Xingjian Jing

The Bio-inspired X-Structure/Mechanism Approach for Exploring Nonlinear Benefits in Engineering

Part I-Nonlinear Stiffness and Nonlinear Damping

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Preface

This book is to present a totally new and systematic method for the design and analysis of beneficial nonlinearity of engineering systems, where nonlinearity is expected to take an important and critical role and thus cannot be simply ignored in structural design, dynamic analysis, and parameter selection. A key issue in the field of nonlinear dynamics is about how to analyze and design potential nonlinearities introduced to or inherent in a system of under study. This is usually a have-to-do task in many practical applications, extensively involved in vibration control, energy harvesting, sensor systems, structural health monitoring, robotic design and control, etc.

This book acts as an up-to-date summary on the most recent development of a cutting-edge method for manipulation and employment of nonlinearity proposed and established recently in the laboratory of nonlinear dynamics, vibration, and control, led by Prof Xingjian Jing, which is known as the X-shaped structure or mechanism (or simply X-structure/mechanism) approach. The method is inspired from animal (or insect) leg/limb skeletons and can provide passive low-cost high-efficiency adjustable and beneficial nonlinear stiffness (high static and ultra-low dynamic), nonlinear damping (dependent on vibration frequency and amplitude) and nonlinear inertia (low static and high dynamic, symmetric or asymmetric) individually or simultaneously.

The X-shaped structure or mechanism is a generic and basic structure or mechanism unit (or design motif) representing a class of beneficial geometric nonlinearity with realizable and versatile linkage mechanism or flexible (or elastic) structural design of different variants or forms (quadrilateral, diamond, polygon, K/Z/S/V/L/W-shape, or others) which all share similar geometric nonlinearity and thus similar nonlinear stiffness/damping properties, easy in design, and convenient to implement.

This book systematically reviews the research background, motivation, essential bio-inspired ideas, advantages of this novel method, beneficial nonlinear properties in stiffness, damping and inertia, potential applications and further development, starting from the most preliminary results which have been developed since 2010.

This book includes several parts. This is the first part of this book, i.e., Part I, which is to present the most fundamental theory and methods of this unique method—the bio-inspired X-structure/mechanism approach, including one chapter for an overall introduction and two sub-sessions, i.e., Nonlinear Stiffness Design with X-Structures/Mechanisms, and Nonlinear damping design with X-Structures/Mechanisms. Part II of this book is to focus on X-structured Linear and Nonlinear Inertia, and Multi-direction Vibration Isolation. Part III is to present an elegant frequency domain method, i.e., the nonlinear characteristic output spectrum (nCOS) function, for analysis and design of nonlinear dynamics with case studies. Part IV of this book introduces case studies in energy harvesting and sensor design based on X-structures, and Part V provides recent advance on a novel approach for energy-saving robust control of nonlinear vibration systems based on X-dynamics and some other applications of the X-structures in robot design and control, and so on.

With these results, readers will be able to have a complete understanding of this series of studies and also have in-depth knowledge about nonlinear dynamic modeling, analysis, engineering design and applications, especially related to the topic of exploring and exploiting nonlinear benefits in dynamic systems.

Hong Kong, China Xingjian Jing

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Chapter 1
Overview of the Bio-inspired X-Structure/Mechanism Approach

1 Fundamental Problems in Applied Design of Nonlinear Systems

In numerous engineering applications, the attainment of an optimal stiffness or damping characteristic is pivotal for superior system efficacy. Deviations in these properties can precipitate a diminution of system functionality, persistent undue oscillations, equipment failures, or in extreme cases, infrastructural impairment or unforeseen calamities (Yu et al. 2002; Ibrahim 2008; Liu et al. 2015; Du et al. 2020).

A multitude of methodologies exists for the attainment of specific stiffness or damping characteristics within a structure or system. Excluding those that employ actively controlled actuators or materials (Li, Zhang, Liu 2017; Wang, Zhang, Zhang et al 2016; Pu, Yuan, Peng 2019; Zhu, Jing, Cheng 2012; Pan, Sun, Jing 2017), passive techniques for the modulation of stiffness or damping are often favored in practical applications. These include well-researched approaches involving the design of metal-materials, shape-memory polymers (Leng, Lan, Liu 2011), and origami configurations (Faber, Arrieta, Studart 2018; Zhai, Wang, Jiang 2018). Nonetheless, the passive strategies currently available for stiffness modification are typically challenging to evaluate analytically with a straightforward and precise mathematical model. This complexity hinders the analysis and design process, thereby affecting the reliability and precision post-manufacture. Particularly for materials exhibiting intrinsic nonlinearity, the absence of an exact analytical model complicates the management of nonlinear responses and the determination of pertinent design parameters, introducing the additional hazard of unforeseen nonlinear dynamic behaviors.

In the domain of system design and control, the strategic utilization of nonlinearities has garnered increasing interest for enhancing dynamic response capabilities, particularly in vibration control and energy harvesting. Nonlinearities are instrumental in refining vibration systems by adjusting the resonant frequency to align with external stimuli (Yu, Gnaganathan, Dukkipati 2002; Ibrahim 2008; Liu, Jing,

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Daley, Li 2015), reducing the resonant frequency without compromising load-bearing capacity (Ding, Chen 2019; Ding, Ji, Chen 2019; Wu, Jing, Bian et al 2015; Wang, Zhou, Xu 2021; Ji, Luo, Ye 2021), diminishing the resonant peak while preserving high-frequency vibration integrity (Jing, Lang and Billings 2011; Jing and Lang 2009), extending the frequency spectrum for vibration mitigation (Jing, Lang and Billings 2008; Xiao, Jing, Cheng 2013; Ibrahim 2008; Liu, Jing, Daley, Li 2015), and conserving active control energy without degrading isolation efficacy (Li, Jing, Li 2019; Pan, Jing, Sun 2018). Harnessing these nonlinear traits can significantly bolster energy harvesting efficiency (Wei, Jing 2017; Li, Ding, Jing et al 2021; Basaran 2022; Tan, Wang, Yan 2021; Xu, Zhang, Zang et al 2021; Mei, Zhou, Yang et al 2021), augment sensor system sensitivity, reliability, and precision (Sun, Jing, Xu et al 2014, 2015), enhance the adaptability of mobile robots to uneven terrains (Li, Jing, Sun et al 2020), and amplify the speed and power of grasping tools or actuation mechanisms (Braun, Chalvet, Dahiya 2018; Kim, Lee, Jung et al 2018). Consequently, specialized structural designs incorporating spring or magnetic elements are being rigorously investigated for their advantageous nonlinear properties in the context of vibration control and energy harvesting (Ibrahim 2008; Liu, Jing, Daley, Li 2015; Wei & Jing 2017; Jing & Vakakis 2019).

In the scenarios described, the fundamental challenge is the conceptualization and realization of advantageous nonlinearities tailored to a system's specific operational demands. The methodology should offer ample adaptability and simplicity in both design and execution. Beyond achieving high performance and cost-effectiveness, the capability for in-situ adjustments stands as a critical metric for evaluating the viability of a solution.

