Trends in Mathematics

Piotr Kielanowski
Daniel Beltita
Alina Dobrogowska
Tomasz Goliński
Editors

Geometric Methods in Physics XL

Workshop, Białowieża, Poland, 2023

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Preface

Last year, we were celebrating the XL Workshop on Geometric Methods in Physics (WGMP), which took place from July 2 to 8, 2023 in Białowieża, our traditional venue. The XII School on Geometry and Physics was held the preceding week, from June 20 to 25, 2023.

The Workshop on Geometric Methods in Physics series began 41 years ago, in 1982, and has continued since then with the sponsorship of the University of Białystok. The School on Geometry and Physics series was initiated in 2012 to accompany the Workshops. Each School offers an introductory series of lectures on cutting-edge areas of research related to the themes of the Workshop. Materials from recent WGMPs, such as the programme, abstracts of the talks and lectures, and participant lists, are posted on the website: https://wgmp.uwb.edu.pl. The website includes also bibliographical information on the *Proceedings* volumes for each year.

Each Workshop highlights a number of main themes. In 2023, the themes were: classical and quantum field theories, infinite-dimensional groups, integrable systems, Lie groupoids and Lie algebroids, noncommutative geometry, operator algebras, quantization, quantum groups, representation theory, and symplectic and Poisson geometry. There were also two special topic sessions: *Facets of the Yang-Baxter Equation* and *Infinite Dimensional Geometry*.

In the context of the anniversary nature of our Workshop in addition of our regular program, we organized a special session in memory of Professor Anatol Odzijewicz, the founder and organizer of all our WGMPs until his untimely death in 2022. The session gathered his close collaborators, friends, and former students, who paid tribute to his achievements.

Wigner Medal is an important prize awarded for outstanding contributions to the understanding of physics and chemistry through precise mathematical tools in the spirit of Eugene Wigner. In spring 2023, the Organizing Committee of the WGMP and the Group Theory and Fundamental Physics Foundation, owner of the rights to the Wigner Medal, reached an agreement on a joint awarding of Wigner Medal. In the framework of this arrangement, the Organizing Committee of WGMP participates in the nomination of the members of the Selection Committee of the Wigner Medal and grants a possibility of the organization of the Wigner Medal ceremony during the Workshop. The Wigner Medal activities are reported in Chap. *Wigner Medal 2023*.

The WGMP series has played an exceptional role in bringing together the two communities of mathematicians and physicists. Participation has grown from the original contingent of mainly eastern-European scholars to become widely international, with speakers from several continents. We hope this collection of articles will provide readers with an overview of the most recent developments in a wide variety of areas, and stimulate interest in the particular threads represented in the School's lectures.

The organizers acknowledge sponsorship of the Workshop by the University of Białystok, including both financial support and permission, to make use of University facilities. We are very grateful for technical assistance during the School provided by the staff of the Faculty of Mathematics of the University of Białystok. The Workshop and School were co-funded by Ministry of Education and Science, Poland program "Doskonała Nauka" under the project "XL Konferencja Workshop on Geometric Methods in Physics" nr DNK/SP/548722/2022. Funding amount 101,970.00 zł, total project value 131,270.00 zł.

Ciudad de México, Mexico — Piotr Kielanowski
Bucharest, Romania — Daniel Beltita
Białystok, Poland — Alina Dobrogowska
Białystok, Poland — Tomasz Goliński
December 2023

Contents

40 Years of the Workshop with Anatol Odzijewicz

Alina Dobrogowska, Tomasz Goliński, Aneta Sliżewska, and Piotr Kielanowski

Abstract We recount the history of the Workshop on Geometric Methods in Physics in *Białowieża* and the role of Professor Anatol Odzijewicz, who was the founder of the workshop and the main organizer for almost 40 years.

Mathematics Subject Classification (2010) 01A70

The first meeting of the series "Workshop on Geometric Methods in Physics" took place in the year 1982 (the picture of the participants is shown in Fig. 1). It was organized in *Białowieża* by Anatol Odzijewicz.[1] Let us quote here one of the participants and later a longtime scientific secretary of the workshop, Dr. Wojciech Lisiecki [4]:

> After Anatol's move to Białystok in 1979 I had little contact with him, so I didn't know how the idea of a conference in Białowieża was born. But I remember very well the first conference in July 1982, still under martial law.
>
> There were ten participants who can be divided into three groups:
>
> - Prominent participants: Professor Maurin, Professor Woronowicz, and Anatol
> - Anatol's students: Andrzej Karpio, Andrzej Kryszeń, and Maciek Horowski
> - People connected to Maurin's chair: Grześ Cieciura, Staszek Janeczko, and Janusz Szmidt.
>
> I could be included either to the second or to the third group.

[1] In our text, we will frequently refer to Professor Anatol Odzijewicz by his first name: *Anatol*.

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Fig. 1 Participants of the first workshop. Anatol is sitting in the first row, second from the right

> There were also two accompanying persons: Professor Maurin's wife Lidia and Staszek Janeczko's fiancée Teresa, and two female physics students from Białystok who took part only in the social events. One of them had a very nice voice and sang in turns with Anatol during the bonfire in the open air museum (skansen), which at that time consisted only of a windmill.

