name	Etymology
10^{18}	E	exa	Greek ex, "beyond"
10^{15}	P	peta	Greek petasos, "outreach"
10^{12}	T	tera	Greek teras, "monster"
10^{9}	G	giga	Latin gigas, "giant"
10^{6}	M	mega	Greek megas, "great"
10^{3}	k	kilo	Greek chilioi, "thousand"
10^{2}	h	hecto	Greek hekaton, "hundred"
10^{1}	da	deca	Greek deka, "ten"
10^{-1}	d	deci	Latin decimus, "tenth"
10^{-2}	c	centi	Latin centum, "hundred"
10^{-3}	m	milli	Latin mille, "thousand"
10^{-6}	μ	micro	Greek mikros, "small"
10^{-9}	n	nano	Greek nanos, "dwarf"
10^{-12}	p	pico	Spanish pico, "small quantity"
10^{-15}	f	femto	Danish femten, "fifteen"
10^{-18}	a	atto	Danish atten, "eighteen"

$$G_p = \frac{P_{\text{out}}}{P_{\text{in}}} \tag{1.9}$$

Expressing this in decibels gives

$$G_{p\text{dB}} = 10\,\log\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \tag{1.10}$$

If the power is lost ($P_{\text{out}} < P_{\text{in}}$), we call it power loss which has a negative value in (dB). For example, if input power is 2 (W) and output power is 4 (W), the power gain is $10\,\log(4/2) = 3$ (dB). Conversely, if the output power is reduced to 0.2 (W), it has a value of $10\,\log(0.2/2) = -10$ (dB).

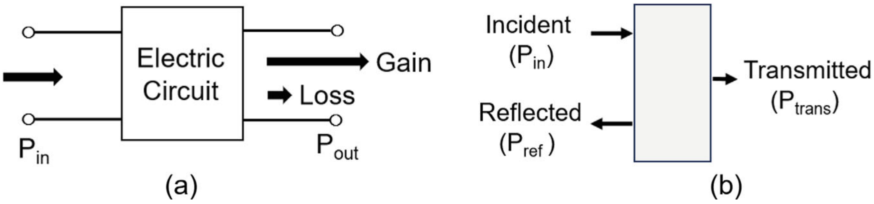

Fig. 1.2 Schematic representation of **a** power gain or power loss in electric circuit and **b** reflection and transmission loss for electromagnetic waves

Reflection and transmission of electromagnetic waves can also be expressed in decibels, as shown in Fig. 1.2b. For the plane wave of incidence with power P_{in}, when the reflected power is P_{ref} and transmitted power is P_{trans}, the reflection loss (RL) and transmission loss (TL) in (dB) are expressed as

$$RL = 10\ \log\left(\frac{P_{\text{ref}}}{P_{\text{in}}}\right) \tag{1.11}$$

$$TL = 10\ \log\left(\frac{P_{\text{trans}}}{P_{\text{in}}}\right) \tag{1.12}$$

The voltage gain G_v is defined as

$$G_v = \frac{V_{\text{out}}}{V_{\text{in}}} \tag{1.13}$$

Expressed in decibels, we get

$$G_{v\text{dB}} = 20\ \log\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right) \tag{1.14}$$

Decibels are also used to compare the absolute values of voltage, current, and power with respect to a reference value. For voltage, dBμV is based on 1 μV as a reference value:

$$\text{dB}\ \mu\text{V} = 20\ \log(\text{volts}/1\ \mu\text{V}) \tag{1.15}$$

Likewise, with a reference value of 1 μA, we use the relative units of dBμA. For power, dBm is often used, indicating the power measurement relative to 1 mW:

$$\text{dBm} = 10\ \log(\text{watts}/1\ \text{mW}) \tag{1.16}$$

The hallmark of decibels is that they can be expressed simply by compressing very large or small values.

1.4 Sinusoidal Harmonic Waves and Phasor

A current that is constant in magnitude and direction regardless of a change in time is called the direct current (DC), and a current that periodically changes in magnitude and direction is called the alternating current (AC). Among alternating currents, the current with a waveform of sine curve is called the sinusoidal wave, which is represented in Fig. 1.3 and expressed as

$$i = I_m \sin(\omega t + \theta) \tag{1.17}$$

where I_m is the amplitude, ω is the angular frequency, and θ is the initial phase.

