Matteo Barigozzi
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Davy Paindaveine *Editors*

Recent Advances in Econometrics and Statistics

Festschrift in Honour of Marc Hallin

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Preface

Marc Hallin has made his career at the Université libre de Bruxelles, where he is now a Professor Emeritus. He was also a Visiting Professor in various universities, including Princeton University, Indiana University, the Université Pierre-et-Marie-Curie (now, Université Sorbonne), the European University Institute and Universidad Carlos III in Madrid. He supervised or co-supervised 25 PhD students, many of whom now have a professor position in Algeria, Belgium, France, Germany, Italy, Japan or Morocco. He acted as a co-Editor-in-Chief of *International Statistical Review* and *Statistical Inference for Stochastic Processes*, and as an Associate Editor of many journals in Econometrics and Statistics, among which the *Journal of the American Statistical Association* and the *Journal of Econometrics*. His research work has been recognized at the highest level through prestigious awards, including, most recently, the *Prix Pierre-Simon de Laplace* from the Société Française de Statistique (2022) and the *Gottfried E. Noether Senior Award* from the American Statistical Association (2022).

Marc's research interests are covering an unusually broad and diverse spectrum of fundamental and applied statistical topics. That versatility is reflected in his list of about 250 publications, with contributions in statistical decision theory, time series, random fields, density estimation, multivariate analysis, panel data, inequalities, high-dimensional and "big data" problems, quantile regression, spectral analysis, data depth, the asymptotic theory of statistical experiments, statistical applications of measure transportation and econometrics. In all those fields, Marc has been promoting nonparametric and semiparametric approaches, which constitute a red thread running through half a century of scientific activity—a period in which our discipline underwent dramatic changes.

Two major themes, which often intersect, are emerging from Marc's otherwise highly diversified publications: (1) distribution-free rank-based and quantile-oriented inference and (2) dynamic factor models for time-series analysis. This volume starts with a commented bibliography that mainly presents his contributions to both these themes. The main part of the book collects 29 chapters, which are authored or co-authored by some of Marc's main collaborators and that are organized into 6 parts:

- Rank- and Depth-Based Methods
- Asymptotic Statistics
- Quantile Regression
- Econometrics
- Statistical Modelling and Related Topics
- High-Dimensional and Non-Euclidean Data

We would like to warmly thank the authors of these 29 chapters, as well as the referees, for their contributions to the present tribute to Marc's career.

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Graz, Austria	Siegfried Hörmann
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Marc Hallin: A Commented Bibliography (from 1972 to 2023)

Matteo Barigozzi, Siegfried Hörmann, and Davy Paindaveine

Abstract We provide a commented bibliography that describes the contributions made by Marc Hallin in the papers he authored or coauthored between 1972 and 2023. Citations using the "name (year)" format correspond to the list of references on page 14, whereas numbers in square brackets refer to Marc's full list of publications to date, which is provided at the end of this text.

1 Rank-Based and Quantile-Oriented Inference

Marc's interest in ranks and quantiles goes back to the mid-1980s. Starting with [126] (actually, [122, 124, 125]), this topic encompasses the entire span of his 50 years of scientific activity and still constitutes an important part of his current research.

Rank-based inference in the mid-1980s was considered an essentially complete theory. After about half a century of intensive development and the seminal contribution of Jaroslav Hájek, as summarized in Hájek and Šidák (1967) and systematized in such monographs as Puri and Sen (1985), research activity in the area had slowed down quite significantly. The theory at that point, however, only was covering a somewhat narrow range of statistical models:

– Essentially limited to the context of general linear models with independent (exchangeable) observations (location, scale, regression, ANOVA, etc.)

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– Restricted (except for methods based on componentwise rankings, which are unsatisfactory on several counts) to the analysis of univariate observations and bivariate dependence/independence
– Making limited use of Le Cam's asymptotic theory of statistical experiments (essentially, only Le Cam's third lemma was used, mainly in order to compute local asymptotic powers and asymptotic relative efficiencies under local asymptotic normality—a practice that goes back to Hájek and Šidák (1967); as for the Bickel, Klaassen, Ritov, and Wellner approach to semiparametric inference problems (Bickel et al. (1993)), it was not available yet.)

