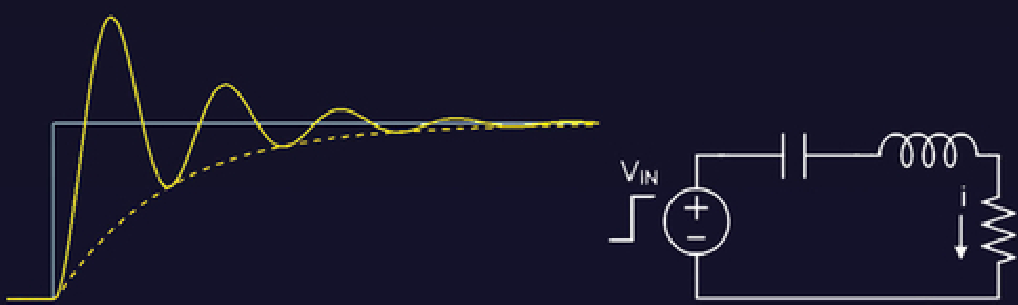
Robert O`Rourke

First and Second Order Circuits and Equations



Technical Background and Insights

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To my wife Evelyn, whose loving support made this book possible.

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About the Author

Robert O'Rourke has an electrical engineering degree from UC Berkeley, including coursework in circuit theory and electromagnetics. Robert has taught electronics and calculus at college level. Robert worked with circuit theory and circuit and system simulation at Analogy, now a part of Synopsys, Inc. During several positions in his career, Robert hired and managed electronic engineers in applications engineering roles that involved circuit theory. Robert has taught circuit and electromagnetic simulation training courses for multiple vendors. As part of Ansys, Robert also created circuit and electromagnetic simulation product training courses for the Ansys Learning Hub.

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The simulations made for this book and the simulation schematics and graphs appearing in this book were made with the QUCS simulator, available from SourceForge.net.

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Part 1

Circuit Elements and Resistive Circuits

1

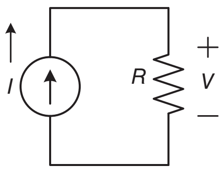
Ohm's Law, Branch Relationships, and Sources

1.1 Chapter Summary and Polarity Reference

1.1.1 Chapter Summary

Ohm's law describes the relationship between voltage, current, and resistance for resistive circuits. This chapter describes Ohm's law and the related circuit elements, resistors, current sources, and voltage sources. This chapter also covers the more general idea of branch relationships, the relation between the voltage across a circuit element and the current through a circuit element, for resistors, voltage sources, and current sources.

Current source and resistor



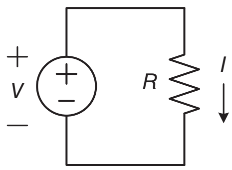
In Figure 1.1a, the current source drives current through the resistor causing a voltage drop across the resistor.

Figure 1.1a Current source driving a resistor.

The ideal independent ***current source*** supplies a fixed amount of current I through the resistor R regardless of the amount of voltage across the source.

The resistor R resists the flow of current through it. The amount of voltage V, that develops across the resistor R, as a result of the current I flowing through the resistor R, is determined by Ohm's law, V equals IR, shown in Equation 1.1a

Voltage source and resistor



In Figure 1.1b, the voltage source across the resistor causes current to flow through the resistor.

Figure 1.1b Voltage source across a resistor.

The ideal independent ***voltage source*** supplies a fixed amount of voltage V across the resistor R regardless of the amount of current through the source.

The resistor R resists the flow of current through it. The amount of current, that flows through the resistor, as a result of the voltage V across the resistor R, is determined by Ohm's law, I equals V over R, shown in Equation 1.1b.

1.1.1.1 Ohm's Law

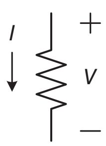


Figure 1.2 Resistor schematic symbol with passive sign convention.

The voltage across a resistor, shown in Figure 1.2, equals the current through the resistor times the resistance of the resistor.

$$V = IR \tag{1.1a}$$

The current through a resistor equals the voltage across the resistor divided by the resistance of the resistor.

$$I = V/R \tag{1.1b}$$

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1.1.1.2 Branch Relationships

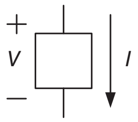


Figure 1.3 Passive sign convention on a generic fictitious schematic symbol.

The branch relationship is the equation describing the relationship between current through a circuit element (in a branch of a circuit) and the voltage across the circuit element. For example, Ohm's law $V = IR$ or $I = V/R$ is the branch relationship of a resistor. Figure 1.3, with a simple square shape, is a generic, non-specific circuit element.

1.1.2 Polarity Reference

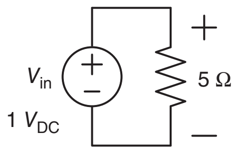


Figure 1.4a 1 volt DC voltage source across a 5 ohm resistor.

