Wavelet Neural Networks

With Applications in Financial Engineering, Chaos, and Classification

Antonios K. Alexandridis • Achilleas D. Zapranis

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To our families

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Preface

Wavelet networks are a new class of networks that combine classic sigmoid neural networks and wavelet analysis. Wavelet networks were proposed as an alternative to feedforward neural networks, which would alleviate the weaknesses associated with wavelet analysis and neural networks while preserving the advantages of each method.

Recently, wavelet networks have gained a lot of attention and have been used with great success in a wide range of applications: financial modeling; engineering; system control; short-term load forecasting; time-series prediction; signal classification and compression; signal denoising; static, dynamic, and nonlinear modeling; and nonlinear static function approximation—to mention some of the most important.

However, a major weakness of wavelet neural modeling is the lack of a generally accepted framework for applying wavelet networks. The purpose of this book is to present a step-by-step guide for model identification for wavelet networks. We describe a complete statistical model identification framework for applying wavelet networks in a variety of ways. Although vast literature on wavelet networks exists, to our knowledge this is the first study that presents a step-by-step guide for model identification for wavelet networks. Model identification can be separated into two parts: model selection and variable significance testing.

A concise and rigorous treatment for constructing optimal wavelet networks is provided. More precisely, the following subjects are examined thoroughly: the structure of a wavelet network; training methods; initialization algorithms; variable significance and variable selection algorithms; model selection methods; and methods to construct confidence and prediction intervals. The book links the mathematical aspects of the construction of wavelet network to modeling and forecasting applications in finance, chaos, and classification. Wavelet networks can constitute a valuable tool in financial engineering since they make no a priori assumptions about the nature of the dynamics that govern financial time series. Although we employ wavelet networks primarily in financial applications, it is clear that they can be utilized in modeling any nonlinear function. Hence, researchers can apply wavelet networks in any discipline to model any nonlinear problem.

Our goal has been to make the material accessible and readable without excessive mathematical requirements: for example, at the level of advanced M.B.A. or Ph.D. students. There is an introduction or tutorial to acquaint nonstatisticians with the basic principles of wavelet analysis, and a similar but more extensive introduction to neural networks for noncomputer scientists: first introducing them as regression models and gradually building up to more complex frameworks.

Familiarity with wavelet analysis, neural wavelets, or wavelet networks will help, but it is not a prerequisite. The book will take the reader to the level where he or she is expected to be able to utilize the proposed methodologies in applying wavelet networks to model various applications.

The book is meant to be used by a wide range of practitioners:

- By quantitative and technical analysts in investment institutions such as banks, insurance companies, securities houses, companies with intensive international activities, and financial consultancy firms, as well as fund managers and institutional investors.
- By those in such fields as engineering, chemistry, and biomedicine.
- By students in advanced postgraduate programs in finance, M.B.A., and mathematical modeling courses, as well as in computational economics, informatics, decision science, finance, artificial intelligence, and computational finance. It is anticipated that a considerable segment of the readership will originate from within the neural network application community as well as from students in the mathematical, physical, and engineering sciences seeking employment in the mathematical modeling services.
- By researchers in identification and modeling for complex nonlinear systems, wavelet neural networks, artificial intelligence, mathematical modeling, and relevant Ph.D. programs.

Supplementary material for this book may be found by entering ISBN 9781118592526 at booksupport.wiley.com.

During the preparation of the book, the help of my (A.K.A.) wife, Christina Ioannidou, was significant, and we would like to thank her for her careful reading of the manuscript.

Canterbury, UK
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October 2013

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1

Machine Learning and Financial Engineering

Wavelet networks are a new class of networks that combine the classic sigmoid neural networks and wavelet analysis. Wavelet networks were proposed by Zhang and Benveniste (1992) as an alternative to feedforward neural networks which would alleviate the weaknesses associated with wavelet analysis and neural networks while preserving the advantages of each method.

Recently, wavelet networks have gained a lot of attention and have been used with great success in a wide range of applications, ranging from engineering; control; financial modeling; short-term load forecasting; time-series prediction; signal classification and compression; signal denoising; static, dynamic, and nonlinear modeling; to nonlinear static function approximation.