In pursuit of this objective, a bio-mimetic approach to structural or mechanical design has been progressively explored, known as the X-shaped structure or mechanism method, or more succinctly, the X-structure or X-mechanism approach. This technique, inspired by the skeletal structure of animal legs/limbs, presents a highly versatile solution for a wide array of engineering applications that require premium nonlinear stiffness and damping characteristics, facilitated through an expedient and dependable structural or linkage mechanism design. Research indicates that the X-structure offers highly customizable nonlinear stiffness, damping, and inertial properties that align with the diverse requisites of engineering applications. These nonlinear attributes are applicable across various fields, including vibration control, energy harvesting, sensor systems, and robotics. This chapter encapsulates a comprehensive overview of this subject, including recent findings, and will also present prospective advancements in the field.

This chapter methodically presents the principal outcomes and progressions of the bio-inspired X-structure/mechanism approach (Jing 2022). Initially, it looks into the overarching concept, the impetus behind the approach, and the procedural methodology. Subsequently, it explores the versatility of design and the liberty in choosing parameters. Following this, it encapsulates the key nonlinear characteristics of the X-structure/mechanism. Thereafter, it provides a concise overview of its prospective applications. The chapter culminates with a summarizing conclusion.

2 The Bio-inspired Idea and Methodology

To seek effective methods for the purpose of efficient employment of beneficial nonlinearity extensively demanded in engineering systems for achieving advantageous performance (in vibration control, energy harvesting and others) with minimum implementation cost and complexity, the X-structure/mechanism is thus proposed and developed with the following bio-inspired idea.

All natural creatures are equipped with superior features over environmental challenges (Fig. 1). Legs or limbs of animals can easily suppress vibration and shock impact, while the osteoporosis bone structures are obviously weak in impact protection due to loosened interlayer connection. Woodpeckers do not suffer from brain damage after daily-repeated 500–600-time drumming a tree with a surprising speed about 20 beats per second and a deceleration up to 1200 g (Yoon and Park 2011; Wang and Cheung 2011). A reason is the special cranial bone structure, which has two outside layer but the interlayer supporting structure does not support outside layers in a normal direction but takes an inclination link like a limb-joint system. Considering human body, it can act as a very good shock absorber to prevent from brain injury caused by vibration and impact, and provide excellent vibration isolation. Noticeably, human body tends to swing arms during walking or jumping, which is a special behavior since the arms play no obvious role in bipedal gait. A study on whole-body vertical angular momentum and ground reaction moment when human walking indicates that arm swinging does improve the walking stability (Spanos et al. 1995; Shimizu et al. 2008).

The observations above motivate a series of innovative studies which well answer the fundamental questions as mentioned before, present new understanding of bio-inspired nonlinear dynamics and their application potential, and consequently lead to the X-structure or mechanism approach established in the past years. To start this series of cutting-edge theory and methods for design of nonlinear dynamics, a key issue is to seek for a generic structural or mechanism design from those observed biological bone structures, animal legs or limbs, which can have supervisor nonlinear features at least in equivalent stiffness and be easy and flexible to implement in practice. Motivated by this idea, the X-shaped structure or linkage mechanism (initially referred to as scissor-like structure or limb-like structure) is therefore proposed (Wu et al. 2015; Sun et al. 2014b), which can now be simply referred to as X-structure/mechanism. The research was initially assigned to a MSC student when the author was with Hong Kong Polytechnic University (Xue 2013), and then followed by PhD students and postdoc research staff in the N.D.V.C lab (Wu et al. 2015; Sun et al. 2014b).

The X-structure/mechanism is shown to be simple and very easy to implement, has no strict material restriction, can have different variants, and take different forms but which all share similar nonlinear geometric relationship between the structural suppression displacement and external exerted force. The nonlinear features provided

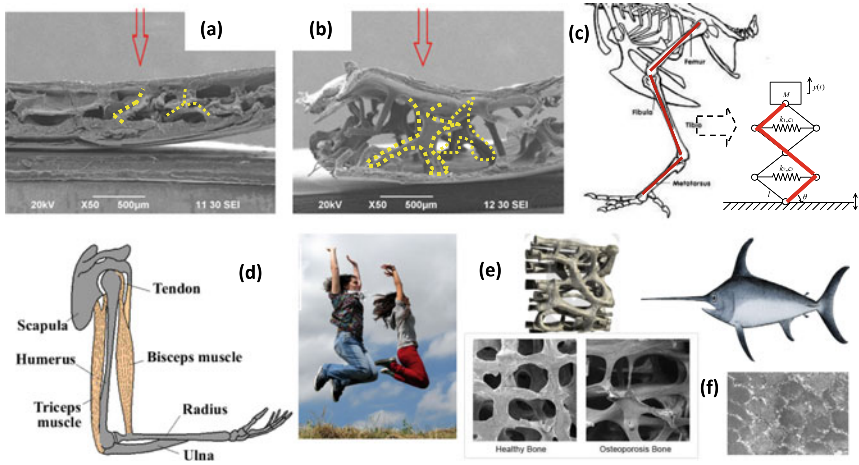


Fig. 1 **a** The SEM image of woodpecker's cranial bone; **b** The hoopoe's cranial bone; **c** Avian leg system; **d**, **e** Human arms, legs, and bone structures; **f** Billfish/swordfish and its head structure. The inside supporting structure of the cranial bone in **a**, **b** or limb bone in **e** does not support outside layers in a normal direction but takes an inclination link like a limb-joint system in **c**, **d**. The limb structures and inside bone structures of animals all take a "Z" shape or polygon shape with spring-like muscles or tissues for supporting or joining. The billfish head bone takes a polygonal structure forming an excellent mounting system for the spear. These structures can successfully absorb or suppress the shock or vibration impact, but have never been systematically explored or employed for vibration isolation/suppression. The osteoporosis bone structures obviously lose impact protection due to weak or loosing inside connection (in the bottom of **e**)

by the X-shaped structure or mechanism are demonstrated to be superior in terms of potential stiffness, damping and inertia properties. These results will all be introduced in the following sections.

3 Flexibility in Structural/Mechanism Design and Implementation

In this section, the flexibility of the X-shaped structure/mechanism approach in design and implementation is demonstrated.

The basic symmetric X-structure (Basic-X). The basic X-shaped structure/mechanism can be designed in the forms as shown in Fig. 2, with a symmetric scissor-like linkage mechanism of different layers (Sun et al. 2014b; Sun and Jing 2016; Dai et al. 2018). The linkages are connected with rotation joints and the overall height of the structure can be designed with different rod length and layers. To employ the geometric nonlinearity, the springs and/or dampers should be connected with the two horizontal joints in each layer, which is of course not the only method for introducing the geometric nonlinearity but the most convenient way as there is no

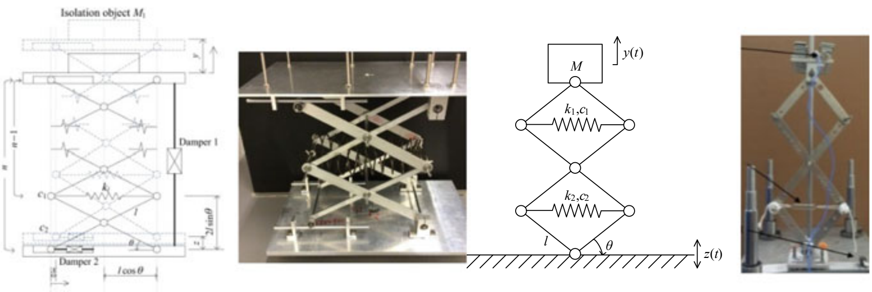


Fig. 2 The basic X-shaped structure or mechanism, taking symmetric 1, 2 or n layers with horizontal springs and/dampers installed in a layer to employ the geometric nonlinearity (Sun et al. 2014b; Sun and Jing 2016)

need of additional connection points and rods. It can be seen that the rod length l, assembly angle θ, spring and damping coefficients, and layer number n can all be design parameters.