In the case of AC circuits consisting of resistance R, inductance L, and capacitance C, the voltage and current always have the same frequency. Therefore, only the magnitude and the relative phase of voltage and current are the main subject in circuit analysis. A vector that expresses the voltage and current of a sinusoidal wave in magnitude and initial phase is called the phasor, which can be represented by the position vector in Fig. 1.3 and expressed as

$$I = I_m \angle \theta \tag{1.18}$$

The phasor greatly facilitates the analysis of alternating current and voltage of sinusoidal wave. Because the phasors are quantities with magnitudes and angles, they can be expressed in complex numbers. Complex numbers are functions consisting of real and imaginary parts, expressed as

$$z = a + jb \tag{1.19}$$

where $j = \sqrt{-1}$, and a is the real part and b is the imaginary part, which can be denoted

$$a = \mathrm{Re}(z),\, b = \mathrm{Im}(z) \tag{1.20}$$

$z^* = a - jb$ is called the conjugated complex for $z = a + jb$. Thus, $z + z^* = 2a$, $z - z^* = j2b$, and $zz^* = a^2 + b^2$.

Complex number is represented by a point in the complex plane of Cartesian coordinate in which x-axis represents real part and y-axis represents the imaginary part, as shown in Fig. 1.4a. The polar form of complex numbers in terms of r and θ is shown in Fig. 1.4b, where r is the absolute value and θ is the argument or phase angle. Thus, the following relations are established:

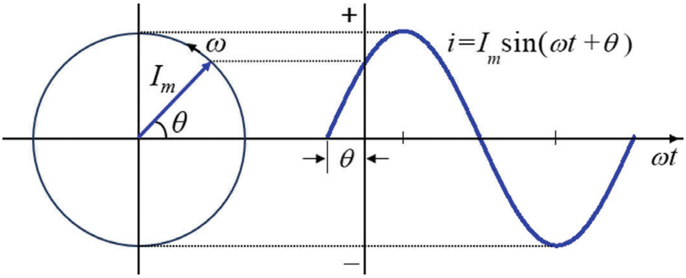

Fig. 1.3 Sinusoidal harmonic wave of current and its vector notation (phasor)

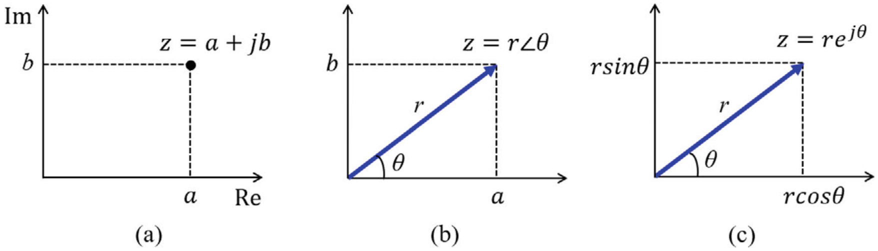

Fig. 1.4 Various representations of complex numbers: **a** rectangular form, **b** polar form, **c** exponential form

$$r = |z| = \sqrt{a^2 + b^2} \tag{1.21}$$

$$\theta = \arg z = \tan^{-1}\left(\frac{b}{a}\right) \tag{1.22}$$

The expression in phasor form is

$$z = r\angle\theta \tag{1.23}$$

A phasor can be expressed in exponential form using the following Euler's formula:

$$e^{j\theta} = \cos\theta + j\sin\theta \tag{1.24}$$

Since $e^{j\theta}$ is a phasor with a magnitude of 1 and a phase angle θ, the phasor can be expressed as an exponential function, as shown in Fig. 1.4c:

$$z = re^{j\theta} = r(\cos\theta + j\sin\theta) \tag{1.25}$$

Complex conjugate $z^* = a - jb$ can be represented in polar and exponential form as $z^* = r\angle - \theta = re^{-j\theta}$.

The argument $e^{j\theta}$ has the meaning of rotating the phasor by θ counterclockwise. Thus, multiplying the imaginary $j(= 1\angle 90°)$, the phasor rotates by $\pi/2$ (rad) counterclockwise, as shown in Fig. 1.5. Likewise, the multiplication of the phasor by $-j$ $(= 1/j = 1\angle - 90°)$ results in the phase shift by $-\pi/2$ (rad), a clockwise rotation.