Wavelet networks are a generalization of radial basis function networks (RBFNs). Wavelet networks are hidden layer networks that use a wavelet for activation instead of the classic sigmoidal family. It is important to mention here that multidimensional wavelets preserve the "universal approximation" property that characterizes neural networks. The nodes (or *wavelons*) of wavelet networks are wavelet coefficients of the function expansion that have a significant value. In Bernard et al. (1998), various reasons were presented for why wavelets should be used instead of other transfer functions. In particular, first, wavelets have high compression abilities, and second, computing the value at a single point or updating a function estimate from a new local measure involves only a small subset of coefficients.

Wavelet Neural Networks: With Applications in Financial Engineering, Chaos, and Classification,
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In statistical terms, wavelet networks are nonlinear nonparametric estimators. Moreover, the universal approximation property states that wavelet networks can approximate, to any degree of accuracy, any nonlinear function and its derivatives. The useful properties of wavelet networks make them an excellent nonlinear estimator for modeling, interpreting, and forecasting complex financial problems and phenomena when only speculation is available regarding the underlying mechanism that generates possible observations.

In the context of a globalized economy, companies that offer financial services try to establish and maintain their competitiveness. To do so, they develop and apply advanced quantitative methodologies. Neural networks represent a new and exciting technology with a wide range of potential financial applications, ranging from simple tasks of assessing credit risk to strategic portfolio management. The fact that neural and wavelet networks avoid a priori assumptions about the evolution in time of the various financial variables makes them a valuable tool.

The purpose of this book is to present a step-by-step guide for model identification of wavelet networks. A generally accepted framework for applying wavelet networks is missing from the literature. In this book we present a complete statistical model identification framework to utilize wavelet networks in various applications. More precisely, wavelet networks are utilized for time-series prediction, construction of confidence and prediction intervals, classification and modeling, and forecasting of chaotic time series in the context of financial engineering. Although our proposed framework is examined primarily for its use in financial applications, it is not limited to finance. It is clear that it can be adopted and used in any discipline in the context of modeling any nonlinear problem or function.

The basic introductory notions are presented below. Fist, financial engineering and its relationship to machine learning and wavelet networks are discussed. Next, research areas related to financial engineering and its function and applications are presented. The basic notions of wavelet analysis and of neural and wavelet networks are also presented. More precisely, the basic mathematical notions that will be needed in later chapters are presented briefly. Also, applications of wavelet networks in finance are presented. Finally, the basic aspects of the framework proposed for the construction of optimal wavelet networks are discussed. More precisely, model selection, variable selection, and model adequacy testing stages are introduced.

FINANCIAL ENGINEERING

The most comprehensive definition of financial engineering is the following: *Financial engineering* involves the design, development, and implementation of innovative financial instruments and processes, and the formulation of creative solutions to problems of finance (Finnerty, 1988). From the definition it is clear that financial engineering is linked to innovation. A general definition of financial innovation includes not only the creation of new types of financial instruments, but the development and evolution of new financial institutions (Mason et al., 1995). Financial innovation is the driving force behind the financial system in fulfilling its primary function: the most efficient possible allocation of financial resources (Ζαπράνης, 2005). Investors, organizations, and companies in the financial sector benefit from financial innovation. These benefits are reflected in lower funding costs, improved yields, better management of various risks, and effective operation within changing regulations.

In recent decades the use of mathematical techniques and processes, derived from operational research, has increased significantly. These methods are used in various aspects of financial engineering. Methods such as decision analysis, statistical estimation, simulation, stochastic processes, optimization, decision support systems, neural networks, wavelet networks, and machine learning in general have become indispensable in several domains of financial operations (Mulvey et al., 1997).

According to Marshall and Bansal (1992), many factors have contributed to the development of financial engineering, including technological advances, globalization of financial markets, increased competition, changing regulations, the increasing ability to solve complex financial models, and the increased volatility of financial markets. For example, the operation of the derivatives markets and risk management systems is supported decisively by continuous advances in the theory of the valuation of derivatives and their use in hedging financial risks. In addition, the continuous increase in computational power while its cost is being reduced makes it possible to monitor thousands of market positions in real time to take advantage of short-term anomalies in the market.

In addition to their knowledge of economic and financial theory, financial engineers are required to possess the quantitative and technical skills necessary to implement engineering methods to solve financial problems. Financial engineering is a unique field of finance that does not necessarily focus on people with advanced technical backgrounds who wish to move into the financial area but, is addressed to those who wish to get involved in investment banking, investment management, or risk management.