Since then, 40 meetings have taken place, one each year with the exception for the pandemic period 2020–2021. The last anniversary meeting took place in July 2023. Since the year 2012, the conference has been accompanied by the School on Geometry and Physics, a satellite event addressed to early career scientists.

There is no doubt that there would be no workshop if not for Professor Anatol Odzijewicz's love for mathematics and physics and the *Podlasie* region from which the idea of the workshop was born. He led the organization of 38 events and almost completed the preparations of the XXXIX by putting together the program and choosing and inviting plenary lecturers [1]. His sudden passing on April 18, 2022, was a great blow to all of his collaborators and students. Inevitably, his death also disrupted the organization process, but the organizing committee decided to continue the effort. As planned, the XXXIX Workshop took place at the Campus of the University of *Białystok* from June 19, 2022, to June 25, 2022, for the first time outside of *Białowieża*.

All earlier Workshops as well as the anniversary XL meeting in the year 2023 took place in *Białowieża*, a small village near the eastern border of Poland at the heart of the last remaining European primeval forest. The lectures were held in a hall belonging to the *Białowieża National Park* (see Fig. 2).

Fig. 2 Workshop participants in the lecture hall. Photo by Tomasz Goliński

The establishing and organization of the *Białowieża* series of workshops was one of the most important accomplishments of Professor Anatol Odzijewicz, and he referred to it as his life's achievement with very personal involvement (see Fig. 3). Organizing the first meeting in 1982, he surely didn't expect that in the future it will become a brand well known in the scientific world. The beginnings were rather humble and difficult, because the 1982 workshop took place under the conditions of the martial law imposed in Poland by the communist regime. The dire conditions of the communist rule and suppression of civil liberties caused many problems; even a simple trip from Warsaw to *Białowieża* required a special permission from the authorities. The first meeting had thus a very local character. The participants were limited to a small group of researchers from the University of Warsaw and its Branch in *Białystok*.

In the first years, the workshop was a regional meeting limited to Polish scientists. Gradually, over the years, scientists from Soviet Union and later also from western countries started to come. In the 1990s, the workshop was already transformed into international scientific event. Let us add that in 1989 the communist government together with trade union "Solidarity" initiated a democratic transformation which resulted in a wave of political changes which swept over the eastern and central Europe freeing it from dictatorships, which culminated in December 1991 with dissolution of the Soviet Union. Incidentally, this also took place in the *Białowieża Forest*, albeit on the other side of the border.

Fig. 3 Participants of the Workshop in Anatol's home in *Białowieża*, 1995

Next years were marked by a rapid economic development of the country which was mirrored by fast increase in the scope of the workshops. The number of participants also increased; for example, for the XXX Workshop in the year 2006, we had a record of 115 attendants.

The years 2019–2021, when restrictions related to the pandemic forced us to cancel two workshops, were a crisis period. However, even in this period, we managed to organize the school for a limited group of participants. This allowed us to continue the meeting even under those difficult conditions. Restrictions notwithstanding, we had guests from *Prague*, *Warsaw*, and *Białystok*. In the subsequent year 2022 when most of the limitations were lifted, we decided to hold a conference in a hybrid form by allowing a virtual participation. However, we were not out of the woods yet. Due to the Russian invasion of Ukraine, the Polish border with Belarus became unstable as well. A wave of illegal immigration from Middle East and North Africa coordinated by the Belarusian regime threatened the eastern border of the European Union. As a defense measure against this form of hybrid warfare, the state of emergency in the areas close to the border was declared severely limiting the freedom of movement. Regretfully, *Białowieża* was within this area. For this reason, Anatol decided to move the XXXIX Workshop to *Białystok* and host it at the Faculty of Mathematics of the University of *Białystok*. He didn't live to see it happen though.

For the XL Anniversary Workshop, with the help of the grant of the Polish Ministry of Science, we managed to return to *Białowieża*.

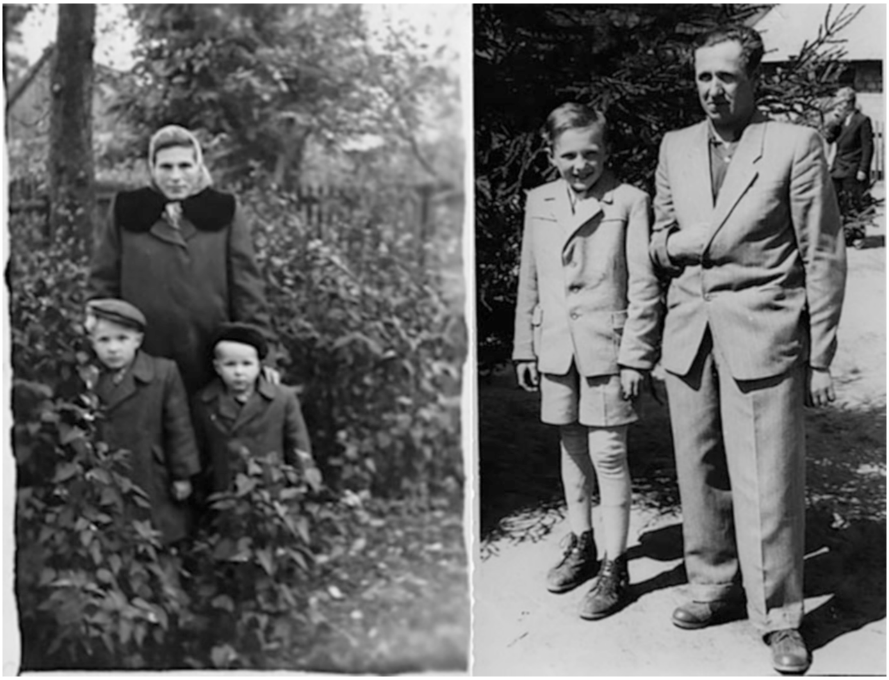

Fig. 4 Young Anatol with parents, with mother Tatiana and younger brother Eliasz on the left and with father Anatol on the right