Fully exploiting the power of Le Cam's theory of locally asymptotically normal experiments (Le Cam (1986)) and the related theory of efficient semiparametric inference (Bickel et al. (1993)), Marc succeeded in removing most of those restrictions and extending the scope of rank-based tests and R-estimation methods into a variety of directions, namely:

– Univariate linear (ARMA) time-series models: the first significant steps in that direction being [126, 201, 167, 205, 208] (see [206, 234, 101, 102, 105] for survey papers)
– Semiparametric versions of more sophisticated nonlinear dynamic models considered, mainly, in econometrics—bilinear, discretely observed diffusions with jumps, AR-ARCH, AR-LARCH, Cox-Ingersoll-Ross processes, duration models, etc. ([141, 137, 138])
– New problems, as independent component analysis (see [165]), skewness in skewed families of densities ([28, 30, 31]), or ([178, 29]) the estimation of cross-information quantities (a crucial step in R-estimation)
– More general concepts of ranks (related to appropriate group invariance or maximal ancillarity arguments), such as signs and ranks for median- or quantile-restricted models ([231, 232]), ranks and some adequate indicators (e.g., in the context of Ornstein-Uhlenbeck processes, see [141], or for unrestrictedly heteroskedastic time series, see [45])
– Multivariate or multiple-output settings: pseudo-Mahalanobis ranks and signs, hyperplane-based ranks and signs ([179, 180, 183, 184, 185, 187, 178]) and, more recently, based on a measure-transportation-based approach, Monge-Kantorovich and center-outward quantiles, ranks, and signs ([32, 112, 115])

A survey of measure transportation as a tool for statistical inference is provided in [109].

1.1 Rank-Based Tests and R-Estimators

Marc can be credited for a systematic introduction of rank-based methods in locally asymptotically normal (LAN) models with parameter $\boldsymbol{\theta}$ driven by a noise or an innovation with unspecified density f. The basic novelty of his approach is a rank-based asymptotic reconstruction $\underset{\sim}{\mathbf{\Delta}}_f^{(n)}(\boldsymbol{\theta})$, via a Hájek representation result, of the tangent-space projection $\mathbf{\Delta}_f^{(n)*}(\boldsymbol{\theta})$ of the central sequence $\mathbf{\Delta}_f^{(n)}(\boldsymbol{\theta})$ under density f. The ranks there are the ranks of the residuals associated with the value $\boldsymbol{\theta}$ of the parameter, which under parameter value $\boldsymbol{\theta}$ are distribution-free. As shown in [235] (see Sect. 1.1.6), the invaluable advantage of these ranks is that the approximate-score version $\underset{\sim}{\mathbf{\Delta}}_f^{(n)}(\boldsymbol{\theta})$ of $\mathbf{\Delta}_f^{(n)}(\boldsymbol{\theta})$ converges in probability, under density f and parameter value $\boldsymbol{\theta}$, to $\mathbf{\Delta}_f^{(n)*}(\boldsymbol{\theta})$. They automatically achieve, as the sample size n diverges to infinity, the desired tangent-space projection, without requiring the case-by-case derivation of least favorable directions, the explicit computation of the tangents, nor any kernel estimation of the actual densities; they also avoid unpleasant ad hoc procedures such as sample splitting. Semiparametric efficiency in the model where $\boldsymbol{\theta}$ is the parameter of interest and the density f of the noise or the innovation is an unspecified nuisance, thus, can be attained at selected f_0 (e.g., the standard Gaussian f_0, which leads to the so-called normal-score or van der Waerden statistics), and with f replaced with some adequate estimator $\hat{f}^{(n)}$ of the actual f, at any density for which LAN holds.

That general strategy applies to a variety of models, yielding locally asymptotically optimal rank tests and R-estimators.