There is a mistaken point of view that financial engineering is accessible only by people who have a strong mathematical and technical background. The usefulness of a financial innovation should be measured on the basis of its effect on the efficiency of the financial system, not on the degree of novelty that introduces. Similarly, the power of financial engineering should not be considered in the light of the complexity of the models that are used but from the additional administrative and financial flexibility that it offers its users. Hence, financial engineering is addressed to a large audience and should be considered within the broader context of the administrative decision-making system that it supports.

FINANCIAL ENGINEERING AND RELATED RESEARCH AREAS

Financial engineering is a very large multidisciplinary field of research. As a result, researchers are often focused on smaller subfields of financial engineering. There are two main branches of financial engineering: *quantitative finance* and *financial econometrics*. Quantitative finance is a combination of two very important and

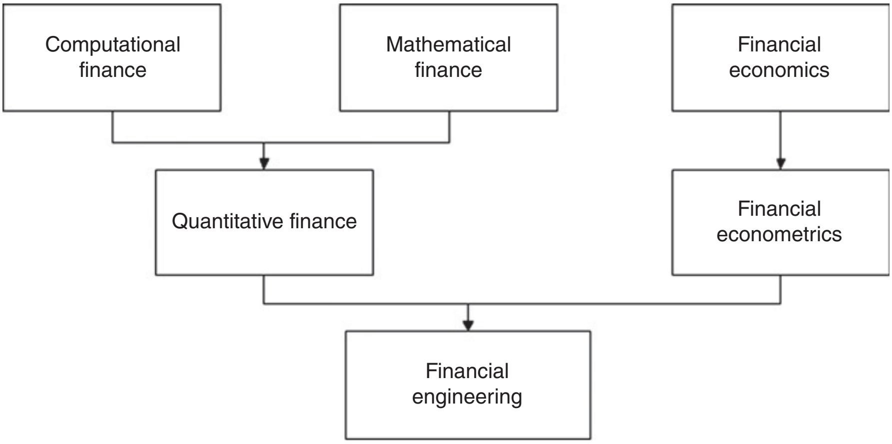

Figure 1.1 *Research areas related to financial engineering.*

popular subfields of finance: *mathematical finance* and *computational finance*. On the other hand, financial econometrics arises from *financial economics*. Research areas related to financial engineering are illustrated in Figure 1.1.

The scientific field of financial engineering is closely related to the relevant disciplinary areas of mathematical finance and computational finance, as all focus on the use of mathematics, algorithms, and computers to solve financial problems. It can be said that financial engineering is a multidisciplinary field involving financial theory, the methods of engineering, the tools of mathematics, and the practice of programming. However, financial engineering is focused on applications, whereas mathematical finance has a more theoretical perspective.

Mathematical finance, a field of applied mathematics concerned with financial markets, began in the 1970s. Its primary focus was the study of mathematics applied to financial concerns. Today, mathematical finance is an established and very important autonomous field of knowledge. In general, financial mathematicians study a problem and try to derive a mathematical or numerical model by observing the output values: for example, market prices. Their analysis does not necessarily have a link back to financial theory. More precisely, mathematical consistency is required, but not necessarily compatibility with economic theory.

Mathematical finance is closely related to computational finance. More precisely, the two fields overlap. Mathematical finance deals with the development of financial models, and computational finance is concerned with their application in practice. Computational finance emphasizes practical numerical methods rather than mathematical proofs, and focuses on techniques that apply directly to economic analyses. In addition to a good knowledge of financial theory, the background of people working in the field of computational finance combines fluency in fields such as algorithms, networks, databases, and programming languages (e.g., C/C++, Java, Fortran).

Today, the disciplinary area of mathematical finance and computational finance constitutes part of a larger, established, and more general area of finance called *quantitative finance*. In general, there are two main areas in which advanced mathematical and computational techniques are used in finance. One tries to derive mathematical formulas for the prices of derivatives, the other one deals with risk and portfolio management.

Financial econometrics is another field of knowledge closely related (although more remote) to financial engineering. Financial econometrics is the basic method of inference in the branch of economics termed financial economics. More precisely, the focus is on decisions made under uncertainty in the context of portfolio management and their implications to the valuation of securities (Huang and Litzenberger, 1988). The objective is to analyze financial models empirically under the assumption of uncertainty in the decisions of investors and hence in market prices. For example, the martingale model for capital asset pricing is related to mathematical finance. However, the empirical analysis of the behavior of the autocorrelation coefficient of the price changes generated by the martingale model is the subject of financial econometrics.