Note that, alternative spring connection methods include any inclined connections between any two points on two different rods respectively, or any connections between one point of any an X-rod and the other point of an external additional part, and they all can produce similar nonlinearity in stiffness or damping included as a special case in the nonlinearity produced by the aforementioned horizontal connection method plus a pure vertical connecting spring (while the latter produces a linear and positive stiffness). The idea can also be extended to any compliant beam or longitudinally elastic beams to act as the X-rods, which eventually produce similar geometric nonlinearity in stiffness and damping, too.

Importantly, to achieve onsite adjustability, except the spring/damper can be directly changed, the spring/damper connection can be designed with an adjustable pre-extension/compression mechanism, and the rod length can also be tuned with a stretchable rod etc. The materials can be metal, stiff plastic, carbon fiber or else. Flexural rods can also be considered for special performance in further studies.

The asymmetric X-structure (Asym-X). Based on the basic symmetric X-shaped structure/mechanism, the rod length can be carefully designed to achieve different asymmetric forms as a new design factor, as shown in Fig. 3. The biological basis for asymmetrical structures comes from that the femur and tibia of animal legs or insect limbs usually take different length and inclination angles. The significance of studying asymmetric structures lies in (1) meeting special space requirement in applications without sacrificing performance (Wu et al. 2015; Hu and Jing 2015), (2) exploring much better performance by tuning structural asymmetry (Wu et al. 2015; Wang et al. 2019; Wang and Jing 2019), and (3) potential applications in period structures or else (Wu et al. 2019). It is shown in Wu et al. (2015), Wang et al. (2019) and Wang and Jing (2019) that the asymmetric factor is useful for achieving much better performance and obtaining the minimum resonant frequency in some special application situations (e.g., low gravity).

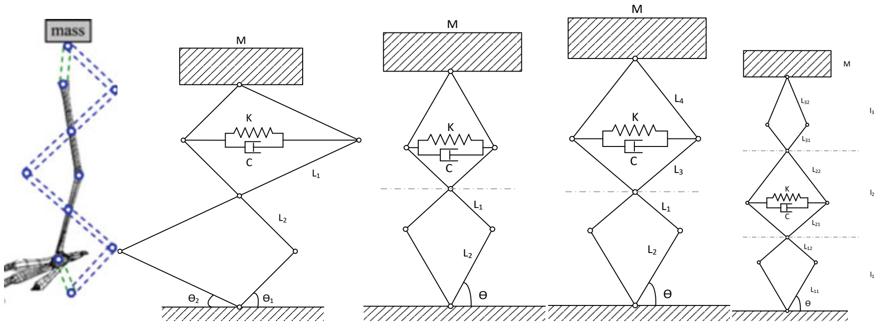


Fig. 3 Asymmetric X-shaped structure/mechanism, in horizontal, vertical, or both, regular or irregular change (Wu et al. 2015; Wang et al. 2019; Wang and Jing 2019), bringing different nonlinear properties and thus nonlinear responses consequently

Half-X or other X-variants. With the structures in Figs. 2 and 3, some other variants can be developed for example polygon-shaped structures (the X-shaped structure can also be regarded as a quadrilateral structure) (Dai et al. 2018; Tian and Jing 2021), half layer structures like a V or K shape and others with/without linkages (Zhao et al. 2021; Gatti 2019). In modelling, the geometrical nonlinearity would be similar and thus they are not focused in our discussions. Importantly, multi-degree freedom systems can also be obtained with each independent layer and demonstrate much more flexibility in performance design and applications (Jiang et al. 2020) as shown in Fig. 4.

Noticeably, the X-shaped structures/mechanisms above provide a very tunable and flexible tool to achieve nonlinear stiffness/damping properties. With this powerful tool, more additional design factors can thus be further considered to obtain some amazing performance. This motivation leads to the following structural designs and associated studies.

Inertia-coupled X-structures (Inertia-X). As mentioned before, arm swing takes an important role in stability control during human body walking and jumping. Inspired by the arm swing, rotation units can thus be introduced into the X-shaped structure to create an alternative and critical design factor for performance improvement. This can be easily done with different manners as shown in Fig. 5 (Liu et al. 2016; Feng and Jing 2019; Feng et al. 2019). It is shown that the X-shaped structure/mechanism reinforces the commonly used rotation units into a powerful and adjustable nonlinear inertia system with much better performance and significantly expanding our understanding.

X-structure/mechanism with special spring arrangements (X-structured variants). For vibration isolation, an enlarged QZS area can ensure a superior isolation performance, while multi-stable nonlinearities are well employed for improving energy harvesting efficiency. The desired nonlinear stiffness can be easily obtained with the X-shaped structure/mechanism via some special spring arrangements as shown in Fig. 6 (Jing et al. 2021). For example, additional horizontal springs can be applied with a controlled contact with the X-structure as shown can achieve specially

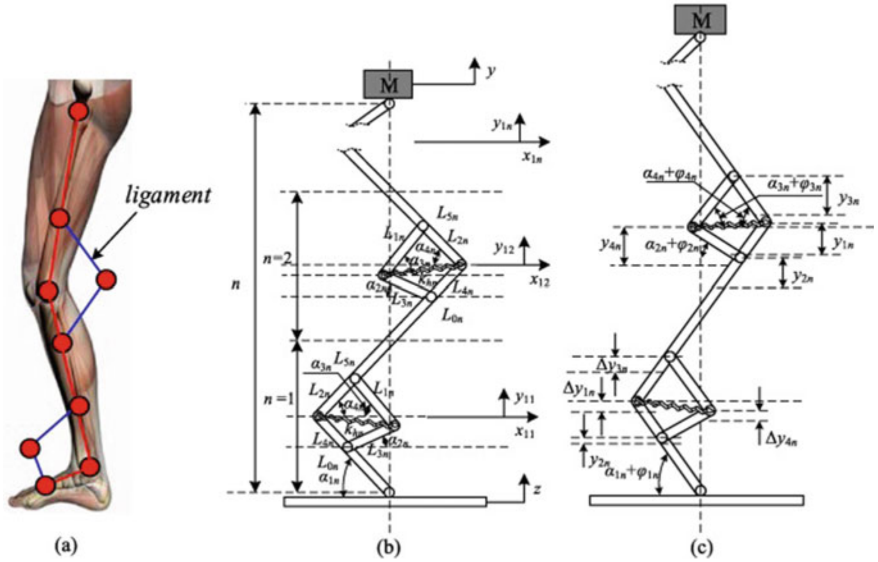


Fig. 4 Multi-degree-of-freedom X-shaped structure/mechanism (**MDOF-X**) with asymmetric units introducing even more flexibility in performance adjustability and practical applications (Jiang et al. 2020)