Let us now take a closer look at the life course of Professor Anatol Odzijewicz, the founder and the main organizer of the workshop. He was born on October 6, 1947, in the small village of *Orla* located some 11 km from *Bielsk Podlaski* and around 60 km from *Białystok* in the eastern part of Poland. In Fig. 4, we show Anatol's childhood pictures with his parents and brother.

Orla used to be a city founded before the year 1507; however, by the end of the nineteenth century, it was degraded to the status of village. In the Interwar period (1918–1939), it had 1500 inhabitants, with majority of Jewish but also Orthodox and Catholic faiths. Of course, the World War II changed the situation drastically. In Orla, we can still find an Orthodox Church, a Roman Catholic Church, and a synagogue. Regretfully, the synagogue is not open for public these days.

In the years 1954–1961, Anatol attended the primary school in *Orla*. Later on, he went to the high school in *Bielsk Podlaski* (1961–1965). In 1965, he started his studies of physics at the University of Warsaw, where he graduated in 1970. In 1975, he obtained his PhD under the supervision of Professor Bogdan Mielnik at the same university. For the next 5 years, he was working at the Department of Mathematical Methods in Physics of the University of Warsaw. In 1979, he moved to *Białystok* where he worked at a local branch of the University of Warsaw. His move to *Białystok* was precipitated by his plans to start a family; at that time, the city of *Białystok* was offering accommodation to scientists in order to promote and encourage growth of the recently founded branch. Anatol had two alternatives: he could move either to *Białystok* or to *Gdańsk*. He chose *Białystok* as an act of local patriotism, since it was a capital of the region of his native village.

Physics and mathematics at this time at the University Branch in *Białystok* were just beginning to develop, so Anatol was maintaining close contact with Warsaw University which included a weekly participation in the seminar of the Department of Mathematical Methods in Physics. Moreover, many scientists from Warsaw were also employed in *Białystok*. Over the time, what started as a Teaching College transformed later into Branch of University of Warsaw and finally on October 1, 1997, became the University of *Białystok*. During this period, Professor Odzijewicz was actively helping to develop local physics and mathematical community. In 1990, he became the head of the Chair of Mathematical Physics, first within the Institute of Physics and then within the Institute of Mathematics. For many years, he was also the dean of the Faculty of Mathematics and Physics or director of the Institute of Mathematics.

While working in *Białystok*, he supervised and educated a group of young scientists (see Fig. 5), both physicists and mathematicians working on topics related to broadly understood mathematical physics. He was a supervisor of seven PhD theses, five in physics:

Fig. 5 Professor Odzijewicz with his team (from the left: Tomasz Goliński, Marzena Szajewska, Maciej Horowski, Aneta Sliżewska, Grzegorz Jakimowicz, Anatol Odzijewicz, Karolina Wojciechowicz, Elwira Wawreniuk, Alina Dobrogowska). Photo by Tomasz Goliński

- Andrzej Karpio (1990)
- Maciej Horowski (1992)
- Alina Dobrogowska (2004)
- Agnieszka Tereszkiewicz (2005)
- Grzegorz Jakimowicz (2006)

and two in mathematics:

- Tomasz Goliński (2010)
- Aneta Sliżewska (2011)

So far, one of his PhD students (A. Dobrogowska) has obtained her habilitation, and several others will apply for it in the near future.

Professor Odzijewicz always said that his main passion was mathematical physics [3]. He was always interested in the topics around the problems of quantization. His main idea was to use the notion of a coherent state to quantize a wide range of Hamiltonian systems. This program led among others to the study of infinite-dimensional Poisson manifolds as natural objects which allow the description of quantum mechanics in the framework of Hamiltonian mechanics and to the reproducing kernels which model physically meaningful transition amplitudes. His other research interests included integrable systems (in particular the factorization method and orthogonal polynomials), operator algebras, and quantum optics.

Another great passion of Professor Odzijewicz was closely related to the workshop. His deep devotion to his native region of *Podlasie* resulted in the idea to highlight and preserve the culture of the region. Being a man of action with a group of his friends, he initiated in 1978 the project of the *Skansen of Wooden Architecture of Ruthenians of Podlasie*. They acquired a piece of land in *Białowieża* on the bank of the river *Narewka* and began moving buildings characteristic of the region there. The first construction moved to the Skansen was a wooden windmill, built in 1925 in the nearby village *Kotły*. Now, after 40 years, there are among others two windmills, cottages, barns, workshops, a well with a well pole, and a chapel. Anatol was deeply involved in the construction of the *Skansen* and personally participated in construction works (see Fig. 6). To commemorate unique role of Anatol in creation of the Skansen, it now bears the name of *Anatol Odzijewicz* and is an important tourist spot in *Białowieża* [2].