1.1.1 Rank-Based Inference for Univariate Time Series

Although some of the earliest rank tests (such as the Wald-Wolfowitz test against serial dependence (Wald and Wolfowitz (1943, 1944)) or some applications (David (1947)) of the runs tests actually address serial dependence issues, time-series problems had not been considered in rank-based inference until 1985 and [126] where Marc, with Jean-François Ingenbleek and Madan Puri, establishes a general Hájek representation result for linear rank statistics of the serial[1] type (see [230] for a proof under minimal assumptions, and [212] for a Berry-Esséen result). That breakthrough opened the door for rank-based inference in:

- Univariate ARMA processes ([201, 167, 62]), single-output linear regression with ARMA errors (equivalently, ARMA processes with a linear trend) [208], random coefficient AR models ([2]), but also long-memory, fractional differentiation, and unit roots ([216, 33, 238, 226])
- Rank-based tests of the cointegration rank in semiparametric error-correction models for cointegration with trends ([227])
- Nonlinear time-series models such as bilinear ([17, 18, 19, 20]) and Ornstein-Uhlenbeck processes ([141]), AR-ARCH and LARCH, or discretely observed diffusions and Lévy processes ([137, 138]), where Gaussian quasi-likelihood

[1] Contrary to the nonserial ones, the scores, in a serial linear rank statistic, involve several consecutive ranks.

estimators are not always consistent and the classical semiparametric tangent-space approach runs into numerical difficulties

All these contributions, however, are dealing with the traditional univariate concept of ranks, based on the strong ordering of the real line. Extending them to a multivariate context was a challenging objective, which Marc successively took on in two steps. A first multivariate extension was obtained in 2002 with the introduction (see [179]) of the so-called Mahalanobis ranks and signs for elliptical models; see Sect. 1.1.2. Then, in 2017 and 2021 ([32, 112]), he was able to relax the assumption of ellipticity by introducing the measure-transportation-based Monge-Kantorovich and *center-outward ranks* and *signs*; see Sect. 1.1.3.

1.1.2 Mahalanobis Ranks and Signs in Elliptical Models (Serial and Nonserial)

Defining ranks and signs based on the Mahalanobis transforms of residuals and their *interdirections*,[2] Marc was able to extend the classical distribution-free and semiparametrically optimal procedures based on traditional univariate ranks to the multivariate context of models with unspecified elliptical noise or innovation process with possibly infinite second-order moments. In a series of papers, most of them with Davy Paindaveine and Thomas Verdebout, he successively considered:

- One-sample location models (in [179], where Mahalanobis ranks are first introduced; see also [181]).
- Testing white noise against VARMA dependence and the adequacy of a VARMA model ([180, 183]).
- VAR order identification, detection of VAR periodicities, and testing linear restrictions in general linear models with VARMA error terms ([184, 159, 185, 186]).
- Tests and R-estimation for shape matrices ([187, 178, 188]) and tests of scatter, covariance, scale, and shape homogeneity ([191, 192]).
- One- and k-sample principal component[3] models ([197, 198]).
- R-Estimation of principal and common principal components ([199]); R-estimation of the mixing matrix in independent component analysis ([165], without the assumption of symmetric component densities).

Some of these contributions are solving long-standing open problems in multivariate analysis, such as the correction ([188, 191]; see [192] for a detailed discussion) of Bartlett's test of homogeneity of variances (here extended, under possibly infinite variances, to homogeneity of scatter) under unspecified densities—previous solutions indeed either were destroying the local power or were losing

[2] A concept introduced by Randles (1989).

[3] The k-sample principal component model is also known as the *common principal component* model; before [198], it only could be analyzed under Gaussian assumptions: see Flury (1984).

the local maximin structure of that celebrated, daily practice tool. Some others are introducing new asymptotic concepts, such as that of *locally asymptotically curved experiments* (see [197, 199]).

1.1.3 A Measure-Transportation Approach to Multivariate Distribution and Quantile Functions, Ranks, and Signs

More recently, Marc ([32], with Victor Chernozhukov, Alfred Galichon, and Marc Henry) proposed a new concept of statistical depth and quantiles based on Monge-Kantorovich measure-transportation ideas. In [112] (with Eustasio del Barrio, Juan Cuesta-Albertos, and Carlos Matrán; see also [38]), this approach is developed, from a statistical decision theory perspective, into a general theory of center-outward distribution and quantile functions, the empirical versions of which yield multivariate concepts of ranks and signs.