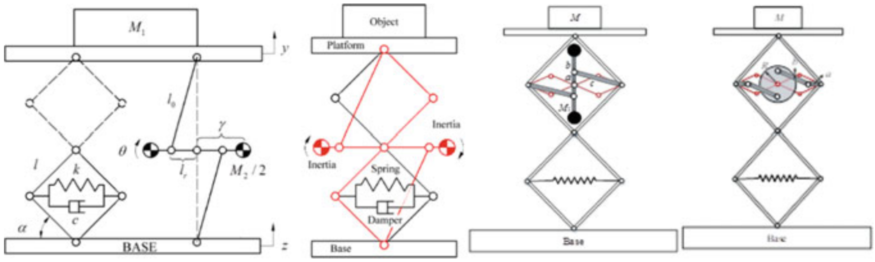


Fig. 5 The X-shaped structure/mechanism with inertial units to create beneficial nonlinear inertia for better performance and expanding understanding (Feng and Jing 2019; Feng et al. 2019). The inertia can be created with commonly-used mechanical components such as leverages, rotation units/discs, or flywheels

improved QZS property, loading capacity and multi-stable nonlinear stiffness. This spring arrangement method with X-structure opens a larger potential for applying the X-shaped structure/mechanism in various engineering systems.

With the results demonstrated in Figs. 2, 3, 4, 5 and 6, it can be seen that the X-shaped structure/mechanism is not only flexible in design and implementation, but also provides a reliable and adjustable manner of great potentials for realizing desired nonlinearities for different engineering requirements. The detailed advantageous nonlinear characteristics will be discussed in the following sections.

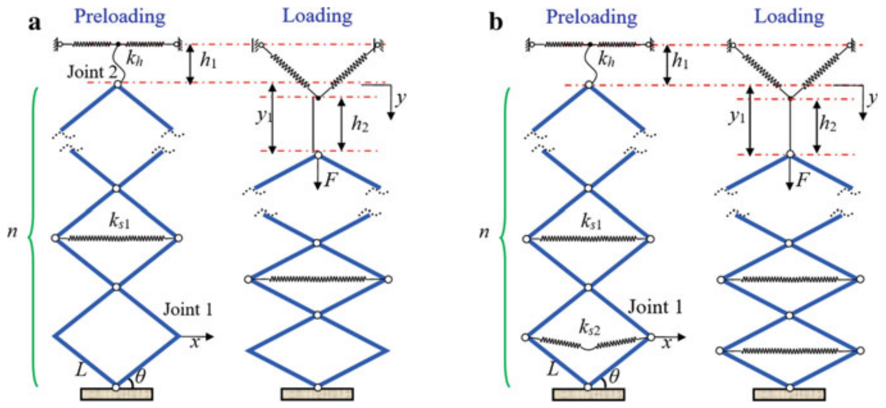


Fig. 6 X-structure/mechanism combined with special arranged springs for achieving more flexibility in stiffness manipulation including enlarged QZS area, enhanced loading capacity and improved multi-stable nonlinearities (Jing et al. 2021)

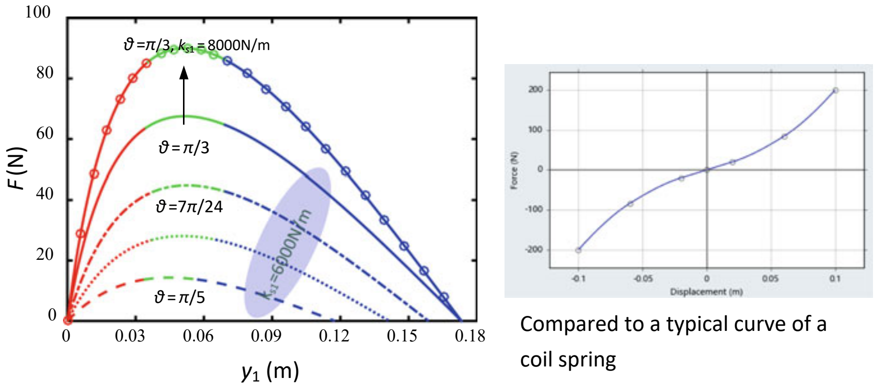
4 Beneficial Nonlinear Characteristics

A. Nonlinear stiffness—a designable and controllable nonlinear degressive stiffness

The nonlinear stiffness created by the X-shaped structures/mechanisms can be summarized in the following several points.

A beneficial degressive stiffness system with adjustable positive, zero and negative ranges and loading capacity. A typical reactive force between the loading force and compression displacement of a 2-layer symmetric Basix-X in Fig. 2 is shown in Fig. 7, with respect to different assembly angle θ and/or applied spring stiffness k (Jing et al. 2021).

In Fig. 7, the stiffness can be well changed from positive stiffness, via QZS area (green) and to negative stiffness. One of the important nonlinear features is the degressive stiffness which is decreasing with more loading force before it goes to the negative area, which completely different from commonly used coil springs in engineering. It is for the first time to show that such a beneficial nonlinear stiffness can be obtained with such a simple and passive mechanism. The positive and negative stiffness can both be tuned for its amplitude, and the loading capacity can be easily changed with different assembly angle θ and/or applied spring stiffness k. For onsite adjustability, this can be done with a pre-extension mechanism which can be used to tune the assembly angle θ, or changing the spring directly. Importantly, it should be emphasized that for a rod length with 5 cm, the 2-layer X-shaped structure can bear with several cm displacement range in the QZS area. This is particularly important for vibration isolation subject to large-stroke vibration excitation. Moreover, the negative stiffness can be well circumvented in practice with simple motion restriction mechanisms.



Compared to a typical curve of a coil spring

Fig. 7 A typical nonlinear stiffness response between loading force and compression displacement with different assembly angle θ and/or applied spring stiffness k (Jing Chai et al. 2021)

The detailed modelling and analysis can be referred to a series of publications, e.g., Wu et al. (2015), Wang et al. (2019), or Jing et al. (2021). The typical reactive force of the X-shaped structure can be shown in Eq. (1) (Wu et al. 2015) with parameters shown in Fig. 8.

$$
\begin{aligned}
f = \frac{k_h}{2}\Bigg[& L_1\cos(\theta_1) + L_2\cos(\theta_2) - \sqrt{L_1^2 - \left(L_1\sin(\theta_1) + \frac{y}{2n}\right)^2} \\
& - \sqrt{L_2^2 - \left(L_2\sin(\theta_2) + \frac{y}{2n}\right)^2}\Bigg] \\
& \left[\frac{L_1\sin(\theta_1) + \frac{y}{2n}}{\sqrt{L_1^2 - \left(L_1\sin(\theta_1) + \frac{y}{2n}\right)^2}} + \frac{L_2\sin(\theta_2) + \frac{y}{2n}}{\sqrt{L_2^2 - \left(L_2\sin(\theta_2) + \frac{y}{2n}\right)^2}}\right] + k_v\frac{y}{n}. \qquad (1)
\end{aligned}
$$

Equation (1) clearly shows that (1) the stiffness is a very complicated nonlinear function of structural parameters and the compression displacement y. The structural asymmetry and ratio of vertical and horizontal spring stiffness can an important role in tuning the nonlinear property. This understanding can help a lot in structural design and parameter selection in engineering practice.

