YUEPING ZHANG

DIFFERENTIAL ANTENNAS

THEORY AND PRACTICE

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Differential Antennas

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Differential Antennas

Theory and Practice

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IEEE Press

WILEY

Published by John Wiley & Sons, Inc., Hoboken, New Jersey.
Published simultaneously in Canada.

Library of Congress Cataloging-in-Publication Data
Names: Zhang, Yueping, author. | John Wiley & Sons, publisher.
Title: Differential antennas : theory and practice / Yueping Zhang.
Description: Hoboken, New Jersey : Wiley, [2024] | Includes index.
Identifiers: LCCN 2024008700 (print) | LCCN 2024008701 (ebook) | ISBN 9781394273089 (hardback) | ISBN 9781394273102 (adobe pdf) | ISBN 9781394273096 (epub)
Subjects: LCSH: Microstrip antennas–Design and construction. | Microstrip antennas–Data processing.
Classification: LCC TK7871.67.M5 Z48 2024 (print) | LCC TK7871.67.M5 (ebook) | DDC 621.382/4–dc23/eng/20240318
LC record available at https://lccn.loc.gov/2024008700
LC ebook record available at https://lccn.loc.gov/2024008701

Cover Design: Wiley
Cover Image: © Andrey Suslov/Shutterstock

Set in 9.5/12.5pt STIXTwoText by Straive, Pondicherry, India

To the memory of my parents

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About the Author

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His current research interests include the development of antenna-on-chip (AoC) technology for very large-scale antenna integration and characterization of chip-scale propagation channels at terahertz for wireless chip area network (WCAN).

Preface

A differential antenna is an antenna with two terminals connecting a differential signal source. A single-ended antenna is an antenna with a single terminal connecting a single-ended signal source. A dipole is such a differential antenna, while a monopole is such a single-ended antenna. The differential antenna has no ground plane, but the single-ended antenna has the ground plane.

A differential antenna can also be an antenna having two single-ended ports. Each single-ended port must connect a single-ended signal source. The two single-ended signal sources applied on the two single-ended ports should have the same amplitude but opposite phases. The single-ended counterpart of such a differential antenna possesses one single-ended port excited by a single-ended signal source, while the other single-ended port maintains an open-circuit configuration. Both such differential and single-ended antennas contain ground planes.

Single-ended antennas show higher gain values and smaller radiator sizes than differential antennas. Therefore, single-ended antennas have dominated the design of antennas for wireless systems. However, it must be stressed that the concept of lower gains and bulkier sizes of differential antennas compared to single-ended counterparts is not always true. It has been demonstrated that differential microstrip patch antennas can possess comparable or even smaller sizes and higher gain values than single-ended microstrip patch antennas.

Differential circuits permit higher linearity and lower offset, leaving them more immune to power supply variations, temperature changes, and substrate noise than single-ended circuits. Consequently, differential circuits are more popular than single-ended circuits in integrated circuit design. Differential circuits naturally call for differential antennas, which is particularly essential in the design of a wireless system-on-chip (SoC) or system-in-package (SiP) device. Differential antennas perfectly interconnect with differential circuits. No lossy balanced/unbalanced conversion circuit (balun) is needed. As a result, the receiver noise performance and transmitter power efficiency are improved. Differential antennas reduce cross-polarized radiation, remove pattern distortion, and produce improved axial ratio for circular polarization.

This book presents the theory and practice of differential antennas for the first time. An effort has been made to give a theoretical treatment of differential antennas while keeping in mind the aspects of practical applications from simple discrete wire to sophisticated integrated designs in antenna-in-package (AiP) or antenna-on-chip (AoC) technologies.

Chapter 1 introduces some basic concepts associated with differential antennas including balanced and unbalanced antennas, even and odd modes, differential and single-ended circuits, mixed-mode *S*-parameters, and typical baluns. The chapter also highlights that there exists the important ratio of 2 : 1 for many differential and single-ended structures.

Differential wire antennas such as the dipole antenna and the loop antenna are the earliest and most basic antennas. Hence, Chapter 2 begins with the dipole antenna and the loop antenna to analyze differential wire antennas. The chapter ends with the Yagi-Uda antenna to highlight directional differential wire antennas. In addition, the chapter briefly considers the single-ended counterparts of differential dipole, loop, and Yagi-Uda antennas to show the relationship between differential and single-ended wire antennas.

