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David Ellerman

Partitions, Objective Indefiniteness, and Quantum Reality

The Objective Indefiniteness Interpretation of Quantum Mechanics

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To the memory of Abner Shimony
—Physicist, Philosopher, and Friend.

Preface

My main formation as a mathematician was in collaboration in the early 1980s with Gian-Carlo Rota of MIT. We wrote a joint paper that took my Erdös number from infinity down to 3. But most of my employment in later years was in Economics, not mathematics.

A big shock came in 1999 just before my retirement when Rota died relatively young at age 66. Some of us who had worked with him wanted to pick up some of his unfinished strands to further develop. One of those strands was a 1996 paper by Rota and some colleagues entitled: “Logic of Commuting Equivalence Relations.” Rota was well aware of the category-theoretic duality between subsets and partitions (or equivalence relations). Since ordinary logic starts with the Boolean logic of subsets (usually presented in the special case of “propositional logic”), Rota had the idea of developing a logic of that dual concept, a logic of partitions or equivalence relations. But the use of the word “logic” in that 1996 paper was an overstatement since there was no known implication operation on equivalence relations or partitions, only the lattice operations of join and meet known in the nineteenth century (e.g., Dedekind and Schröder).

In fact, no new operations of partitions, such as the implication operation, were developed throughout the twentieth century. Eventually, perhaps with a little luck, I was able to define the implication operation on partitions. Soon it became clear that there were, in fact, two algorithms that could be used to define all the Boolean operations on partitions. That was the beginning of partition logic developed in my recent book, *The Logic of Partitions: With Two Major Applications.*

The first major application was again foreshadowed by Rota who emphasized the analogy: $\frac{\text{Probability}}{\text{Subsets}} \approx \frac{\text{Information}}{\text{Partitions}}$. Boole logically developed finite probability theory starting as the quantitative notion of subsets, e.g., the probability of getting a subset S of a finite equiprobable sample space U is just the normalized number of elements: $\Pr(S) = \frac{|S|}{|U|}$. Hence, using Rota’s equivalence, the *logical* notion of information should start with the quantitative notion of the “size” of a partition. The duality of subsets and partitions reveals an underlying duality between elements of subsets and the ordered pairs called the *distinctions* or *dits* of the partition, i.e., ordered pairs in different blocks of a partition.

Then we can finally answer the question raised by Rota's analogy; the quantitative notion of a partition is the size of its set of distinctions, its ditset. The basic logical notion of information in a partition is just the normalized size of its ditset, so the initial definition of *logical entropy* for $\pi = \{B_1, ..., B_m\}$ is

$$h(\pi) = \frac{|\mathrm{dit}(\pi)|}{|U \times U|} = \frac{\left|U \times U - \cup_j \left(B_j \times B_j\right)\right|}{|U \times U|} = 1 - \sum_j \left(\frac{|B_j|}{|U|}\right)^2 = 1 - \sum_j \Pr\left(B_j\right)^2 = \sum_{j \neq k} \Pr\left(B_j\right) \Pr(B_k)$$

with equiprobable points in U. If there is a general probability distribution $p = (p_1, ..., p_n)$ on the points of U, then the logical entropy of π is just the value of product probability measure $p \times p$ on the ditset $\mathrm{dit}(\pi) \subseteq U \times U$. The interpretations of $\Pr(S)$ and $h(\pi)$ are thus also analogous. One random draw from U gets an element of S with the probability $\Pr(S)$, and two random draws from U get a distinction of π with the probability $h(\pi)$. This also (finally) gives a logical definition of information, i.e., information as distinctions.

The quantum version, quantum logical entropy, is one of the topics briefly developed in this book. It has the analogous interpretation, i.e., the quantum logical entropy resulting from a projective measurement of an observable on a state is the probability that in two independent measurements of the same observable on an identically prepared state, different eigenvalues are obtained.

Hence the first major application of partition logic is simply its quantitative version as a logical definition of information analogous to the way Boole approached probability theory as a quantitative version of subset logic. That supplies a much-needed *logical* foundation for information theory (classical and quantum) developed in my 2021 book *New Foundations for Information Theory: Logical Entropy and Shannon Entropy*.

The second major application and the topic of this book is the century-old problem of understanding the reality that quantum mechanics (QM) describes so well. QM was consolidated in the mid-1920s but, over the last century, there has been no agreement on the nature of reality at the quantum level. New so-called "interpretations" are continually being created without any noticeable convergence. Otherwise sane physicists are driven to rather bizarre ideas, e.g., the many-worlds interpretation, when confronted with the "paradoxes" of quantum theory. It is in this intellectual "demolition derby" of quantum interpretations where partition logic and logical entropy offer a new approach to corroborate (in a suitably reformulated manner) an interpretation already promoted by Werner Heisenberg and Abner Shimony, among others.

This new approach first "cuts at the joint" between the mathematics and the physics of quantum mechanics. The mathematics is quite distinctive and different from the mathematical framework of classical physics. The new approach asks:

> Where does the distinctive mathematics of QM come from?

The answer is that the math of QM is the vector space or, particularly, Hilbert space version of the mathematics of partitions. The argument is based, in part, on using a semi-algorithmic procedure, herein called the *Yoga of Linearization* (part of mathematical folklore), to build a translation dictionary between set-level partition math and Hilbert space QM math.

For instance, here is a set-level construction in partition math whose quantum math version is Dirac's notion of a Complete Set of Commuting Observables (CSCO).

Partition-math version: A set $f, ..., g : U \rightarrow \mathbb{R}$ of real-valued numerical attributes on a set $U = \{u_1, ..., u_n\}$ is said to be *complete* (a Complete Set of Compatible Attributes or CSCA) if the join (non-empty intersections of the blocks) of their inverse-image partitions is the partition with all blocks of cardinality one. Then each element u_i of U is uniquely characterized by its ordered set of values for $f, ..., g$.