In Figure 1.4a, 1 V divided by 5 Ω equals 200 mA (0.2 A).

In each of these two direct current (DC) examples, Figures 1.4a and 1.4b, the current in the loop is 200 mA DC, going down through the resistor from + to −, and the voltage across the 5 Ω resistor is 1 V DC.

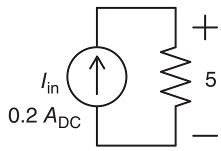
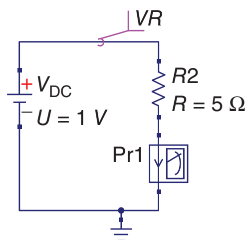


Figure 1.4b 200 mA DC current source driving a 5 ohm resistor.

In Figure 1.4b, 0.2 A multiplied by 5 Ω equals 1 V.

1.1.2.1 DC Voltage Source Polarity Example



Number	Pr1.I	VR.V
1	0.2	1

Figure 1.5 Circuit simulation schematic with avoltage source and a resistor.

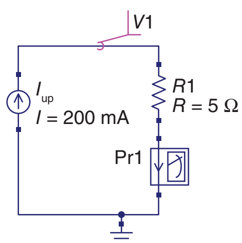
The voltage source in Figure 1.4a applies voltage across the resistor, and the + (plus) and − (minus) signs on the resistor indicate the polarity of the voltage.

Figure 1.5 shows a circuit simulation schematic corresponding to the circuit in Figure 1.4a. The voltage source symbol in Figure 1.5 is specific to DC voltage sources.

There is a current meter in the right-hand leg of the circuit, just below the resistor. The downward arrow in the current meter symbol indicates a reference direction pointing down; the current meter considers clockwise flow of current in the circuit to be positive.

For the DC circuit simulation, there is a table of results in Figure 1.5. VR, measured at the top of the circuit, is positive 1 V, and the current Pr1.I, measured by the current meter, is 200 mA, verifying that current flows downward through the resistor.

1.1.2.2 DC Current Source Polarity Example



Number	Pr1.I	V1.V
1	0.2	1

Figure 1.6 Circuit simulation schematic with a current source and a resistor.

The arrow, pointing up in the current source in Figure 1.4b, indicates the direction of a positive current from the source. Applying Ohm's law, multiplying the current times the resistance, tells us the voltage across the resistor, both the amount and the polarity of the voltage. The + (plus) and − (minus) signs on the resistor indicate the polarity of the voltage.

Figure 1.6 shows a circuit simulation schematic corresponding to the circuit in Figure 1.4b. The downward arrow in the current meter symbol indicates a reference direction pointing down; the current meter considers clockwise flow of current in the circuit to be positive.

In Figure 1.6, the current source arrow points up, telling us that current flows up and out of the current source, and then down through the resistor. As expected from Ohm's law, 200 mA multiplied by 5 Ω yields 1 mV across the resistor. This 1 V result appears in the table under V1.V. The current direction of the current source and the current meter are the same, and the measured current, under Pr1.I, is positive 200 mA.

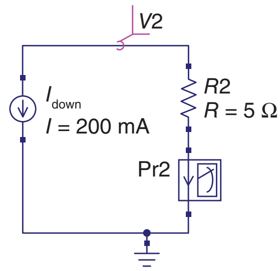
At the top of the schematic in Figure 1.6, there is a V1 marker, indicating a voltage measurement. This voltage is referenced to ground, so it is equivalent to the way schematics show a + (plus) sign above the resistor and a – (minus) sign below the resistor.

In Figure 1.7 the current source arrow points down, telling us that current flows down and out of the current source, and then up through the resistor, the opposite direction from the current source in Figure 1.6.

This makes the measured voltage V2.V –1 V, as it is referenced from the top V2 voltage reference.

As expected from Ohm's law, –0.2 A multiplied by 5 Ω yields – 1 V across the resistor. This result appears under V2.V in units of volts.

In Figure 1.8, the current source arrow indicates counterclockwise circulation of current around the loop. The meter, pointing the same direction in Figure 1.8 as it does in Figure 1.7, indicates opposite circulation direction from the current source and correspondingly the current measured under Pr2.1 is negative 0.2 A.



Number	Pr2.1	V2.V
1	–0.2	–1

Figure 1.7 Circuit simulation schematic with a DC current source, pointed down, and a resistor.

1.1.2.3 Reference Polarity versus Physical Current Flow Direction

It is important to distinguish between actual current flow direction and reference polarity. If we reversed the direction of the current meter in Figure 1.7, the simulation would indicate a positive current value for Pr2.1, but the current still flows counterclockwise in the circuit.