The addition or subtraction of complex numbers can be done in rectangular form. For two complex numbers, $z_1 = a + jb$ and $z_2 = c + jd$, the addition and subtraction are given by

$$z_1 + z_2 = (a + c) + j(b + d) \tag{1.26}$$

$$z_1 - z_2 = (a - c) + j(b - d) \tag{1.27}$$

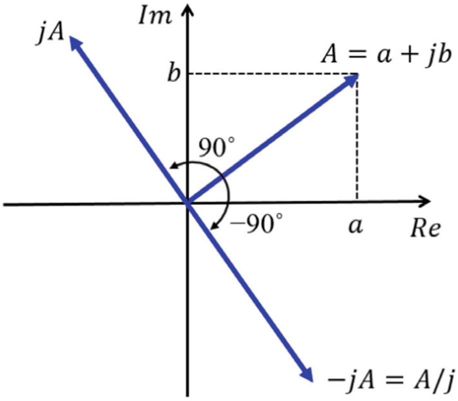

Fig. 1.5 Rotation operator j

The multiplication and division of complex numbers are better carried out using polar form. For two complex numbers, $z_1 = r_1\angle\theta_1$ and $z_2 = r_2\angle\theta_2$, the multiplication and division are given by

$$z_1 z_2 = r_1 r_2 \angle(\theta_1 + \theta_2) \tag{1.28}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle(\theta_1 - \theta_2) \tag{1.29}$$

Power, root, and logarithm of complex numbers are also better carried out using polar form. For a complex number, $z = r\angle\theta = re^{j\theta}$,

$$z^n = r^n \angle n\theta = r^n e^{jn\theta} \tag{1.30}$$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}} \angle \frac{\theta + 2\pi k}{n} = r^{\frac{1}{n}} e^{j\frac{\theta + 2\pi k}{n}} \quad (k = 0, 1, 2, \cdots, (n-1)) \tag{1.31}$$

$$\ln z = \ln r + j(\theta + 2\pi k) \quad (k = 0, 1, 2, \cdots) \tag{1.32}$$

For example, for a given complex number $z = 2\angle\frac{\pi}{3} = 2e^{j\frac{\pi}{3}}$, (a) $z^2 = 4\angle\frac{2\pi}{3} = 4e^{j\frac{2\pi}{3}}$, (b) $\sqrt{z} = \sqrt{2}\angle\frac{\pi}{6} = \sqrt{2}e^{j\frac{\pi}{6}}$ (for $k = 0$) and $\sqrt{z} = \sqrt{2}\angle\frac{7\pi}{6} = \sqrt{2}e^{j\frac{7\pi}{6}}$ (for $k = 1$), (c) $\ln z = \ln 2 + j\left(\frac{\pi}{3}\right)$ (for $k = 0$) and $\ln z = \ln 2 + j\left(\frac{7\pi}{3}\right)$ (for $k = 1$), $\cdots$ for higher values of k.

1.5 Vectors

1.5.1 Vector Operations

A vector is a physical quantity with magnitude and direction. Figure 1.6 shows a vector of $\vec{A}$ of which component is A_x, A_y, A_z in x, y, z Cartesian coordinate, and can be expressed as

$$\vec{A} = A_x a_x + A_y a_y + A_z a_z \tag{1.33}$$

where a_x, a_y, a_z denote the unit vectors along each axis.

Addition and subtraction of two vectors $\vec{A}$ and $\vec{B}$ are carried out component by component,

$$\vec{A} + \vec{B} = (A_x + B_x)a_x + \left(A_y + B_y\right)a_y + (A_z + B_z)a_z \tag{1.34}$$

$$\vec{A} - \vec{B} = (A_x - B_x)a_x + \left(A_y - B_y\right)a_y + (A_z - B_z)a_z \tag{1.35}$$