Chapter 3 deals with the slot antenna. It will be treated as complementary to a dipole antenna based on Booker's seminal paper. An emphasis will be given to extend Booker's relation for differential complementary slot and dipole to single-ended counterparts. Then, the self-complementary antennas will be described in terms of the Mushiake's work. Finally, the Yin-Yang antennas will be discussed.

The first microstrip patch antenna was realized by printing a circular conductor patch on a dielectric substrate with a conductor ground plane and feeding it through two coaxial cables in 1970. It is interesting to mention that the first microstrip patch antenna was also a differential microstrip patch antenna, which is the topic of Chapter 4. As a practical concept for solving many antenna system problems, microstrip patch antennas not only gave birth to a new antenna industry but also enriched antenna theory. It was found for the first time that the ratio of the impedance of a differential microstrip patch antenna to its single-ended counterpart is 4 : 1 rather than 2 : 1. This finding has important implications for impedance matching.

A half-wave microstrip patch antenna operates in the fundamental mode. A virtual electric wall exists along the center line between the two radiating edges. Hence, a physical metal wall was proposed to replace the virtual electric wall to create a quarter-wave microstrip patch antenna or a shorted patch antenna. Although the shorted patch antenna preserves the advantages of a patch antenna with a reduced size, it has a major issue of lower *H*-plane cross-polarization level, which limits the shorted patch antenna from many applications. Chapter 5 focuses on the differential shorted patch antenna, and their variations, aiming to improve H-plane cross-polarization level.

Microstrip slot antennas naturally produce bidirectional radiation patterns, while unidirectional radiation patterns can also be obtained through adding reflectors. Microstrip slot antennas are less sensitive to fabrication tolerances and so can be manufactured at lower cost. In addition, microstrip slot antennas do not add weight and size to the system. Hence, they are suitable for applications where cost, size, and weight are of significance. Most of the reported microstrip slot antennas are single-ended designs. Chapter 6 presents differential microstrip slot antennas and compares them with their single-ended counterparts.

The first microstrip grid array antenna was published in 1981. Chapter 7 deals with the analysis and design of differential microstrip grid array antennas. It explains the principle of operation through careful examination of the current distribution and radiation patterns of the microstrip grid array antenna. It compares the novel with the usual differential excitation technique. It describes the design procedure and various design techniques.

A printed antenna is an antenna printed on one surface of a dielectric substrate where no ground plane is printed directly underneath the radiator on the other surface of the dielectric substrate. There are many types of printed antennas. Chapter 8 covers quasi-Yagi, fractal, and spiral antennas for end-fire radiation, multiband, and wideband operations, respectively.

Chapter 9 describes differential antenna measurements with an emphasis on the balun calibration techniques.

Acknowledgments

The author expresses the highest appreciation to a number of his associates and students who assisted him in completing this book, providing data, figures, tables, and materials related to differential antennas. Particular acknowledgment should go to Drs. Junjun Wang, Mei Sun, Zihong Tu, Bing Zhang, Zihao Chen, Guanghui Xu, Zijian Shao, Jun Shu, Yaowei Hou, Ziyang Zheng, Yulin Fang, Binshan Zhao, Junming Ding, Mrs. Wei Lin, Yang Xue, Zhihao Zhu, and Ms. Lin Zhang.

The author thanks Prof. Jianhuan Sheng of Taiyuan University of Technology, Shanxi, China; Dr. Yeongming Hwang and Prof. Pak Chung Ching of the Chinese University of Hong Kong, Hong Kong, China; Prof. Tung Sang Ng of the University of Hong Kong, Hong Kong, China; and Prof. Manh Anh Do of Nanyang Technological University, Singapore, for their guidance.

The author greatly appreciates the long-term collaboration with Profs. Junfa Mao, Linsheng Wu, Min Tang, Liang Zhou, and Changzhan Gu of Shanghai Jiao Tong University, Shanghai, China; Prof. Wenmei Zhang of Shanxi University, Taiyuan, Shanxi, China; Prof. Guoxin Zheng of Shanghai University, Shanghai, China; Prof. Qingxin Chu of South China University of Technology, Guangzhou, Guangdong, China; Prof. Yilong Lu of Nanyang Technological University, Singapore; and Dr. Duixian Liu of IBM Thomas J. Watson Research Center, New York, United States.

The author is grateful to Prof. Xiaowei Zhu and Ms. Yinyue Zhao of Southeast University, Nanjing, Jiangsu, China, for their contribution to the analysis and design of differential magnetoelectric dipole and array.

