# MAGNETIC RESONANCE IMAGING

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## *Physical Principles and Sequence Design*

*Second Edition*

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# Magnetic Resonance Imaging:
## Physical Principles and Sequence Design

Second Edition

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# Foreword to the Second Edition

Jeffrey L. Duerk

Almost 30 years ago, while a graduate student at Case Western Reserve University, I enrolled in a new course: The Physics of Magnetic Resonance Imaging (PHYS 431) that was being offered by Professors Mark Haacke and Robert Brown. Whole-body MR imaging systems had just emerged on the marketplace and the go-go days of MRI were upon us as numerous companies were dramatically ramping up their research and development efforts in this emerging, yet unproven field. In Cleveland alone, there was Picker International (now Philips Medical Systems) and Technicare (now GE); there were almost 20 MR systems in hospitals and at the manufacturers' facilities. The worldwide need for scientists and engineers with excellent preparation in the underlying physics of MR, signal detection, $k$-space, and a variety of pulse sequences was clear. Ultimately, over the next few years, PHYS 431 class notes were organized into sections, then chapters, and, eventually, 'the green bible' as we know this book today. Within a few years, it was translated into Chinese. For many, the book has served not only as a textbook, but as a sustaining reference on numerous aspects of NMR and MR imaging. Today, 'Haacke, Brown, Thompson and Venkatesan' has reached 2000 citations and counting.

For me, in the intervening 30 years, I went from a student to an industry-based researcher who still remembers fondly those go-go days, the advances and friendships formed, and the definitive impact that MRI provided in patient care. Upon returning to academia, I have seen my own students cut their teeth on this book, move to industry and academic positions, and go on to adopt it in their own courses. This has been repeated across universities, programs, and institutions around the world. The impact of this seminal teaching book is hard to calculate. Like Abragam before it, the book has achieved its authors' goals of sustainable impact and becoming a classic in the field.

Like all things, the field has changed dramatically since its original publication. Scan times of 25 minutes are now replaced by those of 25 milliseconds, or so, using novel sequences, novel trajectories, and constrained reconstruction. Field strengths of 0.15 T, 0.3 T, and 0.5 T are replaced by the now more common field strengths of 1.5 T and 3.0 T, with 7.0 T, 9.4 T, and higher either solidly established or emerging on the horizon. Gradient strengths have increased from the lowly 3 mT/m 30 years ago to 40 mT/m on many systems today and soon some systems will have the capability of 80 mT/m. Topics that have emerged today were not fully formed at the time of the first edition, and, hence, this version is not only greatly anticipated but also fulfills the promise of the original version in providing solid practical and rigorous theoretical underpinnings, and relevant challenging homework questions. Topics like parallel imaging via RF receive coils arrays and numerous other technical insights highlight the additions. The challenge, of course, is how to keep a 'classic' a classic in such a dynamic and rapidly evolving field as MR imaging. As with the early versions, I tip my hat to the authors for their selection of topics and also their patience in allowing 'hot' fields to reach an appropriate level of sustainability and impact before inclusion here.

On behalf of others like me, who grew up (and continue) using 'the green bible,' I want to extend my congratulations and thanks to the authors for this new edition. I anxiously await not only the next generation of discoveries it facilitates but also the next generation of scientists it supports. Well done!

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June 11, 2013

Hiroyuki Fujita

Studying the second edition of this textbook on MRI physics has connected marvelous memories: my beginning graduate school experience, many early and long-lasting friendships, internships and major responsibilities with leading OEMs, returning to direct physics research at my alma mater, and then incubating and growing an MRI manufacturing company, Quality Electrodynamics. QED's success has led to such recognitions as a Presidential and First Lady's guest of honor at the 2012 State of the Union Address and a 2013 Presidential Award for Export. From my own education to the training of my team to the business awards, this big green book started and buttressed it all.

I can echo Professor Jeff Duerk's words about his start in MRI because I too began by enrolling in PHYS 431, but a decade later. By the 1990s, this 'Physics of Imaging' had become a standard CWRU course for a graduate imaging track in both the physics and the biomedical engineering departments. The notes from this course became the primary teaching tool in MRI, which I refer to here as the big green book. Professor Mark Haacke introduced me to MR imaging before he left Case Western Reserve University to start his own company and research institute. Immediately after this, I began my Ph.D. with Professor Robert Brown in hardware design, a key move in view of QED's rf coil products. Thus began a series of prideful career-long collaborations with Professor Brown that continues to this day.