This new approach relies on a monotone (in the sense of *gradients of convex functions*) probability integral transformation to the spherical uniform distribution over the unit ball. That transformation is entirely canonical, and, contrary to halfspace depth contours (which are always convex), its contours account for the "shape" of the underlying distribution—yielding banana- and pear-shaped contours for banana- and pear-shaped distributions, respectively. In the particular case of univariate or spherical/elliptical distributions, that concept produces ranks and signs that coincide with the usual and well-accepted ones, paving the way to a general theory of rank-based inference in multivariate analysis.

Center-outward ranks and signs and the corresponding quantiles, unlike the many other concepts (componentwise ranks, spatial or geometric ranks, Mahalanobis ranks,...) that can be found in the literature, enjoy all the essential structural properties[4] that make traditional ranks a fundamental concept in statistical decision theory and a successful inferential tool. Empirical and population center-outward distribution functions moreover are related by a Glivenko-Cantelli property—the quintessential property of their traditional univariate counterparts. Similarly, the center-outward quantile functions, in sharp contrast with all other concepts developed so far, produce quantile contours and regions with preassigned probability contents that do not depend on the actual underlying distribution while preserving (unlike similar concepts associated with transportation to the unit cube) its symmetry features.

Applications to the long-standing open problem of constructing distribution-free tests for the hypothesis of independence between vectors with unspecified densities have been proposed by Ghosal and Sen (2022), Shi et al. (2022), Deb and Sen

[4] All those properties originate in the fact that the sigma-field of the observations factorizes into the product of the sub-sigma-field of (residual) ranks and signs and the sub-sigma-field of the (residual) order statistic. The latter is sufficient and complete for the nuisance (the unspecified density of the noise driving the model); the former is essentially maximal ancillary (as shown in [112]); by Basu's theorem, they are mutually independent.

(2023), and Marc (with Hongjian Shi, Mathias Drton, and Fang Han) in [246] and [245]. With various coauthors, Marc also develops applications to:

– Optimal rank tests for multiple-output regression and MANOVA[5] in [115] (with Daniel Hlubinka and Šárka Hudecova)
– Rank tests and R-estimators for VARMA models in [139, 157, 229] (with Hang Liu and Davide La Vecchia)
– The measurement of multivariate risk in [16] (with Jan Beirlant, Sven Buitendag, Eustasio del Barrio, and François Kamper)

In [35], Marc, Eustasio del Barrio, and Alberto González-Sanz develop a non-parametric theory for multiple-output quantile regression based on the concept of center-outward quantile. The same concept of quantile is used by Marc and Gilles Mordant in [174] to define multiple-output Lorenz curves and Gini indices. Finally, in [151], Marc, Hang Liu, and Thomas Verdebout propose concepts of distribution and quantile functions, ranks, and signs, for directional variables, that is, observations taking values on hyperspheres.

1.1.4 Regression and Autoregression Rank Scores

Another concept related with rank-based inference is that of (auto)regression rank scores. The concept was introduced in a regression context with independent observations by Gutenbrunner and Jurečková (1992) from a duality argument applied to Koenker and Bassett's celebrated regression quantiles.

The most attractive feature of regression rank scores lies in the fact that, contrary to classical "aligned rank statistics" computed from estimated residuals, regression rank score statistics, without estimating the value of the unspecified regression parameter, asymptotically reconstruct the actual corresponding rank-based statistics—even though exact residuals (hence exact ranks) cannot be computed from the observations. The concept has been extended by Koul and Saleh (1995) to the time-series context, albeit in the Jaeckel style (mixing residuals and their ranks). In [133], Marc and Jana Jurečková are constructing locally asymptotically optimal tests based on such autoregression rank scores derived for linear constraints on the coefficients of an autoregressive model. The related estimation procedures are derived in [6]. In [236] and [136], the technique is applied to the problem of autoregressive order identification and to the problem of testing independence between two autoregressive series with unspecified coefficients. Kolmogorov-Smirnov tests based on autoregression rank scores are constructed in [49], and a very efficient method for estimating the innovation sparsity function (the inverse of the density of the unobservable innovation process at some given quantile) is proposed in [50]. Finally, [134] (joint with Jana Jurečková and Hira Koul) proposes a class of serial statistics which, contrary to Koul and Saleh's, is

[5] The two-sample problem has been considered by Deb et al. (2021).

entirely based on (auto)regression rank scores (thus involving multiple integrals over the quantile ranges of several lagged residuals) and asymptotically equivalent to the corresponding statistic based on the genuine, non-available residual ranks—something the Jaeckel-type statistics cannot achieve.