Fig. 6 Anatol in his youth working on construction of one of cottages in the Skansen

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Wigner Medal 2023

Darlene Wiley-Bohm, Piotr Kielanowski, and Wolfgang P. Schleich

Abstract This chapter provides information about the origins of the Wigner Medal and the selection of the winners after the hiatus caused by the Covid-19 pandemic. The winners of the Wigner Medal 2023 were: Professor Yvette Kosmann-Schwarzbach for 2020 and Professors Iwo Białynicki-Birula and Daniel M. Greenberger for 2022.

Mathematics Subject Classification (2010) 00A99

1 Brief History

The study of the symmetries of physical systems through the analysis of symmetry groups was one of the most important directions in theoretical physics in the 1960s. As part of this research, $SU(3)$ symmetry was discovered, which led to the hypothesis of the existence of quarks. Also space-time symmetries and their relationship to internal symmetries in particle physics were studied. Eugene Paul Wigner was one of the most prominent scientists applying the methods of group theory to physics and received the 1963 Nobel Prize "for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles."

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Arno Bohm of the Department of Physics at the University of Texas at Austin, working on symmetry issues in particle physics, was fascinated by Wigner's great achievements and decided to create an award associated with Wigner's name that would honor scientists who have achieved the highest rated results in the study of the symmetry of physical systems. This is how the idea of the Wigner Medal was born.

In 1972, a periodic conference called the "International Colloquium on Group Theoretical Methods in Physics" was established, where the latest research results on symmetry in physics were presented. The organizer of this conference in 1978 was Arno Bohm, and he was convinced that it would be an excellent opportunity to present a new award at this meeting. The idea had the enthusiastic endorsement of the scientific community, and as a result, with the great support of John Archibald Wheeler, the *Group Theory and Fundamental Physics Foundation* (later referred as *Foundation*) was established to organize and administer the Wigner Medal (later referred as *Medal*). The Foundation included some of the most prominent physicists of the period, as well as scientific institutions from around the world (see below a copy of the list of the founding members).

THE GROUP THEORY AND FUNDAMENTAL PHYSICS FOUNDATION

Chairman: Arno Böhm, Physics Department
The University of Texas at Austin

Vice-Chairman: Morton Hamermesh, Physics Department
The University of Minnesota

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The University of Texas at Austin

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E. J. Konopinski
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W. M. MacDonald
Kazumi Maki
F. A. Matsen
Bernd T. Matthias
Esperanza de Moshinsky
Marcos Moshinsky
Tadao Nakano
Pedro L. Garcia Perez
T. O. Philips
Physical Chemistry Laboratory, ETH Zurich, Switzerland
Humitaka Sato
Marlan Scully
Frederick Seitz
The Solvay Institute, Brussels
SKM, Inc.
Edward Teller
Texas Nuclear
Tracor, Inc.
C. W. Ufford
Instituto de Fisica, Universidad Nacional Autonoma de Mexico
Janette Hegner Wheeler
John Archibald Wheeler

The founding members of *The Group Theory and Fundamental Physics Foundation.*

The efforts of Arno Bohm and the *Foundation's* creators were crowned with success, and Wigner himself became the first awardee of the Wigner Medal which he received from Arno Bohm, the Chairman of the *Foundation* at the International Colloquium on Group Theoretical Methods in Physics in Austin in 1978. On the image to the left there are E.P. Wigner and A. Bohm during the ceremony.

Since then, it has become a tradition to present the Wigner Medal at the International Colloquia on Group Theoretical Methods in Physics, held every two years. However, in 2022 the International Colloquia on Group Theoretical Methods in Physics have dropped its cooperation with the *Foundation*, and since then the Wigner Medal has not been awarded at this conference.

In order to continue the tradition of the Wigner Medal the *Foundation* and the *Organizing Committee* of the *Workshop on Geometric Methods in Physics* decided in 2023 to cooperate in awarding the Wigner Medal. This cooperation consisted of jointly appointing the *Selection Committee* of the Wigner Medal and organizing the Wigner Medal presentation ceremony at the Workshop.

2 Wigner Medal Awardees

Until 2023 the Wigner Medals have been awarded to the following scientists:

1978 E.P. Wigner	1998 M. Moshinsky
1978 V. Bargmann	2000 L. O'Raifeartaigh
1980 I.M. Gel'fand	2002 H.J. Lipkin
1982 L. Michel	2004 E. Inönü
1984 Y. Ne'eman	2006 S. Okubo
1986 F. Gürsey	2010 M. Jimbo
1988 I.M. Singer	2012 C.A. Mead
1990 F. Iachello	2014 J. Zak
1992 J. Wess and B. Zumino	2016 B. Kostant
1996 V. Kac and R.V. Moody	2018 P. Winternitz

Space does not allow us to discuss the achievements of the winners of the Wigner Medal. It suffices to state that the spectrum of their research subjects is very wide—from mathematics to physics, but always with strong elements of symmetry, very much in the spirit of Wigner.

3 Wigner Medals 2023

Due to the Covid-19 pandemic there were no Wigner Medals awarded after 2018. Hence, in 2023 there were medals to be awarded for the years 2020 and 2022.