We illustrate the various subfields of financial engineering by the following example. A financial economist studies the structural reasons that a company may have a certain share price. A financial mathematician, on the other hand, takes the share price as a given and may use a stochastic model in an attempt to derive the corresponding price of a derivative with the stock as an underlying asset. The fundamental theorem of arbitrage-free pricing is one of the key theorems in mathematical finance, while the differential Black–Scholes–Merton approach (Black and Scholes, 1973) finds applications in the context of pricing options. However, to apply the stochastic model, a computational translation of the mathematics to a computing and numerical environment is necessary.

FUNCTIONS OF FINANCIAL ENGINEERING

Financial engineers are involved in many important functions in a financial institution. According to Mulvey et al. (1997), financial engineering is used widely in four major functions in finance: (1) corporate finance, (2) trading, (3) investment management, and (4) risk management (Figure 1.2). In corporate finance, large-scale businesses are interested in raising funds for their operation. Financial engineers develop new instruments or enhance existing ones in order to secure these funds. Also, they are involved in takeovers and buyouts. In trading of securities or derivatives, the objective of a financial engineer is to develop new dynamic trading strategies. In investment management the aim is to develop new investment vehicles for investors.

Examples presented by Mulvey et al. (1997) include high-yield mutual funds, money market funds, and the repo market. In addition, they develop systems for transforming high-risk investment instruments to low-risk instruments by applying techniques such as repackaging and overcollaterization. Finally, in risk management, a financial engineer must, on the one hand, assess the various types of risk of a range of securities and, on the other hand, use the appropriate methodologies and tools to construct portfolios with the desired levels of risk and return. These methodological approaches relate primarily to portfolio insurance, portfolio immunization

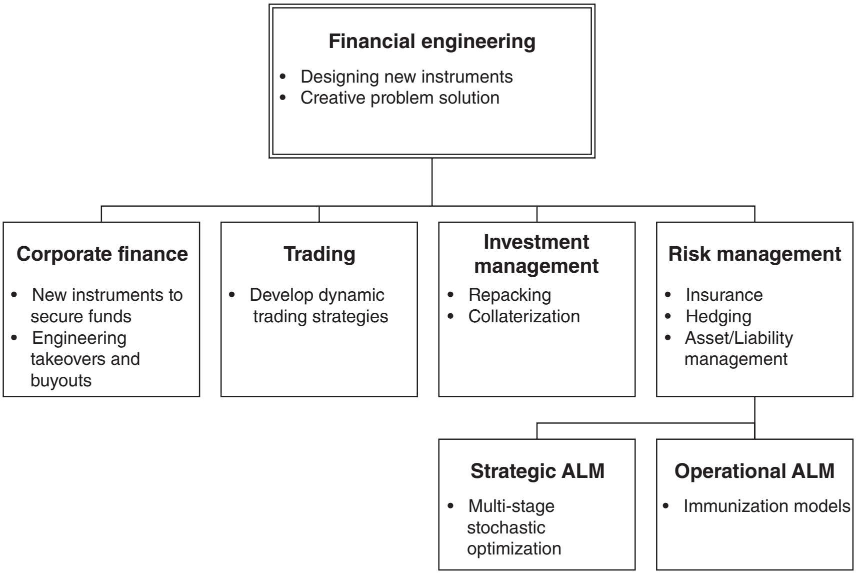

Figure 1.2 *Financial engineering activities according to Mulvey et al. (1997).*

against changes in certain financial variables, hedging, and efficient assets/liability management.

Risk management is a crucial part of corporate financial management. The interrelated areas of risk management and financial engineering find direct applications in many problems of corporate financial management, such as assessment of default risk, credit risk, portfolio selection and management, sovereign and country risk, and financial programing, to name a few.

During the past three decades, a series of new scientific tools derived from the wider field of operations research and artificial intelligence has been developed for the most realistic and comprehensive management of financial risks. Techniques that have been proposed and implemented include multicriteria decision analysis, expert systems, neural networks, genetic and evolutionary algorithms, fuzzy networks, and wavelet networks. A typical example is the use of neural networks by Zapranis and Sivridis (2003) to estimate the speed of inversion within the Vasicek model, used to derive the term structure of short-term interest rates.