With the QZS achieved through the X-shaped structure, a resonant frequency around 1 Hz can be easily obtained in the prototype testing as shown in Wu et al. (2015) and Jing et al. (2021). Compared to traditional QZS system, the resulting QZS can be larger with much more flexible loading capacity and bearing with larger excitation stroke.

Enlarged QZS area. With the stiffness property in Fig. 7, the negative stiffness can be readily exploited with a positive stiffness components such that an enlarged QZS can be obtained (Fig. 9) (Jing et al. 2021). There are several different designs which

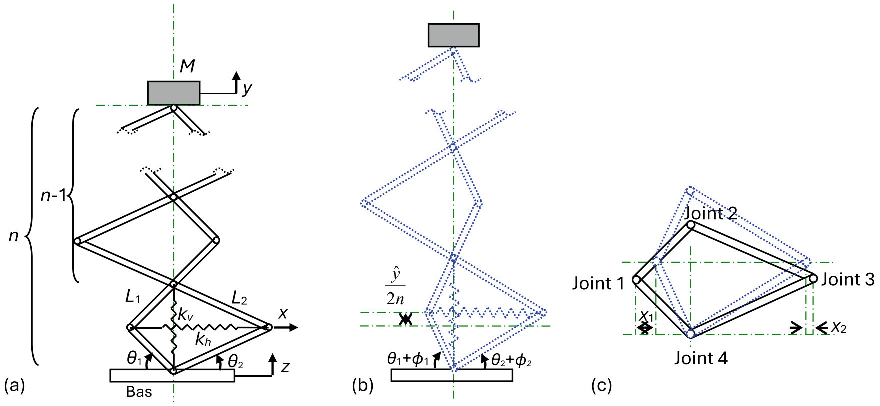


Fig. 8 The bio-inspired structure in modeling (Wu et al. 2015). Both horizontal and vertical springs can be applied for a better adjustability of the nonlinear stiffness property

can be used for this purpose, for example, a vertical spring can be installed with the X-shaped structure (Wu et al. 2015) or an additional horizontal spring with additional fixed support can be employed as those demonstrated in Fig. 6 (Jing et al. 2021).

Multi-stable stiffness. Similarly, multi-stable equilibria can be obtained with additional springs as shown in Fig. 6 or others demonstrated in Jing et al. (2021). The resulting nonlinear stiffness properties can be seen in Fig. 9. Such nonlinear properties could be very beneficial in vibration based energy harvesting or else. More different methods for tuning nonlinear stiffness properties based on the X-shaped structure/ mechanism can be further referred to Jing et al. (2021). It should be the first time for these nonlinear properties and associated adjustability to be demonstrated and obtained in such a flexible and convenient way.

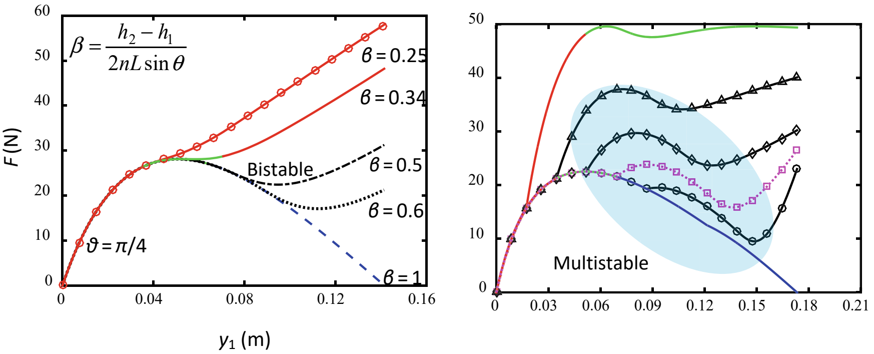


Fig. 9 Enlarged QZS area can be achieved, together with some other properties such as bi-stable states (Jing et al. 2021)

It should also be noted that, for vibration control, the X-shaped structure/mechanism should be tuned to be QZS with only one stable equilibrium in most cases. The multi-stable stiffness of the X-shaped structure/mechanism will be used for other purposes, for example, energy harvesting or sensors.

B. Nonlinear damping—a beneficial nonlinear displacement dependent damping

The other advantageous feature of the X-shaped structures/mechanisms lies in the beneficial nonlinear damping characteristic that could be created simply with such a simple design mechanism. The beneficial nonlinear damping comes from the rotation joints or dampers installed horizontally within a layer of the X-shaped structure due to the geometric nonlinearity (Fig. 10). A simple modelling for the model in Fig. 10 is given as (without detailed explanation about its notations) (Bian and Jing 2019).

$$\ddot{y}_1 + \frac{K}{M}y_1 + \left(\frac{c_1}{M} + \frac{c_2 n_x}{M}\left(\frac{\partial \varphi}{\partial y_1}\right)^2 + \frac{c_3}{M}\left(\frac{\partial x}{\partial y_1}\right)^2\right)\dot{y}_1 + \ddot{z} = 0 \qquad (2)$$

Clearly, the damping effect incurred by rotational friction labelled by c_2 and horizontal damper labelled by c_3 are both nonlinear and dependent on vibration displacement (while c_1 is referred to the air damping vertically).

It is thus shown that the introduced damping through the X-shaped mechanism is dependent on the vertical compression displacement of the structure/mechanism and demonstrated to be bigger around resonant frequencies but smaller at others (see Fig. 11a), and can be well tuned with structural parameters (Bian and Jing 2019). Such displacement-dependent nonlinear property has been theoretically shown as beneficial (Jing et al. 2011; Jing and Lang 2009; Jing et al. 2008; Xiao et al. 2013) but never been achieved in practice with such a simple passive manner before. It can reduce vibration peak but not deteriorate high-frequency vibration. Importantly, when

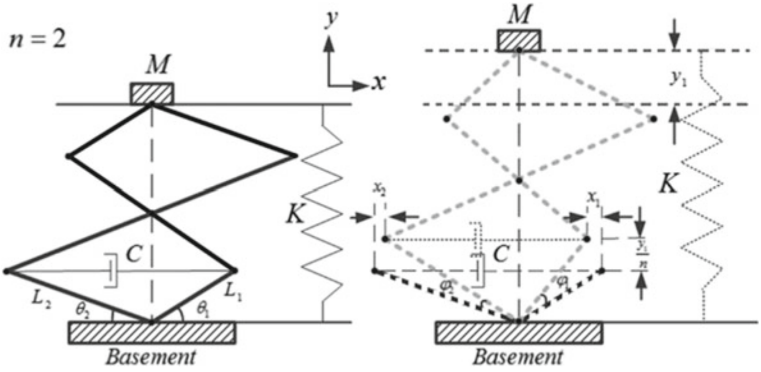


Fig. 10 Beneficial passive nonlinear damping property of the X-shaped structure/mechanism, obtained due to the geometric nonlinearity (Bian and Jing 2019), determined by structural parameters, and very flexible in design