The awardee of the Wigner Medal is determined by the *Selection Committee*. The rules of the Wigner Medal stipulate that the *Selection Committee* of the Wigner Medal consists of five members with four of them nominated by the *Foundation* and the *Organizing Committee* of the Workshop on Geometric Methods in Physics. One member, ex officio, is the Chairman of the *Foundation* or a person representing him.

Obverse of a Wigner Medal

The medals are personalized, and on the obverse of a medal there is an image of Eugene Wigner, and on the reverse there is engraved the name of the awardee and the citation recounting the awardee's achievements.

The *Selection Committee* of the Wigner Medal 2023 consisted of the following members:

Wolfgang P. Schleich (Chairman)—Ulm University, Germany
Daniel Beltiţă—Romanian Academy, Romania
Tomasz Brzeziński—Swansea University, UK and University of Białystok, Poland
Mark Hillery—City University of New York, USA
Piotr Kielanowski—CINVESTAV—Center for Advanced Studies, Mexico City
and they unanimously nominated the following awardees (see picture below):

Wigner Medal 2020	**Yvette Kosmann-Schwarzbach**
for seminal contributions to Poisson geometry	
Wigner Medal 2022	**Iwo Białynicki-Birula and Daniel M. Greenberger**
for fundamental contributions to quantum theory	

Awardees of the Wigner Medals 2020 and 2022, from left to right: Y. Kosmann-Schwarzbach (2020), I. Białynicki-Birula (2022) and D.M. Greenberger (2022) and the reverses of the associated Wigner Medals with the citations: awarded to

Yvette Kosmann-Schwarzbach for seminal contributions to Poisson geometry 2020, awarded to Iwo Białynicki-Birula for fundamental contributions to quantum theory 2022, and awarded to Daniel M. Greenberger for fundamental contributions to quantum theory 2022.

4 Wigner Medal 2023 Ceremonies

The Wigner Medals were awarded at three special events, separate for each awardee, and we now briefly report on each of them.

4.1 *Yvette Kosmann-Schwarzbach*

The event took place on October 24, 2023 in the new building of the Institut Henri Poincaré (IHP) in Paris (France) at the Yvonne Choquet-Bruhat Lecture Hall. It started with the welcome address of Professor Sylvie Benzoni, the director of the IHP. Next, Professor Tomasz Brzeziński from Swansea University and the University of Białystok delivered a laudatio on Professor Kosmann-Schwarzbach emphasizing that *her research has been driven not only by curiosity but also by very deeply rooted desire to find the beauty and the explanation of origins of beauty in the natural world.* The following speaker was Professor Richard Kerner from Sorbonne University, a long-time collaborator of the awardee, who also delivered a laudatio, underlining a wide spectrum of the research subjects of Professor Kosmann-Schwarzbach, and also listing outstanding collaborators. Afterward Professor Piotr Kielanowski from the Center for Advanced Studies in Mexico City was speaking on behalf of the *President* of the *Foundation* about the origin and history of the Wigner Medal, and he presented the Wigner Medal to Professor Kosmann-Schwarzbach heartily congratulating the awardee. Next item was a very inspiring lecture by Professor Eva Miranda from the Universitat Politècnica de Catalunya entitled *Yvette Kosmann-Schwarzbach: Orchestrating the Dance of Brackets and Symmetries*. The ceremony was concluded by a short address by Professor Kosmann-Schwarzbach.

4.2 *Iwo Białynicki-Birula*

The ceremony took place on June 14, 2023, at the Faculty of Physics of Warsaw University (Poland), during a one-day symposium, honoring the 90th birthday of Professor Iwo Białynicki-Birula. The symposium was scientific in nature and thematically related to his lines of research. At the meeting, Professor Białynicki-Birula delivered the lecture *Unifying Classical and Quantum Descriptions of the Penning Trap*, and his daughter, Iwona, presented a talk on *Modeling Reality:*

Then and Now. Next, collaborators and former students of Professor Białynicki-Birula gave a series of talks, emphasizing his impressive achievements in physics. One should also mention the six-minute special recorded address of Professor Anton Zeilinger, the Nobel Prize winner from 2022, listing important scientific accomplishments of Professor Białynicki-Birula. Afterward in the framework of the Wigner Medal ceremony Professor Jan Mostowski delivered the laudatio on Professor Białynicki-Birula, and then Professor Piotr Kielanowski on behalf of the Foundation presented the Wigner Medal to the awardee. The ceremony was concluded by a short address by Professor Białynicki-Birula.

4.3 Daniel M. Greenberger

The event took place on September 28, 2023, in the historic building of the Faculty of Physics of Vienna University (Austria) at Boltzmanngasse 5, during a three-day symposium, honoring the 90th birthday of Professor Daniel M. Greenberger. The symposium gathered his collaborators and friends, including the Nobel Prize winner, Professor Zeilinger, and was organized in a relaxed charming Viennese atmosphere. Professor Zeilinger delivered the laudatio for Professor Greenberger recounting the long friendship and collaboration with the awardee and his profound understanding and important contributions in quantum physics. Professor Piotr Kielanowski on behalf of the Foundation presented the Wigner Medal and congratulated the awardee. The ceremony was concluded by a witty dialogue of Professors Greenberger and Zeilinger.