APPLICATIONS OF MACHINE LEARNING IN FINANCE

Neural networks and machine learning in general are employed with considerable success in primarily three types of applications in finance: (1) modeling for classification and prediction, (2) associative memory, and (3) clustering (Hawley et al., 1990). The use of wavelet networks is shown in Figure 1.3.

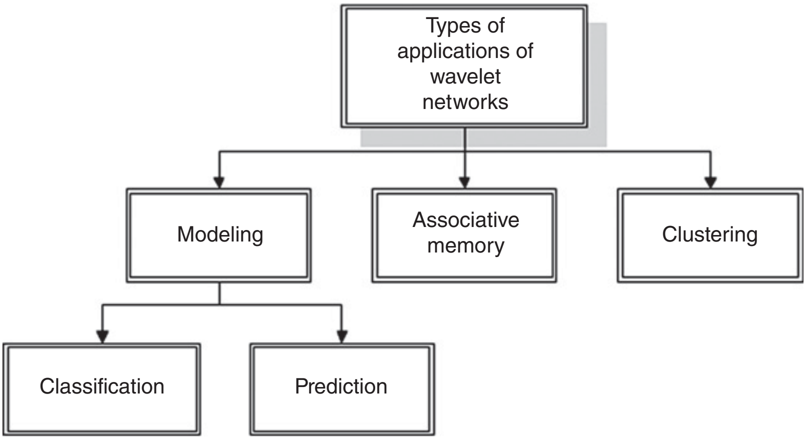

Figure 1.3 *Types of applications of wavelet networks.*

Classification includes assignment of units in predefined groups or classes, based on the discovery of regularities that exist in the sample. Generally, nonlinear non-parametric predictors such as wavelet networks are able to classify the units correctly even if the sample is incomplete or additional noise has been added. Typical examples of such applications are the visual recognition of handwritten characters and the identification of underwater targets by sonar. In finance, an example of a classification application could be the grouping of bonds, based on regularities in financial data of the issuer, into categories corresponding to the rating assigned by a specialized company. Other examples are the approval of credit granting (the decision as to who receives credit and how much), stock selection (classification based on the anticipated yield), and automated trading systems.

The term *prediction* refers to the development of mathematical relationships between the input variables of the wavelet network and usually one (although it can be more) output variable. Artificial networks expand the common techniques that are used in finance, such as linear and polynomial regression and autoregressive moving averages (ARMA and ARIMA). In finance, machine learning is used mainly in classification and prediction applications. When a wavelet network is trained, it can be used for the prediction of a financial time series. For example, a wavelet network can be used to produce point estimates of the future prices or returns of a particular stock or index. However, financial analysts are usually also interested in confidence and prediction intervals. For example, if the price of a stock moves outside the prediction interval, a financial analyst can adjust the trading strategy.

In *associative memory* applications the goal is to produce an output corresponding to the class or group desired, based on one input vector presented in the neural network that determines which output is to be produced. For example, the input vector may be a digitized image of a fingerprint, and the output desired may be reconstruction of the entire fingerprint.

Clustering is used to group a large number of different input variables, each of which, however, has some similarities with other inputs. Clustering is useful for compression or filtering of data without the loss of a substantial part of the information. A financial application could be the creation of clusters of corporate bonds that correspond to uniform risk classes based on data from financial statements. The number and composition of the classes will be determined by the model, not by the user. In this case, in contrast to the use of classification, the categories are not predetermined. This method could provide an investor with a diversified portfolio.

FROM NEURAL TO WAVELET NETWORKS

In this section the basic notions of wavelet analysis, neural networks, and wavelet networks are presented. Our purpose is to present the basic mathematical and theoretical background that is used in subsequent chapters. Also, the reasons that motivated the combination of wavelet analysis and neural networks to create a new tool, wavelet networks, are discussed.

Wavelet Analysis

Wavelet analysis is a mathematical tool used in various areas of research. Recently, wavelets have been used especially to analyze time series, data, and images. Time series are represented by local information such as frequency, duration, intensity, and time position, and by global information such as the mean states over different time periods. Both global and local information is needed for the correct analysis of a signal. The wavelet transform (WT) is a generalization of the Fourier transform (FT) and the windowed Fourier transform (WFT).