1.1.5 Asymptotic Relative Efficiencies (AREs); Chernoff-Savage and Hodges-Lehmann Results

Generalizations of the classical Chernoff-Savage theorem (Chernoff and Savage (1958)) stating that rank tests based on Gaussian scores perform uniformly better than Student tests are established for a number of those extensions: see [95] and [220] for univariate time-series problems, [179, 189], and [180] for nonserial and serial elliptical ones. In time series, for instance, the asymptotic relative efficiencies, with respect to daily practice correlogram-based (pseudo-Gaussian) methods, of the normal-score rank-based procedures developed in [208], are shown to be uniformly larger than one. This should be a strong incentive for bringing ranks into practice in the context.

Marc also has obtained generalizations of the no less famous Hodges and Lehmann "0.864" result (Hodges and Lehmann (1956)). In its original version, this result shows that the lower bound for the asymptotic relative efficiencies, still with respect to Student tests, of Wilcoxon-type methods for location, is 0.864. In a time-series context, with Student replaced by correlogram-based methods, that bound (see [220]) takes a slightly smaller 0.856 value. It is interesting to note that, in higher dimensions (k-dimensional elliptical observations, the Gaussian reference being Hotelling rather than Student), this Hodges-Lehmann bound, with a maximum value of .916 at dimension $k = 2$, is not a monotone function of k; see [179] and [180]. In the same spirit, [222] investigates several extensions of Hodges and Lehmann's "$6/\pi$ result."

1.1.6 Rank-Based Inference and Semiparametric Efficiency

Semiparametric models in which the underlying noise or innovation density plays the role of a nuisance parameter indeed are the general context where rank-based methods naturally come into the picture. A far-reaching result is obtained in [235], where it is shown that conditioning central sequences with respect to the maximal invariants (ranks, for instance) of appropriate generating groups yields the same results as the more traditional tangent-space projections and hence leads to semiparametrically efficient inference.

That result provides a fundamental and very strong justification for considering rank-based inference, by showing that ranks (or more general invariants) actually retain all the information related with the parameter under study, while everything else (typically, an order statistic) only carries information about the nuisance (the underlying unspecified density). Semiparametric efficiency—the best that can be expected in the presence of unspecified densities—thus can be achieved by rank-based procedures; as a corollary, ranks can reach parametric efficiency in a given model if and only if that model is adaptive in the semiparametric sense. And, of course, ranks also bring along the many advantages related to distribution-freeness: exact distributions, unbiasedness, increased robustness, etc.

1.1.7 R-Estimation and the Estimation of Cross-Information Quantities

R-Estimation is another classical topic in rank-based inference. Unlike testing, however, and despite a long history R-estimation, with the exception of linear regression, never really made its way to applications, and a widespread idea is that "ranks are fine for testing but not for estimation."

The reason for this is twofold. Practical reasons first: in contrast with rank-based test statistics, R-estimators do not come under explicit closed forms, but as solutions of optimization procedures involving piecewise constant and nonconvex objective functions; the larger the dimension of the parameter, the trickier the computation of such estimators. Next, the asymptotic variances of R-estimators, which are needed for computing asymptotic confidence regions, typically depend on unknown *cross-information quantities* of the form (in the particular case of regression)

$$\mathcal{J}(f, g) = \int_0^1 \frac{f'(F^{-1}(u))}{f(F^{-1}(u))} \frac{g'(G^{-1}(u))}{g(G^{-1}(u))} du,$$

where F is the distribution function associated with the reference density f used to build the scores, but G and g are the actual unspecified distribution and density functions. Such integrals are not easily estimated. For instance, in the Wilcoxon case, for one-sample location, one has to estimate $\int g^2(z)dz$, where g is the unspecified density of the observations. Unless R-estimators can be computed via some well-behaved alternative minimization (this is the case, for regression—but for regression only, of Jaeckel's approach), R-estimation thus remains a theoretically attractive but practically hardly implementable method. This is particularly regrettable in view of the strong connection established in Sect. 1.1.6, between rank-based methods and semiparametric efficiency.