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Part I
Session in Memory of Anatol Odzijewicz

On the Geometry of Coherent State Maps

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Abstract Given a mechanical system whose phase space $\mathfrak{M}^n$ is equipped with a complex structure J, and a Hermitian line bundle $(E, H) \to \mathfrak{M}$, a coherent state map is an anti-holomorphic embedding $\mathscr{K} : \mathfrak{M} \to \mathbb{CP}\big(\mathscr{M}\big)$ built in terms of (J, H), with $\mathscr{M} = H^0\Big(\mathfrak{M}, L^2\,\mathscr{O}\big(T^{*(n,0)}(\mathfrak{M}) \otimes E\big)\Big)$, such that for any pair of classical states $z, \zeta \in \mathfrak{M}$ the number $\langle \mathscr{K}(z), \mathscr{K}(\zeta)\rangle$ is the transition probability amplitude from the coherent state $\mathscr{K}(z)$ to $\mathscr{K}(\zeta)$. We examine three related questions, as follows: (i) We generalize Lichnerowicz's theorem (on $\pm$ holomorphic maps of finite-dimensional compact Kählerian manifolds) to describe anti-holomorphic maps $\mathscr{K} : \mathfrak{M} \to \mathbb{CP}(\mathscr{M})$ as harmonic maps that are absolute minima within their homotopy classes. (ii) If the phase space is a domain $\mathfrak{M} = \Omega \subset \mathbb{C}^n$ and $E \to \Omega$ is a trivial Hermitian line bundle such that $\gamma = H\big(\sigma_0\,,\,\sigma_0\big) \in AW(\Omega)$ (i.e., γ is an admissible weight), we discuss the use of $K_\gamma(z, \zeta)$ [the γ-weighted Bergman kernel of Ω] *vis-a-vis* to the calculation of the transition probability amplitudes, focusing on the case where $\Omega = \Omega_n$ is the Siegel domain and $\gamma(z) = \gamma_a(z) = \big(\mathrm{Im}(z_n) - |z'|^2\big)^a$, $a > -1$. (iii) We study the boundary behavior of a coherent state map $\mathscr{K} : \Omega \to \mathbb{CP}\big[L^2 H(\Omega_n\,,\,\gamma_a)\big]$.

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1 Coherent State Maps: Odzijewicz's Construction

Let $\mathfrak{M}^n$ be a complex n-dimensional manifold and $\pi : E \to \mathfrak{M}$ a complex line bundle equipped with i) a holomorphic structure $\big\{\phi_\alpha : \pi^{-1}(U_\alpha) \to U_\alpha \times \mathbb{C}\big\}_{\alpha \in I}$ and such that each U_α is the domain of a local complex coordinate system $\big(z^1_\alpha\,,\,\cdots\,,\,z^n_\alpha\big)$ on $\mathfrak{M}$ and with ii) a Hermitian bundle metric H. Let us set $\sigma_\alpha(z) = \phi_\alpha^{-1}(z, 1)$

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for any $z \in U_\alpha$. Let $\mathscr{M} = H^0\big(\mathfrak{M}, L^2\ \mathscr{O}\big(T^{*\,(n,0)}(\mathfrak{M}) \otimes E\big)\big)$ be the space of L^2 holomorphic sections in the holomorphic line bundle $T^{*\,(n,0)}(\mathfrak{M}) \otimes E \to \mathfrak{M}$. Every $s \in \mathscr{M}$ is locally represented as $s\big|_{U_\alpha} = \Psi_\alpha\, \sigma_\alpha \otimes dz^1_\alpha \wedge \cdots dz^n_\alpha$ for some $\Psi_\alpha \in \mathscr{O}\big(U_\alpha\big)$. A L^2 inner product on $\mathscr{M}$ (organizing $\mathscr{M}$ as a Hilbert space) is given by

$$\langle s^1\,,\ s^2\rangle = i^{n^2} \int_{\mathfrak{M}} H^*\big(s^1\,,\ s^2\big), \quad s^1,\ s^2 \in \mathscr{M}, \tag{1}$$

$$H^*\big(s^1\,,\ s^2\big)\big|_{U_\alpha} = \Psi^1_\alpha\, \overline{\Psi}^2_\alpha\, \gamma_\alpha\, dz^1_\alpha \wedge \cdots \wedge dz^n_\alpha \wedge d\overline{z}^1_\alpha \wedge \cdots \wedge d\overline{z}^n_\alpha\,,$$

$$\gamma_\alpha \equiv H\big(\sigma_\alpha\,,\ \sigma_\alpha\big), \quad \alpha \in I.$$