The big green book provided a foundation for all the CWRU graduate students I knew in MRI. My good friends, Mike Thompson and Norman Cheng, have been promoted from beginning students to achieving co-authorship. Like myself, they studied the course material, and went on to be teaching assistants, lecturers, and faculty. We could see the influence of the venerable physics textbooks such as Jackson's *Classical Electrodynamics (*which influenced the QED name as well!) and Kittel's just-in-time *Thermal Physics* homework. It is satisfying to notice how the 'critical thinking' goals that are so much in the current educational news — connecting new work to old, applying theory to practice, taking a first step even if the second is not yet clear, finding alternatives if one path fails, looking at whether a solution makes sense, and learning to collaborate — are strongly reflected in this textbook's many problems and the lead-up to them. I am currently serving on the U.S. Manufacturing Council committee tasked with advising the Secretary of Commerce. Science education is recognized as having ever-increasing significance when discussing national policies necessary to improve the current and future workforce in America. I feel that the big green book is emblematic as a key tool in connecting physics and math to imaging science, biology, and chemistry, and preparing the student for an important high-tech career. This is a real teaching tool well received by many generations of CWRU students.

The rf material added in the second edition, especially in the new Chapter 28 on parallel imaging, is really welcome. Over the past decade, my industrial colleagues and I have constantly referenced this textbook and its chapters on signal-to-noise and contrast analysis, sequence parameters, and especially rf software and hardware topics. The green book is evident on many shelves of our company and, wherever we visit, we often observe that it is so beat-up a new edition is really needed. It is not just the expanded rf treatment that will stimulate folks to replace their (dilapidated) first editions.

As the writer of a foreword in this new edition, I perhaps have the responsibility to revisit the original words by Professor Felix Wehrli to report on what has been added. The book continues to be most appropriate for the physics and engineering graduate and advanced undergraduate classes, with the first two years of math and science in typical technical university curricula as sufficient prerequisites. On the face of it, this second edition is not changed terribly much, with one chapter added to the original twenty-seven. Besides new material in Chapters 17 and 28, however, there seem to be countless improvements found throughout the text and, particularly, the problems. With the new material taking us to a 1000-page book, the authors can be forgiven the omission of some topics such as diffusion tensor weighted imaging. I do know in practice they tend to emphasize hardware better by teaching the material in Chapter 27 earlier in the semester, and I suggest other instructors adopting this text do the same. In my world one can understand why I believe this is a very good suggestion.

Professor Wehrli spoke of 'exceptional didactic skill' and predicted '*Magnetic Resonance Imaging: Physical Principles and Sequence Design* is likely to become the daily companion of the MRI scientist and a reference standard for years to come.' I believe his prediction came true. Over the past decade, the book has exceeded 5,000 printed copies and been cited thousands of times, more than those from a number of standard physics textbooks. Its sales have stayed constant right up to the present time. I expect that the new edition will, in the words of Professor Duerk, keep this classic a classic in the coming decade. With my close connections to the authors, customers and contacts in industry and academia often ask me if I know anything about any new edition. After 14 years, I'm thrilled to tell them the second edition is finally here!

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June 28, 2013

# Foreword to the First Edition

Jeffrey L. Duerk

I heard my first lecture on an emerging field in medical imaging known as Nuclear Magnetic Resonance Imaging in 1983 as an electrical engineering graduate student at The Ohio State University. I was captivated and soon moved to Cleveland, a city then considered by many to be a United States center for the development of MR imaging and where both Picker International and Technicare were located a few miles apart. After studying many manuscripts, books and 'primers,' I enrolled in a new Physics and Biomedical Engineering course at Case Western Reserve University denoted by PHYS/EBME 431: The Physics of Medical Imaging, taught by Prof. E. Mark Haacke. In large part, the present book has grown and evolved from the class notes and lectures from this course's offering over the years.

The power of Magnetic Resonance Imaging (MRI) in the diagnostic arena of patient care is unquestionable. A multitude of books exist to assist in the training/teaching of clinicians responsible for interpreting MR images. Since joining the faculty of Case Western Reserve University almost a decade ago, I have been asked by graduate students, new industry hires, and fellow professors (from both CWRU and institutions throughout the world), if there was a book I could recommend which would provide sufficient depth in physics and MR imaging principles to serve as either a textbook or a complete tutorial for basic scientists. In my opinion, there were none which could provide the basic scientist with the tools to understand well the physics of MRI and also understand the engineering challenges necessary to develop the actual acquisitions (known as pulse sequences) which ultimately lead to the images. While there were no single sources available, I implored all to be patient. Today, I believe that patience has been rewarded as the book has arrived.