Finally, the author pays tribute to his wife Aihua Wei, daughter Yu Zhang, and son Yaohua Zhang for their love, support, patience, sacrifice, and understanding.

1

Introduction

1.1 Background

During 1864 and 1867, James Clerk Maxwell established his theory of electromagnetism, which predicted that electric and magnetic fields travel through space as waves moving at the speed of light.[1] Between 1886 and 1889, Heinrich Rudolf Hertz conducted a series of experiments that demonstrated the existence of electromagnetic waves and validated Maxwell's theory.[2] By the mid-1890s, the scientific and technical foundation had been laid for Guglielmo Giovanni Maria Marconi to develop wireless telegraphy systems.[3] At the turn of the 20th century, wireless telegraphy began to be used commercially and wireless telephony was also demonstrated, indicating that the wireless age came.[4] Since then, antennas that made wireless communication possible have opened up many other possibilities.

1.2 Balanced and Unbalanced Antennas

A dipole antenna commonly consists of two identical conductive elements such as metal wires or rods. A loop antenna is usually made of a coil of metal wire or another electrical conductor. A dipole antenna was used as a transmitting antenna and a loop antenna as a receiving antenna by Hertz for the discovery of electromagnetic waves. Hence, the dipole and loop antennas are the earliest antennas. Figure 1.1 shows photos of Hertz's sphere-loaded dipole and loop antennas [1].

1 https://en.wikipedia.org/wiki/James_Clerk_Maxwell
2 https://en.wikipedia.org/wiki/Heinrich_Hertz
3 https://en.wikipedia.org/wiki/Guglielmo_Marconi
4 https://en.wikipedia.org/wiki/Reginald_Fessenden

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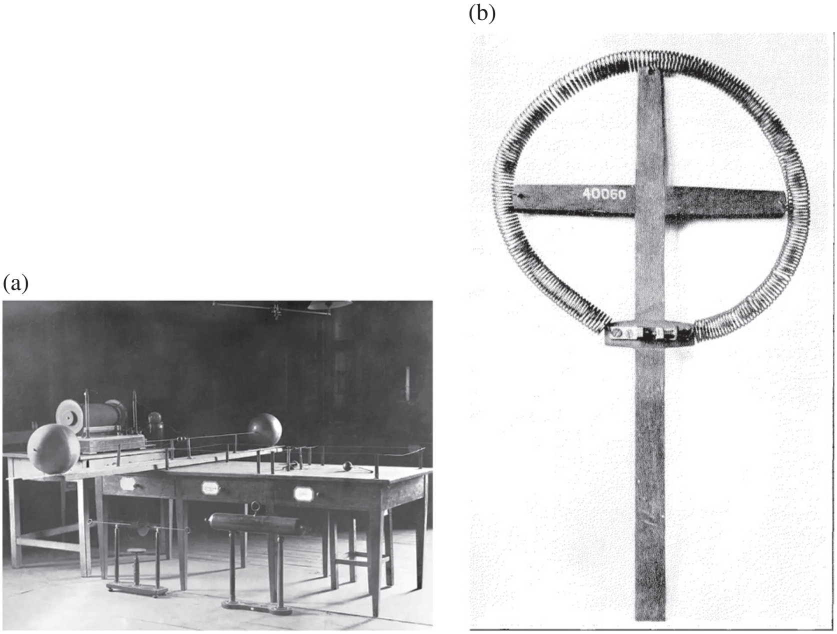

Figure 1.1 Photos of (a) Hertz's dipole and (b) loop antennas. *Sources:* (a) Heinrich Hertz/Public Domain. (b) Rollo Appleyard/Wikimedia Commons/Public domain.

A monopole antenna normally consists of a straight metal wire or rod, often mounted perpendicularly over a conductive surface, called a ground plane. The monopole antenna was invented in 1895 and patented in 1896 by Marconi [2]. He found that the monopole antenna could cover longer distances than the dipole antenna in his radio transmission experiments. Figure 1.2 shows a photo of the monopole antenna setup by Marconi himself at Shanghai Jiaotong University (SJTU) on 8 December 1933.[5]

Antennas can be balanced or unbalanced. A dipole antenna is balanced for its structural symmetry about the feed point, while a monopole antenna is unbalanced for its structural asymmetry about the feed point. It is noted that the old terms of balanced and unbalanced antennas cause confusion. For example, if a dipole antenna is installed parallel to the Earth's surface, it is indeed balanced. However, if the dipole antenna is installed vertically to the Earth's surface, it is unbalanced because one arm of the dipole antenna is closer to the Earth's surface than the other. In addition, if a dipole antenna is fed off-center, the dipole antenna is obviously unbalanced, but the feeding source is balanced. To avoid confusion,