The dot (scalar) product $\vec{A} \cdot \vec{B}$ of two vectors is expressed as

$$\vec{A} \cdot \vec{B} = AB\ \cos\theta_{AB} = A_x B_x + A_y B_y + A_z B_z \tag{1.36}$$

where θ_{AB} is the angle between two vectors $(0 \le \theta \le \pi)$, as shown in Fig. 1.7a. The cross (vector) product of two vectors $\vec{A}$ and $\vec{B}$ is expressed as

$$\vec{A} \times \vec{B} = AB\ \sin\theta_{AB} a_n \tag{1.37}$$

or

$$\begin{aligned} \vec{A} \times \vec{B} &= \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\ &= \left(A_y B_z - A_z B_y\right)a_x + (A_z B_x - A_x B_z)a_y \\ &\quad + \left(A_x B_y - A_y B_x\right)a_z \end{aligned} \tag{1.38}$$

where a_n is the unit vector normal to the plane containing $\vec{A}$ and $\vec{B}$, as shown in Fig. 1.7b. The direction of a_n is that of the advance of right-hand screw from $\vec{A}$ to $\vec{B}$. Thus, $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$.

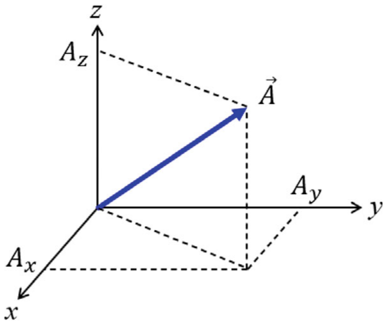

Fig. 1.6 Vector notation in Cartesian coordinate

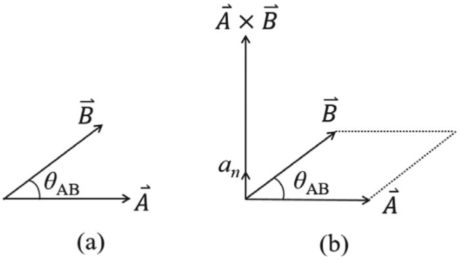

Fig. 1.7 Geometrical representation of vector multiplication: **a** dot product, **b** cross product

1.5.2 Line, Surface, and Volume Integrals

Given a vector field $\vec{A}$ and a path L, as shown in Fig. 1.8, the line integral of vector $\vec{A}$ along the path L is defined as

$$\int_L \vec{A} \cdot \mathrm{d}\vec{l} = \int_L A \cos\theta \, \mathrm{d}l \tag{1.39}$$

where θ is the angle between $\vec{A}$ vector and tangential component of differential curve $\mathrm{d}\vec{l}$. In the case of integrating along a closed curve, which is called the circulation of $\vec{A}$ around path L, Eq. (1.39) is expressed as $\oint_L \vec{A} \cdot \mathrm{d}\vec{l}$.

Given a vector field $\vec{A}$ passing through the curved surface S, as shown in Fig. 1.9, the flux Ψ through S is obtained by the surface integral defined as

$$\Psi = \int_S \vec{A} \cdot \mathrm{d}\vec{S} = \int_S A \cos\theta \, \mathrm{d}S \tag{1.40}$$

where θ is the angle between $\vec{A}$ and unit normal to S. For a closed surface, Eq. (1.40) becomes

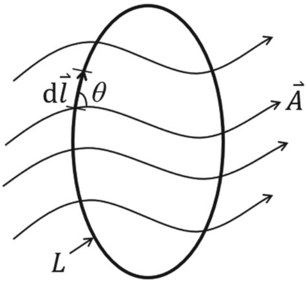

Fig. 1.8 Line integration of vector field $\vec{A}$ along the path L

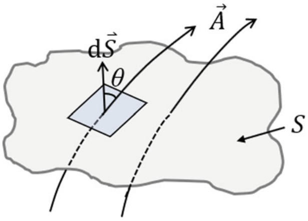

Fig. 1.9 Surface integration of vector $\vec{A}$ through S

$$\Psi = \oint_S \vec{A} \cdot d\vec{S} \tag{1.41}$$

The integration of scalar amount per unit volume (e.g., the charge density ρ_v) over the volume is called the volume integral, and the total charge Q is obtained by the following equation:

$$Q = \int_v \rho_v dv \tag{1.42}$$

1.5.3 Del Operator

Del operator, denoted by ∇, has the meaning of a vector differential operator and is represented in rectangular coordinate:

$$\nabla = \frac{\delta}{\delta x} a_x + \frac{\delta}{\delta y} a_y + \frac{\delta}{\delta z} a_z \tag{1.43}$$

This operator is used as follows:

(a) ∇V means the gradient of the scalar V and is expressed as