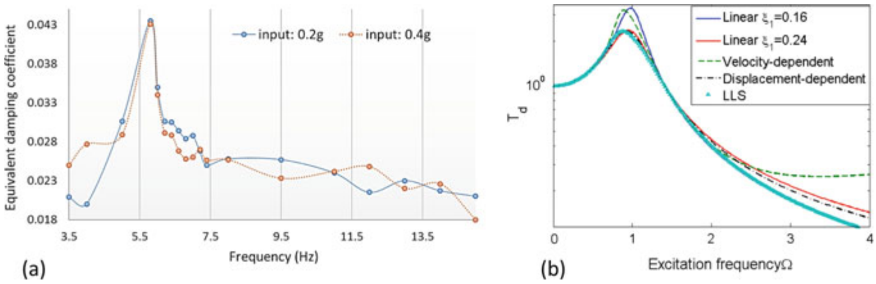


Fig. 11 Beneficial nonlinear damping effect (Bian and Jing 2019). **a** Measured equivalent damping effect with different frequency; **b** Comparisons with other linear or nonlinear damping, showing the X-shaped structure (or Limb-like structure, LLS) has much better damping effect dependent on displacement and velocity in a beneficial nonlinear way

vibration excitation is stronger, the nonlinear damping effect is becoming better and robust to parametric and excitation change. All these demonstrate superior properties over other linear or nonlinear counterparts (Fig. 11b) (Bian and Jing 2019), and show that the X-shaped structure (or Limb-like structure, LLS) has much better damping effect dependent on displacement and velocity in a beneficial nonlinear way.

Much more detailed discussions about the damping effect of the X-shaped structures can be referred to Bian and Jing (2019). Because of such a beneficial nonlinear damping property, together with the flexibility in design and implementation and ultra-low resonant frequency which can be achieved, nonlinear tuned mass dampers can be designed in a more powerful way. This will be discussed in the potential application of Sect. 5.

C. Nonlinear inertia (or equivalent mass)—a unique nonlinear inertia system

With rotation units deliberately introduced into the X-shaped structure/mechanism as discussed before (Fig. 5), even more interesting nonlinear effects in terms of equivalent mass or inertia can therefore be obtained. A critical question is about the equivalent mass or inertia of an engineering system, that is, whether the equivalent mass should be maintained to be constant or changed in a beneficial way. Considering vibration control, the idea comes from again from biological motion, like human body walking or animal jumping. Arm swing or tail swing provides new motivations to proceed with such a new topic as studied in Liu et al. (2016); Feng and Jing (2019); Feng et al. (2019) and Liu and Jing (2015).

It is for the first time shown in Feng and Jing (2019); Feng et al. (2019) that a unique and beneficial nonlinear inertia characteristic can be obtained through a simple leverage mechanism coupled with the X-shaped structure/mechanism (see Fig. 12), which can significantly reduce the resonant frequency due to its very special nonlinear characteristic, that is, smaller inertia (equivalent mass) for smaller vibration displacement but much bigger inertia (equivalent mass) for stronger vibration displacement; and have obvious advantages over its linear counterparts (Fig. 13).

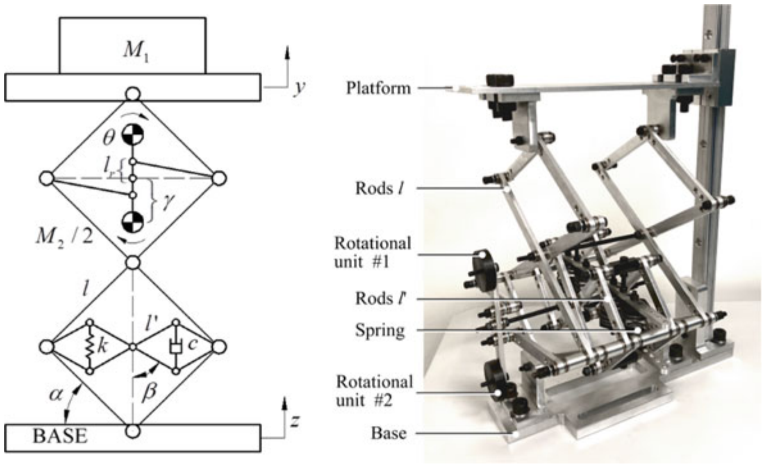


Fig. 12 Arm-swing inspired anti-vibration systems based on the X-shaped mechanism (Feng et al. 2019), revealing a unique nonlinear inertia property very beneficial to vibration isolation

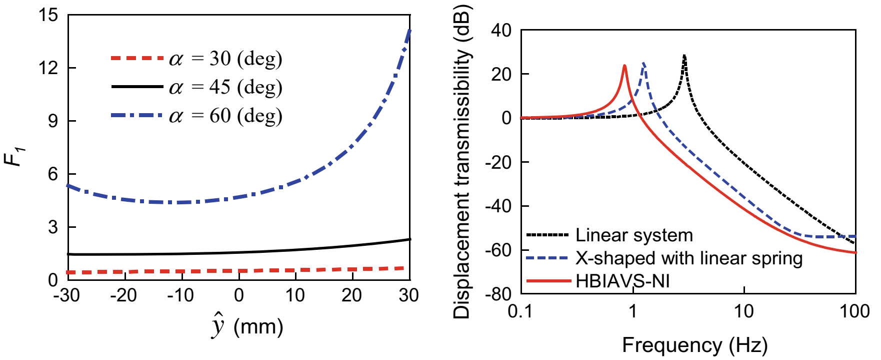


Fig. 13 Nonlinear inertia and its effect (Feng et al. 2019). **a** The nonlinear inertia characteristic demonstrates smaller inertia for smaller vibration displacement while much larger inertia for stronger vibration displacement; **b** The X-inertia system overperforms those linear systems with linear inertia (doted), and X-shaped structure without inertia (dash)

In modelling, it can be seen that the nonlinear inertia effects are not only reflected in the equivalent mass but also in a special term—inertia incurred conservative force (see Eq. (3) with parameters referring to Fig. 12), which can act as a very excellent storage pool for excessive interactive force between excitation input and output platform (Fig. 14). The later implies that this nonlinear force reduces the interactive force in compression but increases the interactive force in extension such that this reactive force can be maintained relatively stable compared to its linear counterparts, and thus help reduce vibration.

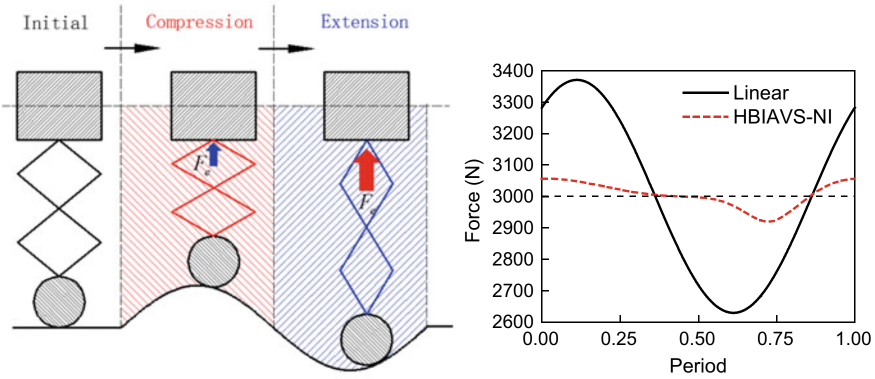


Fig. 14 A special nonlinear effect incurred by the inertial unit coupled with the X-shaped structure, which can well balance the interactive force during a vibration cycle (Feng et al. 2019)

$$\begin{aligned} &M_1\ddot{y} + M_2\gamma^2\left(\frac{\partial\theta}{\partial\hat{y}}\right)^2\ddot{\hat{y}} + M_2\gamma^2\frac{\partial\theta}{\partial\hat{y}}\frac{\partial^2\theta}{\partial\hat{y}^2}\frac{\partial\hat{y}}{\partial y}\dot{\hat{y}}^2 + k(x+l_s)\frac{\partial x}{\partial\hat{y}}\frac{\partial\hat{y}}{\partial y} \\ &+ (M_1+M_2)g = -c\frac{\partial x}{\partial\hat{y}}\frac{\partial\hat{y}}{\partial t} \end{aligned} \tag{3}$$