While much has changed in the field since my introduction to it in the early 1980's (e.g., the 'N' in NMR Imaging and the company Technicare are both gone), the power of this book is that many central concepts in MRI are rather more permanent, and their coverage here is superb. The influence by such notable predecessors as Abragam, Slichter, and Ernst is, at times, unmistakable. Mostly, the personal descriptive and analytical teaching style of Drs. Haacke, Brown, Thompson and Venkatesan builds an understanding of new concepts while clarifying old ones from the solid foundation provided earlier in the text. Another particular advantage of this book is that the notation is consistent, and located in a single reference; the readers do not have to overcome notational differences among our predecessors or difficulties in separating fundamental concepts from advanced material. Importantly, virtually every homework problem in the text has been designed to emphasize a central concept crucial to MRI. When I page through the book, I am often able to find the same derivations in the homework questions as in my log-books from the early part of my career in MRI. Insights from the authors are present throughout the text as well as within the problems; they provide those less experienced with glimpses (which later become illuminating flashes (no pun intended)) into how MR physics and sequences work and how they can be taken advantage of in the application to new ideas.

While I have been a co-instructor for EBME/PHYS 431 for a number of years and have used drafts of this book as the textbook, by now it bears little resemblance to the class notes of the initial offering in 1985. For that matter, the field of MRI has exploded with new techniques, new applications and far greater understanding and analysis of the innumerable aspects of the MRI hardware and software on image quality. I have benefited from my long friendships with Drs. Haacke and Brown, and the more recent ones with Drs. Thompson and Venkatesan. If you were to walk into the CWRU MRI Laboratory today, you would find no less than five drafts of this book on the shelves. The greatest tribute to these authors in their efforts to compile an important comprehensive treatise on the physics of MRI and MR sequence design can be heard in our research group's discussions of new imaging techniques (and likely those in the future at other institutions world-wide) when someone beckons 'Grab 'Haacke and Brown'!'

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January 20, 1999

Felix W. Wehrli

Haacke et al.'s new book spans a significant portion of proton MRI concerned with the design of MRI pulse sequences and image phenomenology. The work, designed for the physicist and engineer, is organized in twenty-seven chapters. The first eight chapters deal with the fundamentals of nuclear magnetic resonance, most of which is based on the classical Bloch formalism, except for a two chapter excursion into quantum mechanics. This portion of the book covers the basic NMR phenomena, and the concepts of signal detection and data acquisition. Chapters 9 and 10 introduce the spatial encoding principles, beginning with one-dimensional Fourier imaging and its logical extension to a second and third spatial dimension. Chapter 11 treats continuous and discrete Fourier transforms, followed in Chapters 12 and 13 by sampling principles, filtering and a discussion of resolution. Chapter 14 may be regarded as the opening section of the book's second part exploring more advanced concepts, beginning with treatment of non-Cartesian imaging and reconstruction. In Chapter 15, the properties of signal-to-noise are dealt with in detail including a discussion of the important scaling laws, followed, in Chapter 16 by a return to a more advanced treatment of rf pulses, along with such concepts as spatially varying rf excitation and spin-tagging. Chapter 17 is dedicated to the various currently practiced methods for water-fat separation, and in Chapters 18 and 19, the authors delve into the ever-growing area of fast imaging techniques. Chapter 18 is entirely dedicated to steady-state gradient-echo imaging methods to which the authors have themselves contributed a great deal since the inception of whole-body MRI. Chapters 19 and 20 address echo train methods focusing on EPI, $T_2^*$ dephasing effects and the resulting artifacts, ranging from intravoxel phase dispersion to spatial distortion. Chapter 21 is a brief introduction to the physics underlying diffusion-weighted imaging and pertinent measurement techniques. Chapter 22 treats the quantification of the fundamental intrinsic parameters, spin density, $T_1$ and $T_2$. Chapters 23 and 24 deal with the manifestions of motion and flow in terms of the resulting artifacts and their remedies, followed by a broad coverage of the major angiographic and flow quantification methods. The topic of Chapter 25 is induced magnetism and its various manifestations, including a discussion of its most significant application — brain functional MRI exploiting the BOLD phenomenon. In Chapter 26, the authors return to pulse sequence design, reviewing the design criteria for the most important pulse sequences and discussing potential artifacts. The final Chapter 27, at last, discusses hardware in terms of magnets, rf coils and gradients.