Inspired by Le Cam's one-step estimation technique, Marc, with Hannu Oja and Davy Paindaveine in [178], proposes, in the context of the estimation of shape matrices, a one-step R-estimator based on the same rank-based central sequence as the semiparametrically efficient rank tests described in Sect. 1.1. The problem with this one-step R-estimator is that (irrespective of the problem under study) it explicitly depends on the same unknown cross-information quantity $\mathcal{I}(f, g)$ as above. It is shown in [178] and [29] how an ingenious one-dimensional local likelihood maximization argument, exploiting the LAN structure of the experiment under study, provides a consistent estimation of that quantity. Under this one-step form, R-estimators in principle can be constructed and computed for the broad range of models considered in [235].

This one-step method also has been successfully applied to:

- The estimation ([224]) of regression coefficients in linear models with stable errors; unlike the much-studied OLS estimator, this R-estimator is root-n consistent (see below for details).
- The estimation ([199]) of principal component and common principal components (this yields the only available estimators of common principal components retaining consistency in the absence of Gaussian assumptions).
- The estimation ([165]) of mixing matrices in independent component analysis.
- The estimation ([137, 138]) of sophisticated time-series models such as non-Gaussian discretely observed Ornstein-Uhlenbeck processes, discretely observed diffusions with jumps, AR-ARCH, AR-LARCH, Cox-Ingersoll-Ross processes, duration models, etc., for which Gaussian quasi-likelihood methods typically lead to poor and non-robust results and numerically difficult implementations.

In most of these problems, the dimension of the parameter to be estimated is relatively large, and a method involving a one-dimensional optimization step only is quite welcome. The asymptotic performances of those R-estimators, moreover, are particularly good when based on data-driven reference densities, such as skew-t ones with estimated skewness and degrees of freedom accounting for the skewness and kurtosis of the actual unspecified density (which does not have to be skew-t).

R-Estimation in regression models with infinite-variance stable errors is of particular importance. These models, indeed, are nicely LAN with root-n contiguity rates. Classical OLS estimators, however, fail miserably and are not even rate-optimal. Considerable efforts have been made, in the literature on extremes, to renormalize them in some appropriate fashion; but the result is not even rate-optimal! Starting from a LAD preliminary, [224] proposes R-estimators based on stable scores that remain root-n consistent under the whole family of stable distributions, irrespective of their asymmetry and tail index. With the LAD estimator, these R-estimators are the only rate-optimal ones available in the literature.

1.2 Depth- and Quantile-Based Inference

1.2.1 Multiple-Output Halfspace Depth Regression

Before the introduction, in [32] and [112], of measure-transportation-based quantiles, Marc (with Davy Paindaveine and Miroslav Šiman) in an *Annals of Statistics* paper with discussion ([193, 194]) established an unexpected link between the concept of halfspace depth introduced by Tukey, and a directional version of Koenker and Bassett's celebrated concept of quantile regression. That connection brings to halfspace depth the benefits, inherited from the L_1 nature of quantiles, of such results as a Bahadur representation and the asymptotic normality of depth hyperplane coefficients, as well as those of linear programming computation. The same approach also can be adopted in multiple-output regression, where it produces nested *depth regression tubes* (see [152]) with an interpretation of multiple-output quantile regression surfaces. The concept is particularly attractive in the construction of multivariate growth charts (see [241] for applications) and multivariate outlier detection. Indeed, medical doctors typically consult single-output growth charts which only can diagnose marginal outliers, while a nowhere marginally outlying observation clearly can be a multivariate outlier.

A more direct approach to a multiple-output generalization of Koenker and Bassett's concept is taken in [217], where ellipsoids, rather than hyperplanes, are characterized via the minimization of the expected check function, leading to elliptical linear regression quantiles. The delicate issue there consists in showing that the resulting minimization can be turned into a convex optimization problem.