It should be emphasized that the nonlinear inertial characteristic is relatively a new topic which could be explored more in further studies for different design mechanisms and different nonlinear forms. Note also in Eq. (3) that there are 4 nonlinear terms including nonlinear inertia $M_2\gamma^2\left(\frac{\partial\theta}{\partial\hat{y}}\right)^2$, nonlinear stiffness $k(x+l_s)\frac{\partial x}{\partial\hat{y}}\frac{\partial\hat{y}}{\partial y}$, nonlinear damping $c\frac{\partial x}{\partial\hat{y}}\frac{\partial\hat{y}}{\partial t}$, and nonlinear inertia-incurred conservative force $M_2\gamma^2\frac{\partial\theta}{\partial\hat{y}}\frac{\partial^2\theta}{\partial\hat{y}^2}\frac{\partial\hat{y}}{\partial y}\dot{\hat{y}}^2$. For more discussions, please refer to Feng et al. (2019).

Recent studies further reveal that nonlinear inertia can exhibit three different types of nonlinearities with varying performance within a vibration cycle (Zhu et al. 2023; Jing et al. 2023), which implies another very promising R&D area for further exploration.

5 Potential Applications

In this section, several benchmark applications are briefly introduced and discussed.

A. **Passive vibration isolation (X-mount)**

The X-shaped structures/mechanisms can be well applied to various engineering systems for passive vibration isolation with an adjustable property. A benchmark application is the anti-vibration mounting technology which is greatly demanded in engineering for safety, quality, precision, noise and vibration control (Yu et al. 2001; Ibrahim 2008; Liu et al. 2015). One critical issue of such mounting technology is to achieve high-quality vibration isolation in a very compact design with tunable stiffness to meet space and changing load requirements. For this purpose, an X-cube technology is proposed in Jing et al. (2021) and demonstrated its supervisor performance (Fig. 15).

It can be seen that a resonant frequency about 1 Hz can be easily obtained with such an X-mount and it can bear with relatively large vibration excitation in displacement up to 3–5 cm for a rod length 10 cm cube design.

Moreover, multi-degree of freedom (DoF) vibration isolation can also be done with the X-structure approach. A steward platform with purely passive X-structured legs is presented in Hu and Jing (2018), several multi-DoF vibration isolation platforms with coupling inertia are shown in Liu et al. (2016), Liu and Jing (2015), and multi-direction vibration isolation is also preliminarily studied in Chai and Jing (2021), Sun and Jing (2015) or others. Clearly, more versatile vibration isolation platforms and applications with advantageous and expected performance can thus be obtained with this similar method.

B. **Passive vibration absorption (X-absorber)**

Due to the superior passive nonlinear damping characteristics that can be achieved with the X-shaped structures/mechanism, an X-structured tuned mass damper (X-absorber) is proposed and discussed in Bian and Jing (2021) (see also Fig. 16). It is shown that the X-absorber can have some advantageous properties, including flexible to adjust resonant frequency, wider frequency band coverage in vibration suppression, less sensitive to parameter changes and excitation changes, and more robust to inherent nonlinearity of master structures etc.

The X-absorber can be applied in the vertical direction as shown in Fig. 16 or horizonal direction (Bian and Jing 2021), very flexible up to practical requirements.

C. **X-dynamics Based Active Vibration Control**

Nonlinear benefits in active vibration control or others have been extensively explored in the past years. However, nonlinear benefits are not only demonstrated in a better vibration control performance but much more. Some studies have thus been conducted in Xiao and Jing (2015, 2016) and Zhang and Jing (2020, 2021a, b) and it is for the first time systematically shown that the following advantages are very much preferable as well in practice (Fig. 17):

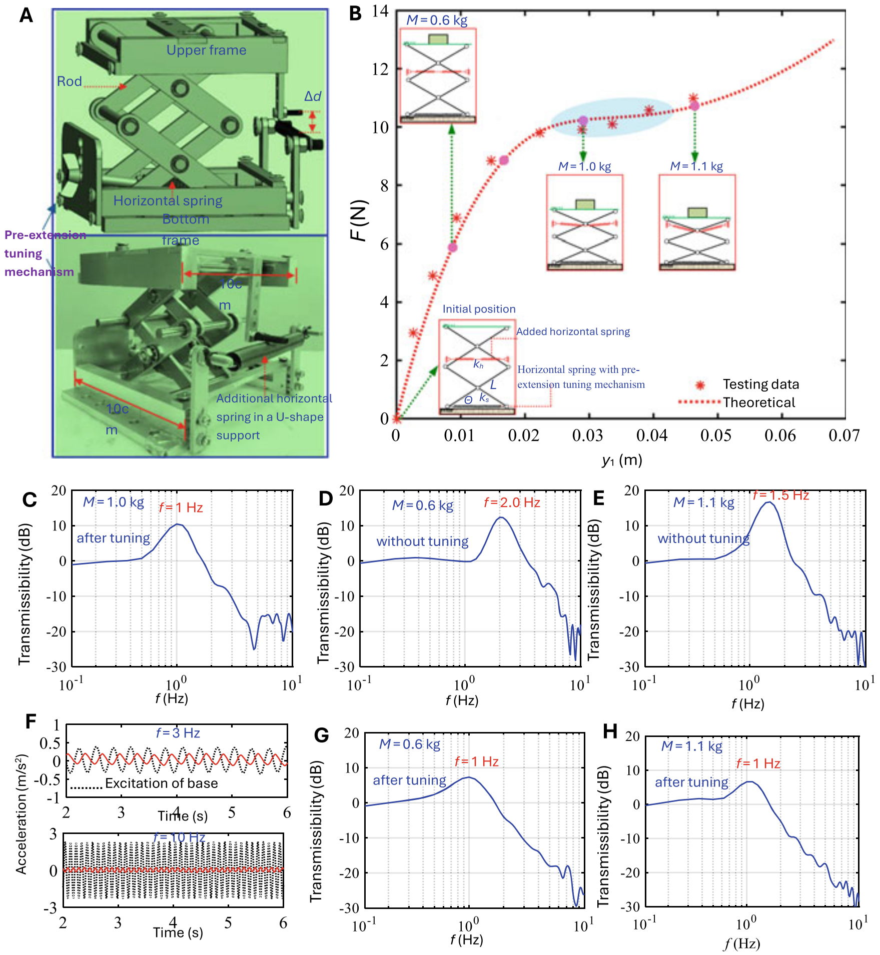


Fig. 15 Experimental results of the X-cube (Jing et al. 2021). **a** The prototype is manufactured with lightweight aluminum material with rotation joint and 2 two-layer X-shaped mechanisms in parallel. There is a pre-extension tuning mechanism at the bottom for adjusting the extension state of the horizontal spring $k_s = 6680$ N/m, and an additional horizontal spring $k_h = 160$ N/m supported by a U-shaped frame is used to construct the **X-aSpr-aS** mechanism. **b** Experimental and theoretical force–displacement curves of the X-cube indicating a large quasi-zero-stiffness area. **c** Vibration transmissibility for $M = 1$ kg exactly located in the quasi-zero stiffness area showing an excellent vibration isolation performance. **d**, **e** Vibration transmissibility for $M = 0.6$ and 1.1 kg with unmatched performance due to the system is not working in the quasi-zero-stiffness state. **g**, **h** Vibration transmissibility for $M = 0.6$ and 1.1 kg after stiffness tuning indicating excellent performances. **f** Experimental time response of the upper platform and the base excitation at different frequencies