Fourier Transform The attempt to understand complicated time series by breaking them into basic pieces that are easier to understand is one of the central themes in Fourier analysis. In the framework of Fourier series, complicated periodic functions are written as the sum of simple waves represented mathematically by sines and cosines. More precisely, Fourier transform breaks a signal down into a linear combination of constituent sinusoids of various frequencies; hence, the Fourier transform is decomposition on a frequency-by-frequency basis.

Let $f: \mathbb{R} \to \mathbb{C}$ be a periodic function with period $T > 0$ that satisfies

$$\|f\|^2 = \int_{-\infty}^{\infty} |f|^2\, dt < \infty \tag{1.1}$$

Then its FT is given by

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} e^{-2\pi i\omega t} f(t)\, dt \tag{1.2}$$

and its Fourier coefficients are given by

$$c_n = \int_{-\infty}^{\infty} e^{-2\pi i \omega_n t} f(t)\, dt \tag{1.3}$$

where $\omega_n = n/T$ and

$$e^{2\pi i \omega_n} = \cos 2\pi\omega_n + i \sin 2\pi\omega_n \tag{1.4}$$

In a common interpretation of the FT given by Mallat (1999), the periodic function $f(t)$ is considered as a musical tone that the FT decomposes to a linear combination of different notes c_n with frequencies ω_n. This method allows us to compress the original signal, in the sense that it is not necessary to store the entire signal; only the coefficients and the corresponding frequencies are required. Knowing the coefficients c_n, one can synthesize the original signal $f(t)$. This procedure, called *reconstruction*, is achieved by the inverse FT, given by

$$f(t) = \int_{-\infty}^{\infty} e^{2\pi i \omega t} \hat{f}(\omega)\, d\omega \tag{1.5}$$

The FT has been used successfully in a variety of applications. The most common use of FT is in solving partial differential equations (Bracewell, 2000), in image processing and filtering (Lim, 1990), in data processing and analysis (Oppenheim et al., 1999), and in optics (Wilson, 1995).

Short-Time Fourier Transform (Windowed Fourier) Fourier analysis performs extremely well in the analysis of periodic signals. However, in transforming to the frequency domain, time information is lost. When looking at the Fourier transform of a signal, it is impossible to tell when a particular event took place. This is a serious drawback if the signal properties change a lot over time: that is, if they contain nonstationary or transitory characteristics: drift, trends, abrupt changes, or beginnings and ends of events. These characteristics are often the most important part of a time series, and Fourier transform is not suited to detecting them (Zapranis and Alexandridis, 2006).

Trying to overcome the problems of classical Fourier transform, Gabor applied the Fourier transform in small time "windows" (Mallat, 1999). To achieve a sort of compromise between frequency and time, Fourier transform was expanded in windowed Fourier transform or short-time Fourier transform (STFT). WFT uses a window across the time series and then uses the FT of the windowed series. This is a decomposition of two parameters, time and frequency. Window Fourier transform is an extension of the Fourier transform where a symmetric window, $g(u) = g(-u)$, is used to localize signals in time. If $t \in \mathbb{R}$, we define

$$f_t(u) = \bar{g}(u - t) f(u) \tag{1.6}$$

Expression (1.6) reveals that $f_t(u)$ is a localized version of f that depends only on values of $f(u)$. Again following the notation of Kaiser (1994), the STFT of f is given by

$$\tilde{f}(\omega, t) = \hat{f}_t(\omega) = \int_{-\infty}^{\infty} e^{-2\pi i\omega u}\, \bar{g}(u-t) f(u)\, du \tag{1.7}$$

It is easy to see that by setting $g(u) = 1$, the SFTF is reduced to ordinary FT. Because of the similarity of equations (1.2) and (1.7), the inverse SFTF can be defined as

$$f(u) = C^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{2\pi i\omega u} g\,(u-t) \tilde{f}\,(\omega, t)\; d\omega\, dt \tag{1.8}$$

where $C = \|g\|^2$.

As mentioned earlier, FT can be used to analyze a periodic musical tone. However, if the musical tone is not periodic but rather is a series of notes or a melody, the Fourier series cannot be used directly (Kaiser, 1994). On the other hand, the STFT can analyze the melody and decompose it to notes, but it can also give the information when a given note ends and the next one begins. The STFT has been used successfully in a variety of applications. Common uses are in speech processing and spectral analysis (Allen, 1982) and in acoustics (Nawab et al., 1983), among others.