Let $z \in \mathfrak{M}$ and $\alpha \in I$ such that $z \in U_\alpha$, and let us set $\delta^\alpha_z(s) = \Psi_\alpha(z)$ for any $s \in \mathscr{M}$. By a result of K. Gawedzki (cf. [21])

$$\big|\Psi_\alpha(z)\big| \le C_\alpha \|s\| \tag{2}$$

hence, $\delta^\alpha_z : \mathscr{M} \to \mathbb{C}$ is continuous. By the Riesz representation theorem, there is $k_{z,\overline{\alpha}} \in \mathscr{M}$ such that

$$\delta^\alpha_z(s) = \big\langle s,\ k_{z,\overline{\alpha}}\big\rangle = i^{n^2} \int_{\mathfrak{M}} H^*\big(s,\ k_{z,\overline{\alpha}}\big)$$

where

$$k_{z,\overline{\alpha}}\big|_{U_\beta} = \overline{K_{\alpha\overline{\beta}}\,(z,\ \cdot\,)}\, \sigma_\beta \otimes\ dz^1_\beta \wedge \cdots \wedge dz^n_\beta\,,$$

$$H^*\big(s,\ k_{z,\overline{\alpha}}\big) = K_{\alpha\overline{\beta}}(z,\ \cdot\,)\, \Psi_\beta\, \gamma_\beta\, dz^1_\beta \wedge \cdots \wedge dz^n_\beta \wedge d\overline{z}^1_\beta \wedge \cdots \wedge d\overline{z}^n_\beta\,.$$

Let $\Omega \subset \mathbb{C}^n$ be an open set. A *weight* on Ω is a Lebesgue measurable function $\gamma : \Omega \to (0,\ +\infty)$. The set of all weights on Ω is denoted by $W(\Omega)$ (a Banach manifold modeled on $L^\infty(\Omega,\ \mathbb{R})$, cf. [32]). Let $L^2(\Omega,\ \gamma)$ consist of all measurable functions $\Psi : \Omega \to \mathbb{C}$ such that $\|\Psi\|_\gamma < \infty$ where

$$(\Psi,\ \Phi)_\gamma = \int_\Omega \Psi(z)\, \overline{\Phi(z)}\, \gamma(z)\, d\,\mu(z), \quad \|\Psi\|_\gamma = (\Psi,\ \Psi)^{1/2}_\gamma\,,$$

(μ is the Lebesgue measure on $\mathbb{R}^{2n}$) and let us set $L^2H(\Omega,\ \gamma) = \mathscr{O}(\Omega) \cap L^2(\Omega,\ \gamma)$. A weight $\gamma \in W(\Omega)$ is *admissible* if i) the evaluation functional $\delta_z : L^2H(\Omega,\ \gamma) \to \mathbb{C}$, $\delta_z(\Psi) = \Psi(z)$ and ii) $L^2H(\Omega,\ \gamma)$ is a closed subspace of $L^2(\Omega,\ \gamma)$. Let $AW(\Omega)$ be the set of all admissible weights (an open subset of $W(\Omega)$, cf. [32]). If $\mathfrak{M} = \Omega \subset \mathbb{C}^n$, $\phi : E \simeq \Omega \times \mathbb{C}$ (a vector bundle isomorphism), σ_0 is the (globally defined) holomorphic frame $\sigma_0(z) = \phi^{-1}(z,\ 1)$, and $\gamma = H(\sigma_0\,,\ \sigma_0) \in AW(\Omega)$, then

$$H^0\big(\Omega, L^2\,\mathscr{O}\big(T^{*\,(n,0)}(\Omega)\otimes E\big)\big) \simeq L^2 H(\Omega\,,\,\gamma)$$

(a Hilbert space isomorphism). When $\gamma = H\big(\sigma_0\,,\sigma_0\big)$ satisfies $\gamma^{-a} \in L^1(\Omega)$ for some $a > 0$, a simple proof to Gawedzki's lemma (2) was given by Z. Pasternak-Winiarski, [31]. As a remarkable feature of Pasternak-Winiarski's proof, it relies on the relationship between holomorphic and subharmonic functions, rather than a power series argument.[1] Indeed, for each $z \in \Omega$, let $r > 0$ such that $B_{2r}(z) \subset \Omega$. By Corollary 2.1.15 in [26], p. 75, if $\Psi \in \mathscr{O}(\Omega)$, then $|\Psi|^P$ is subharmonic for every $P > 0$. Let $p = (1+a)/a > 1$ and $P = 2/p$. Then, for every $\Psi \in L^2 H(\Omega,\,\gamma)$ and every $\zeta \in B_r(z)$

$$|\Psi(\zeta)|^{2/p} \le \frac{1}{\operatorname{Vol}\big[B_r(\zeta)\big]}\int_{B_r(\zeta)} |\Psi(w)|^{2/p}\, d\,\mu(w) \le$$

(by Hölder's inequality with $1/p + 1/q = 1$, hence $q = 1 + a$)

$$\le \frac{1}{\operatorname{Vol}\big[B_r(\zeta)\big]}\Big(\int_{B_r(\zeta)} |\Psi|^2\,\gamma\; d\,\mu\Big)^{1/p}\Big(\int_{B_r(\zeta)} \gamma^{-q/p}\; d\,\mu\Big)^{1/q}$$

yielding

$$|\Psi(\zeta)| \le C\,\|\Psi\|_\gamma\,, \quad C = \frac{1}{\operatorname{Vol}\big[B_r(\zeta)\big]^{(1+a)/(2a)}}\Big(\int_\Omega \gamma^{-a}\; d\,\mu\Big)^{1/(2a)}.$$