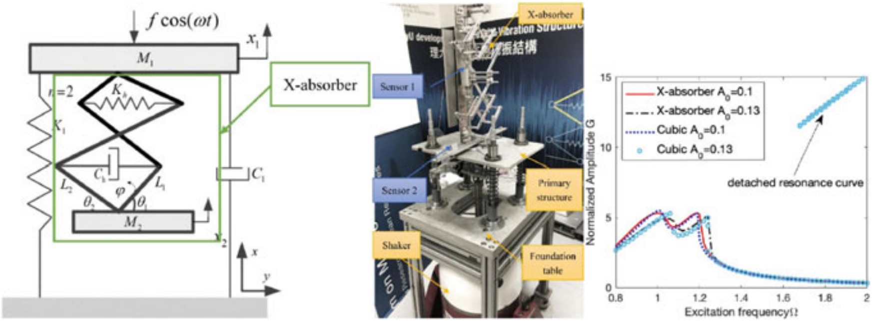


Fig. 16 The X-absorber, demonstrating more advantageous performance over its counterparts

(a) Sufficient consideration of inherent nonlinearities can considerably reduce the burden in active controller development and a simple linear controller could achieve very competitive performance (Xiao and Jing 2015, 2016);

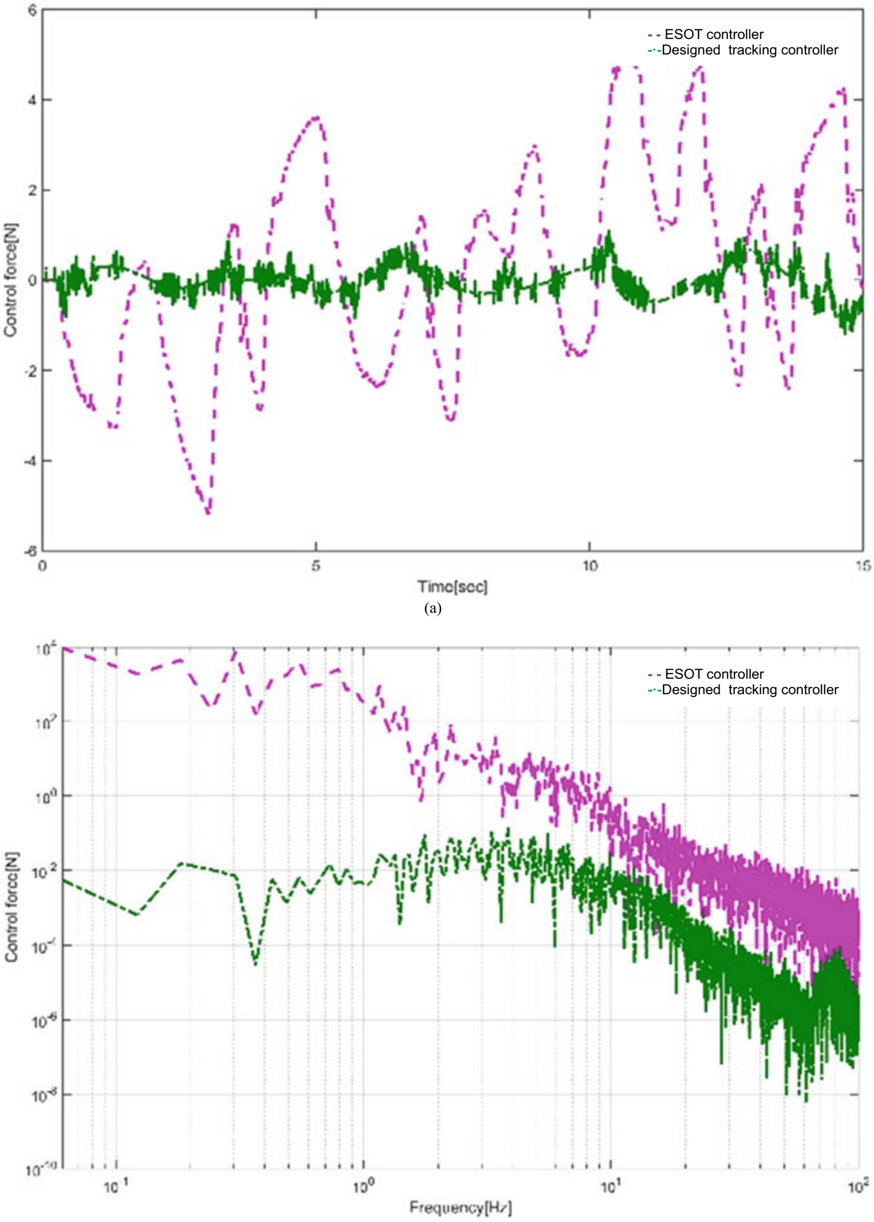
(b) Noticeably, intentionally introduced nonlinearity in a carefully designed control scheme can significantly save practical control efforts (e.g., save energy cost) but without sacrificing vibration control performance, which is well vindicated in experimental testing (Zhang and Jing 2020, 2021a, b).

D. X-structured Energy harvesting systems

Because of the advantages of the X-shaped structures/mechanisms as discussed before, X-structures based energy harvesting systems can then be developed and demonstrate much better energy harvesting efficiency. The nonlinear damping effect (Liu and Jing 2016, 2017), ultra-low and adjustable resonant frequency (Li and Jing 2019), flexibility in structural design (Wei and Jing 2017), and multi-stable stiffness property (Li and Jing 2021) can all be employed for improving harvesting performance. Prototypes and experimental validation demonstrate that these results can lead to a series of innovative technologies of great application potential (Li and Jing 2019, 2021) (Fig. 18).

E. X-structured sensor systems

Another innovative application of the X-shaped structure approach is the design of X-structured sensor systems for measuring absolute vibration displacement. Due to the ultra-low resonant frequency that can be achieved, a fixed vibration-free point could be obtained at least for a wider frequency range above the resonant frequency and in single or multiple DOF. Employment of this advantage, innovative sensor systems for measuring absolute vibration displacement can therefore be developed with the X-shaped structure or mechanisms and several prototypes for the first time validated the feasibility and sensitivity of this novel concept (Sun et al. 2014a, 2015; Pan et al. 2018; Li et al. 2017; Jing et al. 2016). It is shown that, with appropriate



(a)

(b)

Fig. 17 Control efforts in an active vehicle suspension system compared to a traditional controller (Zhang and Jing 2021b), indicating an advantageous energy saving performance