Extending the Fourier Transform: The Wavelet Analysis Paradigm

As mentioned earlier, Fourier analysis is inefficient in dealing with the local behavior of signals. On the other hand, windowed Fourier analysis is an inaccurate and inefficient tool for analyzing regular time behavior that is either very rapid or very slow relative to the size of the window (Kaiser, 1994). More precisely, since the window size is fixed with respect to frequency, WFT cannot capture events that appear outside the width of the window. Many signals require a more flexible approach: that is, one where we can vary the window size to determine more accurately either time or frequency.

Instead of the constant window used in WFT, waveforms of shorter duration at higher frequencies and waveforms of longer duration at lower frequencies were used as windows by Grossmann and Morlet (1984). This method, called *wavelet analysis*, is an extension of the FT. The fundamental idea behind wavelets is to analyze according to scale. Low scale represents high frequency, while high scales represent low frequency. The wavelet transform (WT) not only is localized in both time and frequency but also overcomes the fixed time–frequency partitioning. The new time–frequency partition is long in time at low frequencies and long in frequency at high frequencies. This means that the WT has good frequency resolution for low-frequency events and good time resolution for high-frequency events. Also, the WT adapts itself to capture features across a wide range of frequencies. Hence, the WT can be used to analyze time series that contain nonstationary dynamics at many different frequencies (Daubechies, 1992).

In finance, wavelet analysis is considered a new powerful tool for the analysis of financial time series, and it is applied in a wide range of financial problems. One example is the daily returns time series, which is represented by local information such as frequency, duration, intensity, and time position, and by global information such as the mean states over different time periods. Both global and local information is needed for a correct analysis of the daily return time series. Wavelets have the ability to decompose a signal or a time series on different levels. As a result, this decomposition brings out the structure of the underlying signal as well as trends, periodicities, singularities, or jumps that cannot be observed originally.

Wavelet analysis decomposes a general function or signal into a series of (orthogonal) basis functions called *wavelets*, which have different frequency and time locations. More precisely, wavelet analysis decomposes time series and images into component waves of varying durations called wavelets, which are localized variations of a signal (Walker, 2008). As illustrated by Donoho and Johnstone (1994), the wavelet approach is very flexible in handling very irregular data series. Ramsey (1999) also comments that wavelet analysis has the ability to represent highly complex structures without knowing the underlying functional form, which is of great benefit in economic and financial research. A particular feature of the signal analyzed can be identified with the positions of the wavelets into which it is decomposed.

Recently, an increasing number of studies apply wavelet analysis to analyze financial time series. Wavelet analysis was used by Alexandridis and Hasan (2013) to estimate the systematic risk of CAPM using wavelet analysis to examine the meteor shower effects of the global financial crisis. Similarly, one recent research strand of CAPM has built an empirical modeling strategy centering on the issue of the multiscale nature of the systematic risk using a framework of wavelet analysis (Fernandez, 2006; Gençay et al., 2003, 2005; Masih et al., 2010, Norsworthy et al., 2000; Rabeh and Mohamed, 2011). Wavelet analysis has also been used to construct a modeling and pricing framework in the context of financial weather derivatives (Alexandridis and Zapranis 2013a,b; Zapranis and Alexandridis, 2008, 2009).

Moreover, wavelet analysis was used by In and Kim (2006a,b) to estimate the hedge ratio, and it was used by Fernandez (2005), and In and Kim (2007) to estimate the international CAPM. Maharaj et al. (2011) made a comparison of developed and emerging equity market return volatility at different time scales. The relationship between changes in stock prices and bond yields in the G7 countries was studied by Kim and In (2007), while Kim and In (2005) examined the relationship between stock returns and inflation using wavelet analysis. He et al. (2012) studied the value-at-risk in metal markets, while a wavelet-based assessment of the risk in emerging markets was presented by Rua and Nunes (2012). Finally, a wavelet-based method for modeling and predicting oil prices was presented by Alexandridis and Livanis (2008), Alexandridis et al. (2008), and Yousefi et al. (2005). Finally, a survey of the contribution of wavelet analysis in finance was presented by Ramsey (1999).

Wavelets A wavelet ψ is a waveform of effectively limited duration that has an average value of zero. The WA procedure adopts a particular wavelet function called a *mother wavelet*. A *wavelet family* is a set of orthogonal basis functions generated