The arrow pointing down, in the current source in Figure 1.8, indicates positive current going down from the current source and then up through the resistor. Using the same + (plus) and – (minus) signs on the resistor as a reference polarity for the resistor, we would get a negative voltage.

In Figure 1.7 the reference direction for the current is down, indicated by an arrow, in both schematics (Figures 1.7 and 1.8). The current polarity will be measured relative to this counter clockwise reference direction.

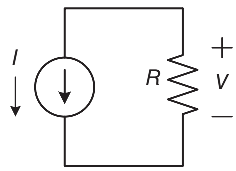


Figure 1.8 Schematic with a current source, pointed up, and a resistor.

1.2 Branch Relationships and *I*–*V* Characteristics

1.2.1 Circuit Element Branch Relationships

1.2.1.1 Ohm's Law is a Resistor's Branch Relationship

Ohm's law $V = IR$, describing the behavior of a resistor, is the branch relationship for a resistor. Figure 1.9 shows how voltage V can be expressed as a function of I (voltage as a dependent variable) or current can be expressed as a function of V (current as the dependent variable).

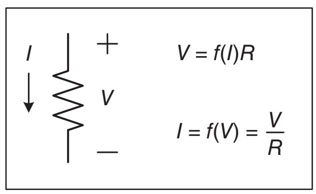


Figure 1.9 Resistor schematic diagram and ohm's law expressions where current or voltage are dependent.

1.2.1.2 Capacitor and Inductor Branch Relationships

The branch relationships for capacitors and inductors involve first derivatives and integrals, as shown in Figure 1.10. The branch relationships for capacitors and inductors will be studied in more detail in subsequent chapters on reactive elements.

Branch Relationships for R, L, and C

Circuit element	Current as a function of voltage	Voltage as a function of current
Resistor	$I = \frac{V}{R}$	$V = IR$
Inductor	$I = \frac{1}{L}\int_0^T V\,dt + I_0$	$V = L\frac{di}{dt}$
Capacitor	$I = C\frac{dV}{dt}$	$V = \frac{1}{C}\int_0^T I\,dt + V_0$

Figure 1.10 Table showing schematic symbols and branch relationships for resistor, inductor, and capacitor.

A circuit elements' defining equation (model) typically describes the current through the element in terms of the voltage across the element.

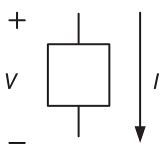


Figure 1.11a Fictitious generic circuit schematic symbol.

Figure 1.11a shows an example of a generic (non-specific) circuit element's schematic symbol, including a chosen sign convention for the voltage across it and the current through it.

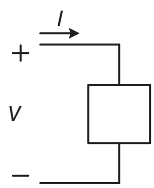


Figure 1.11b Fictitious generic circuit schematic symbol.

Drawn differently than Figure 1.11a, Figure 1.11b shows the same sign convention as Figure 1.11a for the voltage across a circuit element and the current through it that make up an I–V characteristic.

1.2.2 *I–V* Characteristic Plots

The branch relationship is sometimes called the I–V (current-voltage) characteristic of a circuit element. The I–V characteristic can be plotted with current on the vertical (dependent) axis and voltage on the horizontal (independent) axis. Some publications put voltage the vertical axis and current on the horizontal axis.

Resistors obeying Ohm's law are linear devices and this linearity appears as a straight line in the I–V characteristic.

1.2.2.1 *I–V* Characteristics for Circuit Elements

An I–V characteristic of a circuit element is the equation and graph of the current through the circuit element as a function of the voltage across the circuit element. (The shape of the curve in Figure 1.12 is a fictitious generic example.)

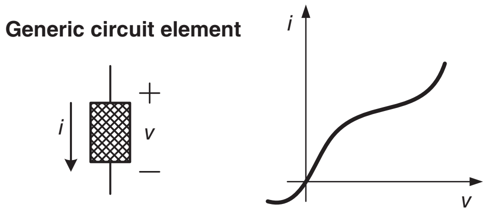


Figure 1.12 Fictitious generic circuit schematic symbol.

1.2.2.2 Resistor *I–V* Characteristic

A linear resistor's I–V characteristic is Ohm's law v equals iR. The I–V characteristic line, shown in Figure 1.13, is straight because there is a linear relationship between current and voltage. The slope of the line is 1 over R (1 over R equals G equals conductance).

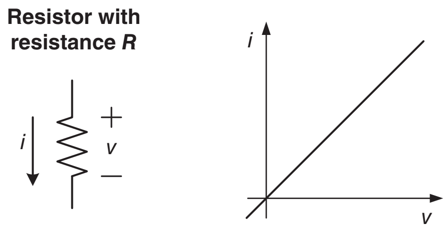


Figure 1.13 Resistor schematic symbol, marked with passive sign convention, adjacent to its linear I-V characteristic graph.