Quantum-math version: A set $F, ..., G$ of commuting observables on a Hilbert space V is said to be *complete* (a Complete Set of Commuting Observables or CSCO) if the join (non-zero intersections of the eigenspaces) of their direct-sum decompositions (DSDs) of eigenspaces is the direct-sum decomposition with all subspaces of dimension one. Then each eigenvector v_i in the set of simultaneous eigenvectors spanning V is uniquely characterized by its ordered set of eigenvalues for $F, ..., G$.

Each version is essentially a word-for-word translation using the following translation dictionary. Many of the tables in the book are additions to the translation dictionary to show how the QM math (right side of the table) is the vector space, and particularly Hilbert space version of the math of partitions (left side of the table).

Partition math	Quantum math
Real-valued attributes on a set	Observable ops. on a Hilbert space
Attributes defined on same U	Observables that are commuting
Domain U of the attributes	Basis simult. eigenvectors of comm. ops.
Inv.-image partition of attribute	DSD of eigenspaces of an observable
Join of inverse-image partitions	Join of DSDs of commuting observables
Cardinality of subset in partition	Dimension of subspace in DSD
Partition blocks of cardinality 1	DSD subspaces all of dimension 1
Values of attributes on $u_i \in U$	Eigenvalues of simultaneous eigenvectors
$u_i \in U$ given by attrib. values	Eigenvectors given by eigenvalues

Illustration of QM math being Hilbert space version of partition math

The next step in the argument is to ask:

What basic concepts are represented at the logical level by partitions?

The answer is the concepts described in various vocabularies as indistinctions versus distinctions, indefiniteness versus definiteness, indistinguishability versus distinguishability, equivalence versus inequivalence, or difference versus identity. These pairs of concepts, described by different words in different contexts, might be referred to as the "identity & difference concepts." The *logic* of those identity & difference concepts is the logic of partitions (or equivalence relations) on a set.

Then to interpret QM, we ask:

> What is the essential non-classical concept in QM?

The answer is the notion of superposition (entanglement being a particularly vexing special case). But that non-classical notion has been emphasized from the beginning (e.g., by Dirac) so,

> Why has there been so little progress in understanding the reality behind the notion of superposition?

The answer lies in the mathematics of QM itself. A Hilbert space is a vector space over the complex numbers $\mathbb{C}$, and the complex numbers are the natural mathematics to describe waves, i.e., the polar representation of a complex number is an amplitude and phase of a wave. In fact, QM was often called "wave mechanics" and the "wave function" is a commonly used mathematical tool to represent the quantum state. Hence, superposition has usually been interpreted simply like the addition of waves—just as water waves might add and interfere with each other in the classroom ripple tank model of the double slit experiment. But after a century of looking, no physical reality has been found for the wave functions—much to the dismay of Erwin Schrödinger who invented the wave equation bearing his name. The wave functions were "probability waves" which are not physical entities at all.

Indeed, the math of QM is formulated using the complex numbers for reasons that have nothing to do with waves, namely that the complex numbers are the algebraically complete extension of the real numbers so that the real-valued quantum observables will then have a complete set of eigenvectors. The whole wave interpretation of QM math was mistakenly giving an ontological importance to the wave-like computational artifacts present in any vector space over the complex numbers.

> How to escape this conundrum that the wave-like math is not reflected in quantum level ontology?

What is needed is a *totally different interpretation of superposition* (than the mathematically correct but ontologically misleading addition of vectors that can be interpreted as waves). And *that* new interpretation is supplied by the mathematics of partitions. At the simple logical level, a partition is made up of blocks of elements of the underlying set. Each block is an equivalence class that says, according to this partition, these elements in the block are equated, blobbed, blurred, and cohered together with no distinctions between them—since the distinctions are between different blocks. Thus, the blocks (or equivalence classes) with two or more elements are the logical version of a superposition of eigenstates in QM math. The block is indefinite or indistinct on the differences between elements (or eigenstates)—and definite on commonalities. The elements in the set (or eigenstates in the QM version) represent (not different particles but) different states of a particle that are equated, blobbed together, or cohered together in the superposition. That is the *non-wave indefiniteness reinterpretation of superposition* that corroborates an interpretation of QM proposed by Heisenberg, Shimony, and others. The key idea of this version of superposition is that a particle can be in an objectively indefinite state like a particle in a superposition state of "here" and "there", i.e., it is "not definitely here and not definitely there, but definitely not anywhere else."

Quantum theorists constantly use the wave-math without finding any physical waves, and they (mostly) recognize the reality of indefinite states. That is, when the quantum state is a superposition in the basis of the observable being measured, then it is widely recognized that the quantum state does not have a definite value before the measurement which causes the quantum jump into a definite state. And the set-level version of (projective) measurement is just the partition join operation from partition logic. Heisenberg, Shimony, and others then extrapolate that notion of an indefinite state to the whole of quantum-level reality. The quantum world is Indefinite World. And the set-level mathematics to represent definiteness versus indefiniteness is the math of partitions with the math of QM being the Hilbert space version of that partition math. That new partition-math approach to the Heisenberg-Shimony Objective Indefiniteness Interpretation of QM is the topic of this book.

In addition to the influence of Gian-Carlo Rota and Abner Shimony (my undergraduate advisor at MIT '65), I would like to acknowledge the assistance of the late Larry Harper, Brian Linard, John DePillis, and Tom Payne who were the members of the "Schmooze Group" of retired professors studying quantum mechanics at the University of California at Riverside.

Ljubljana, Slovenia David Ellerman
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