When compared to the center-outward quantile regression described in Sect. 1.1.3, halfspace depth regression suffers the same drawbacks as halfspace depth versus center-outward quantile regression: no control over the (conditional) probability content of (conditional) quantile regions which, moreover, are necessarily convex. The latter, thus, may seem preferable, but they also are computationally more demanding. A 2017 survey of multiple-output quantile regression can be found in [218].

1.3 Quantile-Oriented Spectral Analysis

In a somewhat different direction, [36] is introducing a new type of quantile-related periodogram where the traditional least squares regression of the observations X_t on sines and cosines is replaced with quantile regression in the Koenker and Bassett spirit, yielding a distinct cross-periodogram for every couple (τ_1, τ_2) of quantile orders. This leads to a rank-based periodogram kernel with interesting properties and suggests the definition of a population copula-based cross-spectral density kernel. Contrary to traditional spectral densities, copula-based spectra do not require any moment conditions to exist and are able to account for any features of the bivariate joint distributions of the couples (X_t, X_{t-k}), $k \in \mathbb{Z}$. Rank-based periodograms and copula-based cross-spectral densities are invariant under continuous order-preserving marginal transformations of the series under study, so that the approach very neatly separates the marginal features of the underlying process and the serial dependence ones.

Consistent and asymptotically normal estimators are obtained in [36] for a smoothed version of those rank-based periodograms. These results, however, are only pointwise with respect to $\tau \in [0, 1]$. A closely related rank-based periodogram kernel, consistently estimating, after due smoothing, the same copula-based cross-spectral densities, is studied in [239], where uniform asymptotics are carefully established. The main difficulty there lies in the fact that the ranks involved are not computed from exchangeable observations. In [26] and [25], locally stationary versions of the same spectral concepts, requiring the definition of a new concept of local stationarity, are introduced. The associated integrated spectrum is studied in [65].

2 Time-Series Analysis

2.1 Time-Series Models with Time-Varying Coefficients

Marc's early interest in time series was focused on ARMA models with time-varying coefficients. In [87], he proposed a complete solution to the spectral factorization problem of nonstationary q-dependent processes, based on an original theory of matrix-valued continued fractions (see also [92]), followed, in [90], by a complete discussion of the invertibility properties of the resulting moving-average models and the impact of these properties on optimal forecasting. The periodic case is studied in [22] and [24]. In [121], with Jean-François Ingenbleek, he similarly developed a Yule-Walker theory for autoregressive models with time-varying coefficients.

2.2 Nonparametric Analysis of High-Dimensional Time-Series Data: Dynamic Factor Models

Another important domain of Marc's research activity in nonparametric statistics is the analysis of high-dimensional time series. Such time series appear in a huge variety of applications, but Marc's contributions were motivated, mainly by his collaboration with econometricians, both in macroeconometrics (Mario Forni, Marco Lippi, Lucrezia Reichlin, Paolo Zaffaroni) and in financial econometrics (Matteo Barigozzi, David Veredas, Stefano Soccorsi, Carlos Trucíos). Datasets in those areas typically come under the form of very large panels of interrelated time series; "large" here means several hundreds to one thousand. A parametric approach in dimension n, even for the simplest VAR(1) model, requires $n(3n+1)/2$ parameters, that is, for $n = 1{,}000$, a hopeless parameter space of dimension 1,500,500 .... A nonparametric approach is thus the only reasonable attitude.

Inspired by Brillinger's concept of dynamic principal components, [54] (with more than 2,300 Google Scholar citations) introduces a generalized dynamic factor model (GDFM) method by which the spectral density matrix of the n-dimensional process under study is decomposed into two components: the "common component," of low rank, characterized by exploding (as $n \to \infty$) dynamic eigenvalues,[6] and the "idiosyncratic component," with uniformly bounded dynamic eigenvalues. The intuitive interpretation is that the "common component" is driven by a small number of shocks (the "market shocks") that are hitting almost all time series in the panel and cause exploding eigenvalues, whereas the "idiosyncratic" shocks only affect a small number of series, yielding mild cross-correlations and hence bounded dynamic eigenvalues.