1.2.2.3 Non-resistor *I–V* Example

Diodes have a nonlinear exponential relationship between the voltage across them and the current through them, shown in Figure 1.14 and given by Equation 1.2.

$$i(v) = i_s e^{(qv/kT-1)} \tag{1.2}$$

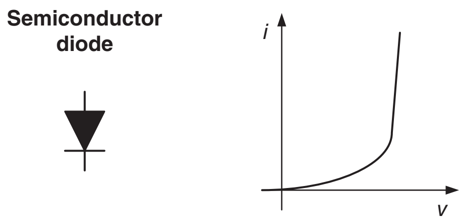


Figure 1.14 Diode schematic symbol and nonlinear *IV* characteristic.

1.2.3 Circuit Elements Models and Schematic Symbols

1.2.3.1 Circuit Elements are Model Descriptions and Schematic Symbols

In electronics, a circuit element is a description (model) and a schematic symbol of a physical device in an electric or electronic circuit, as shown in Figure 1.15 for a resistor. Resistors, capacitors, inductors, opamps (operational amplifiers), transistors, and current sources are all examples of circuit elements. Branch relationships represent the defining model of a lumped circuit element such as a resistor, inductor, or diode.

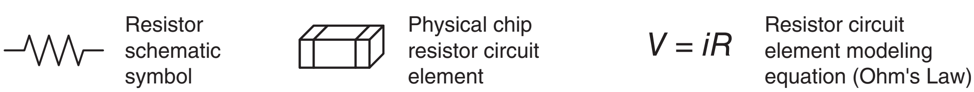


Figure 1.15 Resistor circuit element visual summary including the resistor schematic symbol, a drawing of a chip resistor, and Ohm's law, the branch relationship, and ideal lumped element modeling equation for a resistor.

1.2.3.2 Circuit Schematic Symbols

A circuit schematic symbol represents the circuit element in a schematic of the circuit, as shown in Figure 1.16.

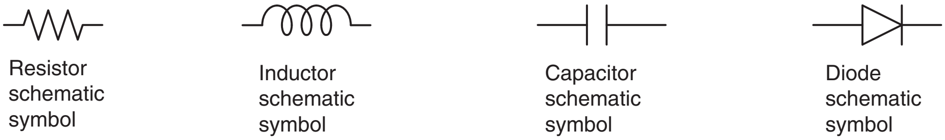


Figure 1.16 Examples of common circuit element schematic symbols.

1.3 Ohm's Law, Resistance, and Resistors

1.3.1 Resistor and Conductor Equations

1.3.1.1 Resistors

Figure 1.17 Resistor schematic symbol.

A resistor, shown in Figure 1.17, is a circuit element which resists the flow of electrical current. An applied voltage is required in order to pass current through a resistor. The resistance R is the ratio of the voltage applied to the current that flows through the resistor. This ratio relationship is called Ohm's law. The unit of resistance is the ohm (Ω).

1.3.1.2 Ohm's Law

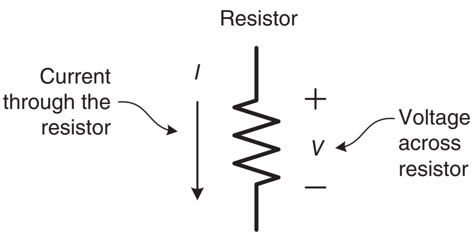


Figure 1.18 Resistor schematic symbol labeled with passive sign convention for visualizing Ohm's law.

Ohm's law states that the voltage across a resistor, shown in Figure 1.18, equals the current through the resistor times the resistance R of the resistor.

Equivalently, Ohm's law states that the current through the resistor equals the voltage across the resistor divided by the resistance R.

$$V = IR \tag{1.3}$$

$$IG = \frac{V}{R} = GV \tag{1.4}$$

$$R = \frac{V}{I} \tag{1.5}$$

Equations 1.3, 1.4, and 1.5 are all statements of Ohm's law, which describes the behavior of a resistor. V is the voltage across the resistor. I is the current through the resistor. R is the resistance and G is conductance, the inverse of resistance, shown in Equation 1.6. The unit of conductance is the siemen. It used to be called the mho.

Ohm's law assumes that the resistor is linear, that is, that the resistance of the resistor does not vary as a function of time, current, or voltage.

$$G = \frac{1}{R} = \frac{I}{V} \tag{1.6}$$

1.3.1.3 Ohm's Law Notation with Time-Dependent Functions

Ohm's law is also valid for currents and voltages that vary as a function of time. (But the resistance is still constant with respect to time, voltage, and current.)