This book is the result of a monumental five-year effort by Dr. Haacke and his coauthors to generate a high-level, comprehensive graduate and post-graduate level didactic text on the physics and engineering aspects of MRI. The work clearly targets the methodology of bulk proton imaging, deliberately ignoring chemical shift resolved imaging or treatment of biophysical aspects such as the mechanisms of relaxation in tissues. Understanding the book requires college-level vector calculus. However, many of the basic tools, such as Fourier transforms and the fundamentals of electromagnetism, are elaborated upon either in dedicated chapters or appendices. The problems interspersed in the text of all chapters are a major asset and will be appreciated by student and teacher alike.

There is no doubt that the authors have succeeded in their effort to create a textbook that finally fills a need which has persisted for years. Haacke et al.'s book is, in the reviewer's assessment, the most authoritative new text on the subject, likely to become an essential tool for anyone actively working on MRI data acquisition and reconstruction techniques, but also for those with a desire to understand MR at a more than superficial level. The work is a rare synthesis of the authors' grasp of the subject, and their extensive practical experience, which they share with the reader through exceptional didactic skill.

The book has few flaws worth mention at all. First, not all chapters provide equal coverage of a targeted topic in that the book often emphasizes areas in which the authors have excelled themselves and thus are particularly experienced. Such a personal slant, of course, is very much in the nature of a treatise written by a single group of authors. On the other hand, the coherence in terms of depth of treatment, quality of illustrations and style, offered here, is never achievable with edited books. A case in point of author-weighted subject treatment is fast imaging, which is heavy on steady-state imaging. The following chapter on echo-train imaging almost exclusively deals with EPI and only secondarily with RARE and its various embodiments. Likewise, diffusion is treated only at its most fundamental level with little mention of anisotropic or restricted diffusion, or diffusion tensor imaging. Though the suggested reading list is helpful, a division into historic articles and those more easily accessible to the student would have been helpful since many of the historic papers cited would have to be retrieved from the library's storage rooms provided they are available at all. Finally, an introduction to the imaging hardware earlier (rather than as the last chapter) would help the novice bridging the gap between theory and instrumentation. None of the above, however, should detract from the book's high quality and practical usefulness.

In summary, the authors need to be congratulated on a superb product; a text vital to those concerned with MRI at a rigorous level. *Magnetic Resonance Imaging: Physical Principles and Sequence Design* is likely to become the daily companion of the MRI scientist and a reference standard for years to come.

Felix W. Wehrli, Ph.D.
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Editor-in-Chief, *Magnetic Resonance in Medicine*

February 9, 1999

This book is dedicated to our parents:

William James Brown
Florence Elizabeth Brown

Shih-Tai Cheng
Tuan-Yu Cheng

Helena Doris Haacke
Ewart Mortimer Haacke

Robert Thompson
Mary Christina Thompson

Ramasubramaniam Venkatesan
Saroja Venkatesan

# Preface to the Second Edition

In the second edition of this book, we have made more improvements and corrections in texts, equations, and homework problems than we can count, enhanced some chapters with new material, added a sizable new chapter, and updated a number of figures in various chapters. In particular, this includes a proof of the equal numbers in discrete Fourier transform pairs in Sec. 12.2.4, the correct interpretation of the $T_2^*$ filter effect on resolution in Sec. 13.5, revised materials throughout Ch. 16, new material on off-resonance excitation principles in Sec. 17.2.2, optimizing contrast in short-$T_R$ steady-state incoherent imaging in Sec. 18.1.2, a special discussion relating the 2D DFT with a 1D DFT as originally proposed by Professor Peter Mansfield in the 1970s in Sec. 19.9, a rigorous derivation of reducing a 3D dataset to 2D in Sec. 20.3.5, and an introduction to parallel imaging in Ch. 28.

Over the past decade, we indeed followed up our statement of motivation made in the preface to the first edition by teaching hundreds of graduate students and advanced undergraduate students at our home universities. We are aware of many other classes at other universities where the first edition of this book played an important role. MRI education continues to be our primary goal, but we have been gratified by the book's value as a research reference. Limitations remain and, alas, important topics are still missing. There are certain other MRI books that have since appeared and to which we enthusiastically refer the interested reader; we have added them to our suggested readings. To address missing topics, newly emerging topics, and amendments and corrections to the second edition, we have set up a website for students, teachers, and researchers. We are posting the many exam problems and optional homework examples developed in our years of teaching and we offer contacts with lecturers to compare solutions. However, students should try to solve these problems by themselves!