That approach is essentially model-free (on this particular point, see [148]); apart from second-order stationarity, the existence of spectral densities, and the presence of a finite (but unspecified) number of diverging spectral eigenvalues, no constraints are put on the data-generating process, and the GDFM decomposition follows as a canonical representation result. This is in sharp contrast with other factor model decompositions considered in the literature, where the covariance matrix $\mathbf{\Gamma}_0$ is assumed to decompose into the sum of a low-rank matrix plus a sparse one—an assumption which is static—it does not take into account the serial dependence features of the data. That GDFM method had a significant impact in the econometric community.

In a series of papers joint with Marco Forni, Mario Lippi, and Lucrezia Reichlin, a consistent estimation procedure is constructed ([54]), consistency rates are studied ([57]), and a forecasting strategy is proposed ([58]); [55, 4], and [56] establish the applicability of the method to real macroeconometric data. The problem of determining the number of independent common shocks underlying the data is treated by Marc and Roman Liška [147], based on information criterion techniques with a clever tuning of penalty terms; some financial applications are considered in [247, 161, 249], and [221].

Up to that point, the methods, based on Brillinger's theory, which involves two-sided filters, had relatively poor performances at the end of the observation period, and hence it was of little help in forecasting problems. That problem found an elegant solution in [59] and [60] which, building on recent results for reduced-rank stochastic processes, proposes and studies a strictly one-sided alternative to the original Forni-Hallin-Lippi-Reichlin (2000) method.

In collaboration with Matteo Barigozzi, Marc also developed general dynamic factor model methods for a nonparametric study of volatilities in high dimension ([8, 9, 10, 11, 13]). Complete asymptotic results (consistency, rates, asymptotic normality of the estimators, etc.) are obtained in [15]. The locally stationary case is considered in [14]. Extensions to time series of functional data have been obtained ([177, 248]) with, as a first step, a functional extension of Brillinger's theory of dynamic principal components ([66], in collaboration with Siegfried Hörmann and Lukasz Kidziňski).

The Forni-Hallin-Lippi-Reichlin dynamic factor methods and their refinements are implemented on a daily basis by a number of economic and financial institutions, including several central banks and national statistical offices, who are using

[6] Those dynamic eigenvalues are those of spectral density matrices; they are functions of the frequency.

it in their current analysis of the business cycle (among them, the European Central Bank, the Federal Reserve, the National Bank of Switzerland, the Banca d'Italia, etc.). A real-time coincident indicator of the EURO area business cycle (EuroCOIN), based the same method, is published monthly by the London-based Centre for Economic Policy Research and the Banca d'Italia: see http://www.cepr.org/data/EuroCOIN/. Also based on that method, a similar monthly index is established for the US economy by the Federal Reserve of Chicago.

2.3 Miscellaneous Problems in Time Series and Random Fields

Besides time-varying models and high-dimensional time series, Marc also worked on several classical time-series-related topics:

– Local asymptotic normality of VARMA processes with a linear trend in [61] and [185]
– Information-theoretical identification of the order of a time-series model in [51] and adaptive estimation of the lag parameter of a long-memory process in [216]
– M-Estimation in AR(p) models under nonstandard conditions in [48]
– Testing non-correlation and non-causality between VARMA time series in [213] and [214]
– Kernel density estimation for linear processes in [219], for random fields in [27]
– Local linear spatial regression in [155] and local linear spatial quantile regression in [156]

3 Miscellaneous Problems in Mathematical Statistics

Finally, indulging in excursions out of his main subjects, Marc also contributed to various problems in mathematical statistics, among which:

– Fisher information singularities in families of skew densities ([145, 146])
– Local asymptotic normality and the power of sphericity tests in high-dimensional spiked models ([242, 243])
– The construction of pseudo-Gaussian tests for linear parameter restrictions in LAN models ([190]) and the non-Gaussian asymptotics of likelihood ratio tests for the homogeneity of covariances [104])
– Optimal detection of random regression coefficients ([1, 53])
– Bounded completeness and the denseness of likelihood ratios in [237]
– Distributional approximation and bounds in [40, 41, 42, 43, 170, 44, 212, 46]
– The existence and regularity of monotone measure-preserving maps in [38] and [64]

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