The *coherent state map* is

$$\mathscr{K} : \mathfrak{M} \to \mathbb{CP}(\mathscr{M}), \quad \mathscr{K}(z) = \big[k_{z,\,\overline{\alpha}}\big], \quad z \in \mathfrak{M},$$

where $\alpha \in I$ is picked up such that $z \in U_\alpha$ and $[s]$ denotes the projective ray through $s \in \mathscr{M} \setminus \{0\}$. See also [5, 6]. Let us consider the globally defined (0, 2)-tensor field g on $\mathfrak{M}$ such that

$$g\big|_{U_\alpha} = \sum_{j,k=1}^{n} \frac{\partial^2\, K_{\alpha\overline{\alpha}}(z,\ z)}{\partial z_\alpha^j\, \partial \overline{z}_\alpha^k}\, dz_\alpha^j \odot d\overline{z}_\alpha^k\,, \quad \alpha \in I.$$

By a foundational result due to A. Odzijewicz [30], the coherent state map $\mathscr{K}$ is an anti-holomorphic embedding if and only if $\mathscr{K}$ is one to one and g is positive definite. If this is the case, g is a Kählerian metric (the *Bergman metric* of $\mathfrak{M}$) so that classical states of a mechanical system whose phase space is $\mathfrak{M}$ may be quantized (within the

[1] The typical proof of (2) in the case $\gamma \equiv 1$ (leading to the Bergman kernel of Ω, cf., e.g., [23]) is to represent $\Psi \in \mathscr{O}(\Omega)$ in power series in a polidisc $P(z,\,\epsilon) \subset \Omega$, and profit from the fact that monomials of the form $(\zeta - z)^\alpha$, $\alpha \in \mathbb{Z}_+^n$, are mutually orthogonal in $L^2\big(P(z,\,\epsilon)\big)$.

quantization scheme proposed by A. Odzijewicz, cf. *op. cit.*) only when $\mathfrak{M}$ meets the topological requirements needed to support globally defined Kählerian metrics. See also [29]. Given a set E, let $\mathscr{F}(E)$ denote the space of all functions $f : E \to \mathbb{C}$. Let

$$\mathscr{H} = H^0\big(\mathfrak{M},\ C^\infty\big(T^{*\,(n,0)}(\mathfrak{M}) \otimes E\big)\big)$$

[a Hilbert space with the inner product (1)]. For every $\alpha \in I$, we consider

$$T_\alpha : \mathscr{H} \to \mathscr{F}\big(U_\alpha\big), \quad \big(T_\alpha\, s\big)(z) = \langle s,\, k_{z,\overline{\alpha}} \rangle\,, \quad s \in \mathscr{H}, \quad z \in U_\alpha\,.$$

By a result of A. Odzijewicz (cf. [30]) $T_\alpha s \in \mathscr{O}\big(U_\alpha\big)$ for every $\alpha \in I$ if and only if $\mathscr{K} : \mathfrak{M} \to \mathbb{CP}(\mathscr{M})$ is anti-holomorphic. For every $s \in \mathscr{H}$, the holomorphic functions $T_\alpha s$ glue up to a unique holomorphic section Ts in $T^{*\,(n,0)}(\mathfrak{M}) \otimes E$ such that $(Ts)\big|_{U_\alpha} = T_\alpha s$ for any $\alpha \in I$, thus yielding a linear operator:

$$T : \mathscr{H} \to H^0\big(\mathfrak{M}, L^2\ \mathscr{O}\big(T^{*\,(n,0)}(\mathfrak{M}) \otimes E\big)\big)$$

(the *Bergman-Gawedzki-Odzijewicz projection*). Note that $Ts = s$, i.e., T reproduces the square integrable E-valued holomorphic n-forms. The kernel $\mathscr{N}(T)$ is a closed subspace of $\mathscr{H}$; hence, the range $\mathscr{R}(T)$ may be organized[2] as a Hilbert space with the inner product:

$$\langle s^1\,,\, s^2 \rangle_{\mathscr{R}(T)} = \langle P\sigma^1\,,\ P\sigma^2 \rangle\,, \quad \sigma^j \in T^{-1}\big(s^j\big), \quad j \in \{1,\, 2\},$$

where $P : \mathscr{H} \to \mathscr{H} \ominus \mathscr{N}(T)$ is the orthogonal projection. Then

$$\|s\|_{\mathscr{R}(T)} = \inf \Big\{ \|\sigma\|_{\mathscr{H}}\ :\ \sigma \in T^{-1}(s) \Big\}.$$

Cf. also [2].

2 Kählerian Structure of $\mathbb{CP}(\mathscr{M})$

Let $\mathscr{M}$ be a separable Hilbert space. The present section reviews the complex structure J and the Fubini-Study metric g on the complex projective space $\mathscr{P} = \mathbb{CP}(\mathscr{M}) = \big(\mathscr{M} \setminus \{0\}\big)/\mathbb{C}^*$ (cf. [35]). For every point $p \in \mathscr{P}$, let Φ be a normalized representative of p, i.e., $p = [\Phi]$ and $\|\Phi\| = 1$. One sets

$$V^\perp_{[\Phi]} = \big\{ \Psi \in \mathscr{M}\ :\ \big\langle \Phi,\ \Psi \big\rangle = 0 \big\}, \quad \tilde{U}_{[\Phi]} = \mathscr{M} \setminus V^\perp_{[\Phi]}\,,$$

[2] By a result of S. Saitoh, [34].