5 https://museum.sjtu.edu.cn/

Figure 1.2 Photo of Marconi and his monopole antenna taken at SJTU on 8 December 1933. *Source:* Shanghai Jiao Tong University/Wikimedia Commons/Public domain.

the new terms of differential and single-ended antennas are adopted in this book. The new terminology is based on feeding sources rather than antenna structures. A differential antenna is an antenna fed with a differential or an equivalently differential signal source. A dipole antenna is a differential antenna. A single-ended antenna is an antenna fed with a single-ended signal source. A monopole antenna is a single-ended antenna.

1.3 Even and Odd Modes

Three principal types of transmission lines are microstrip, stripline, and coplanar waveguide (CPW). They are widely used in modern wireless systems. A transmission line pair can be formed with any of them. Figure 1.3 shows the transmission line pair in a microstrip structure. We use it as an example to describe the even and odd modes of propagation.

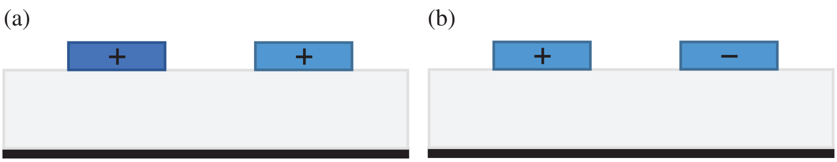

Figure 1.3 Illustration of the transmission line pair in a microstrip structure. (a) Even and (b) odd modes.

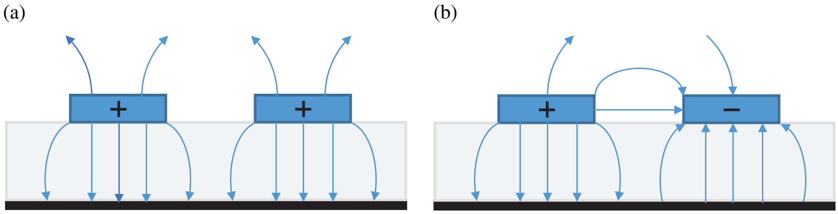

Figure 1.4 Illustration of the electric field distributions (a) for the even and (b) for the odd modes.

The even mode is the mode corresponding to both lines having the same potential V to the ground plane and carrying the identical current in the same direction. The odd mode is the mode corresponding to both lines having opposite potentials V and $-V$, relative to the ground plane and carrying the identical current in the opposite directions. Figure 1.4 shows a sketch of electric field lines for the two modes. Note a magnetic wall exists in the even mode whereas an electric wall exists in the odd mode. The wall separates the whole structure into two identical half structures.

The even mode has an associated characteristic impedance Z_{0e}, which can be calculated from the half structure of the even mode. Similarly, the odd mode corresponds to a characteristic impedance Z_{0o}, which can be calculated from the half structure of the odd mode. It should be able to figure out from the field distributions in Figure 1.4 that Z_{0e} is larger than Z_{0o}.

1.4 Differential and Single-Ended Circuits

A differential circuit deals with the difference between two input signals, while a single-ended circuit accepts a single input signal. Figure 1.5 shows the schematic diagrams of differential and single-ended bipolar junction transistor (BJT) amplifiers. We will use them as examples to illustrate the responses of differential and single-ended circuits to differential and common-mode signals.

Note that both V_1 and V_2 are two input signals. The differential-mode input signal is defined as

$$V_{dm} = \frac{V_1 - V_2}{2} \tag{1.1}$$

and the common-mode input signal as

$$V_{cm} = \frac{V_1 + V_2}{2}. \tag{1.2}$$

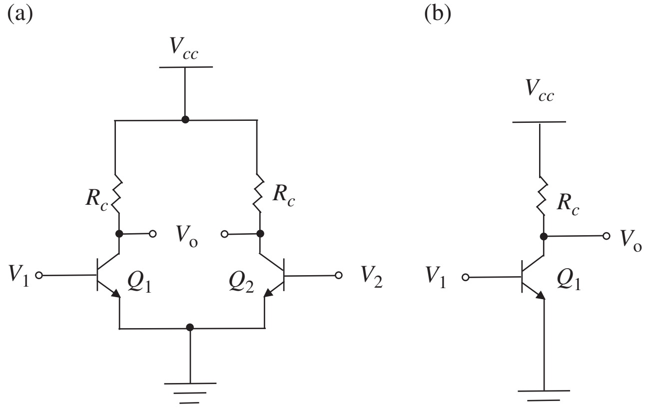

Figure 1.5 Schematic diagrams of (a) differential and (b) single-ended BJT amplifiers.

Thus, the two input signals V_1 and V_2 can be expressed as

$$V_1 = V_{cm} + V_{dm} \tag{1.3}$$

and

$$V_2 = V_{cm} - V_{dm}. \tag{1.4}$$

Assuming that the common-mode signal is an interference, and the differential-mode signal is desirable, the single-ended amplifier will amplify both the interference and desirable signal, while the differential amplifier has the advantage of amplifying only the desirable signal and rejecting the interference.

1.5 An Important Ratio

There exists an important ratio between many differential and single-ended structures. For a differential antenna such as a half-wavelength dipole, it has an input impedance

$$Z_d = 73.1 + j42.5\,\Omega, \tag{1.5}$$

while for a single-ended antenna such as a quarter-wavelength monopole, it has an input impedance

$$Z_s = 36.5 + j21.3\,\Omega. \tag{1.6}$$

Hence, the impedance ratio of the differential dipole to the single-ended monopole is 2 : 1, and so is the size ratio.

Let us turn to the differential and single-ended amplifiers as shown in Figure 1.5. The differential amplifier consists of two transistors and two resistors. The area is assumed to be A_d. The input resistance R_d is given by

$$R_d = \frac{2V_T}{I_B}\ \Omega \tag{1.7}$$

where V_T is the thermal voltage and I_B is the base bias current. The single-ended amplifier consists of one transistor and one resistor. The area is assumed to be A_s. The input resistance R_s is given by

$$R_s = \frac{V_T}{I_B}\ \Omega \tag{1.8}$$

Hence, the ratio of 2 : 1 is also true for the circuits. In Chapter 4, we will show that this important ratio is not always correct.

1.6 Mixed-Mode *S*-Parameters

S-parameters refer to the scattering parameters of a microwave network. Taking a three-port network as an example, the *S*-parameters in a matrix form are expressed as

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \tag{1.9}$$

where the variable a_i represents a power wave incident to port i and the variable b_j a power wave reflected from port j. If each port is terminated in the reference impedance Z_0, the *S*-parameters of the three-port vector network are defined as

$$S_{ij} = \frac{b_i}{a_j} \tag{1.10}$$

where i and j are from 1 to 3 and a_{j-1} and a_{j-2} should be set to zero. *S*-parameters are measured with a two-port vector network analyzer.

Mixed-mode *S*-parameters are used for the characterization of differential structures [3]. Let us reconfigure the three-port network as one single-ended port (port 1) and one differential port (ports 2 and 3). Assuming that a single-ended signal exists at the single-ended port, a differential-mode signal, and a common-mode signal exist at the differential port, we use the nomenclature *s*, *d*, and *c* to represent the single-ended, differential, and common modes, respectively, and we obtain the mixed-mode S-parameters in a matrix form as

$$\begin{bmatrix} b_{s1} \\ b_{d2} \\ b_{c2} \end{bmatrix} = \begin{bmatrix} S_{ss11} & S_{sd12} & S_{sc12} \\ S_{ds21} & S_{dd22} & S_{dc22} \\ S_{cs21} & S_{cd22} & S_{cc22} \end{bmatrix} \begin{bmatrix} a_{s1} \\ a_{d2} \\ a_{c2} \end{bmatrix}. \tag{1.11}$$

Mixed-mode S-parameters are measured with a four-port vector network analyzer. They can also be calculated using the single-ended S-parameters measured by a two-port network analyzer [3], for example,

$$S_{ss11} = S_{11}, \tag{1.12}$$

$$S_{sd12} = S_{ds21} = \frac{1}{\sqrt{2}}(S_{21} - S_{31}), \tag{1.13}$$

$$S_{sc12} = S_{cs21} = \frac{1}{\sqrt{2}}(S_{21} + S_{31}), \tag{1.14}$$

$$S_{dd22} = \frac{1}{2}(S_{22} - S_{23} - S_{32} + S_{33}), \tag{1.15}$$

$$S_{dc22} = \frac{1}{2}(S_{22} + S_{23} - S_{32} - S_{33}), \tag{1.16}$$

$$S_{cd22} = \frac{1}{2}(S_{22} - S_{23} + S_{32} - S_{33}), \tag{1.17}$$

$$S_{cc22} = \frac{1}{2}(S_{22} + S_{23} + S_{32} + S_{33}). \tag{1.18}$$

For a differential antenna, there are two single-ended ports, which can only form one differential or common-mode port. So, the mixed-mode S-parameters for the differential antenna are

$$S_{dd11} = (S_{11} - S_{12}) = S_{d11} \tag{1.19}$$

and

$$S_{cc11} = (S_{11} + S_{12}) = S_{c11}. \tag{1.20}$$

For an array with two differential antenna elements, there are four single-ended ports [4], which can form two differential ports or two common-mode ports. So, the mixed-mode S-parameters for the array are

$$S_{mm} = \begin{bmatrix} S_{dd} & S_{dc} \\ S_{cd} & S_{cc} \end{bmatrix} = \begin{bmatrix} S_{dd11} & S_{dd12} & S_{dc11} & S_{dc12} \\ S_{dd21} & S_{dd22} & S_{dc21} & S_{dc22} \\ S_{cd11} & S_{cd12} & S_{cc11} & S_{cc12} \\ S_{cd21} & S_{cd22} & S_{cc21} & S_{cc22} \end{bmatrix} \tag{1.21}$$

where

$$S_{dd} = \begin{bmatrix} S_{dd11} & S_{dd12} \\ S_{dd21} & S_{dd22} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} S_{11}-S_{13}-S_{31}+S_{33} & S_{12}-S_{14}-S_{32}+S_{34} \\ S_{21}-S_{23}-S_{41}+S_{43} & S_{22}-S_{24}-S_{42}+S_{44} \end{bmatrix}, \tag{1.22}$$

$$S_{dc} = \begin{bmatrix} S_{dc11} & S_{dc12} \\ S_{dc21} & S_{dc22} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} S_{11}+S_{13}-S_{31}-S_{33} & S_{12}+S_{14}-S_{32}-S_{34} \\ S_{21}+S_{23}-S_{41}-S_{43} & S_{22}+S_{24}-S_{42}-S_{44} \end{bmatrix}, \tag{1.23}$$

$$S_{cd} = \begin{bmatrix} S_{cd11} & S_{cd12} \\ S_{cd21} & S_{cd22} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} S_{11}-S_{13}+S_{31}-S_{33} & S_{12}-S_{14}+S_{32}-S_{34} \\ S_{21}-S_{23}+S_{41}-S_{43} & S_{22}-S_{24}+S_{42}-S_{44} \end{bmatrix}, \tag{1.24}$$

$$S_{cc} = \begin{bmatrix} S_{cc11} & S_{cc12} \\ S_{cc21} & S_{cc22} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} S_{11}+S_{13}+S_{31}+S_{33} & S_{12}+S_{14}+S_{32}+S_{34} \\ S_{21}+S_{23}+S_{41}+S_{43} & S_{22}+S_{24}+S_{42}+S_{44} \end{bmatrix}. \tag{1.25}$$

1.7 Balun

A balun is a three-port device, which bridges between the single-ended and the differential structures. It is succinctly defined by the required (ideal) *S*-parameters as

$$S_{11} = 0, \tag{1.26}$$

$$S_{12} = -S_{13} = S_{21} = -S_{31}. \tag{1.27}$$

Note that the single-ended input port is matched to the line characteristic impedance (usually 50 Ω) and the two single-ended output ports will provide the signals of equal amplitude and opposite phase. Also note that the two single-ended output ports are not necessarily matched, may or may not be isolated, and there may be a different return loss for differential and common mode signals.

A balun can take many forms. Three typical baluns that will be used in the design examples of this book are discussed as follows. Figure 1.6 shows a reactive balun. It consists of a reactive 3-dB power divider, a line with the length of l, and another line with the length of $l + \lambda/2$. A third line that has the length of $\lambda/4$ and the characteristic impedance of $\sqrt{2}Z_0$ acts as an impedance transformer to provide impedance matching between the input and output lines of

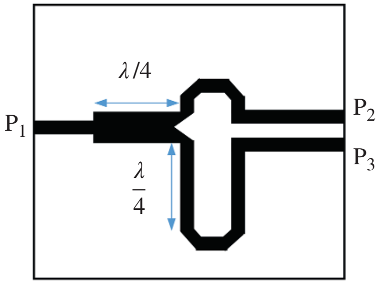

Figure 1.6 Reactive balun.