Ștefan Cantaragiu

# Microwave Numerical Solutions

Springer

# Microwave Numerical Solutions

Ştefan Cantaragiu

# Microwave Numerical Solutions

Springer

Ştefan Cantaragiu
Information Science and Technology
Academy of Romanian Scientists
Bucharest, Romania

APA

ISBN 978-3-031-61208-4 ISBN 978-3-031-61209-1 (eBook)
https://doi.org/10.1007/978-3-031-61209-1

© The Editor(s) (if applicable) and The Author(s), under exclusive license to Springer Nature Switzerland AG 2024
This work is subject to copyright. All rights are solely and exclusively licensed by the Publisher, whether the whole or part of the material is concerned, specifically the rights of reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.
The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.
The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

If disposing of this product, please recycle the paper.

# Foreword by Piet van Genderen

This book presents a wide range of issues relevant for components and circuit elements in microwave engineering.

The greater part of the book is dedicated to the analysis of shielded microstrips. Although their complexity in fabrication is higher than conventional microstrips, they have definite merits in reduced electromagnetic interference, reduced crosstalk, and a more predictable and stable characteristics. These merits are the more important; the higher is the level of integration of composite circuits and the higher is the radiofrequency. Both properties (level of integration and increasing frequency) are driving microwave engineering research and development. The author develops in great mathematical detail and rigor the properties of the quasi-TEM electromagnetic fields such that not only the magnitude of various components of the field can be computed but also important properties like dispersion and the occurrence of distinct wave modes.

Additional features offered by the book concern the modelling of a variety of other circuit elements and methods of analyzing circuit behavior.

While a great deal of the text is addressing the mathematics of analysis of the electromagnetic field of shielded microstrips, the additional support offered by the software package enables the reader to obtain more quantitative insight for self-proposed architectures of these devices. The same holds for a wide variety of other circuit elements. This package adds to opportunities for understanding of the electromagnetic behavior of advanced and complicated structures. This is a perspective to welcome this modern book, authored by Dr. Cantaragiu, who is a recognized scholar in this domain!

Genderen

Delft University of Technology
Delft, The Netherlands

Piet van Genderen

# Foreword by George Lojewski

This book provides a relevant outlook to the challenges of electromagnetic field behavior in non-homogeneous media, like the shielded microstrip line. Furthermore, the book features a set of interactive program applications designed to help the study.

It is a common fact that, unfortunately, as the operating frequency is pushed higher and higher, the performance of traditional transmission lines degrades, while their fabrication complexity and cost increase. A competitive solution for this problem is represented by the shielded microstrip lines.

I am privileged to read in advance Dr. Cantaragiu's new book on the dynamic behavior of the electromagnetic field in the shielded microstrip line, and in other microwave circuits. This book is a result of a very long period of author's research and teaching activities.

The author not only draws the full picture of the state of the art, but also presents many elements of his impressive and ingenious research progress in the investigated domain.

Dr. Cantaragiu's book is thought of as a modern, complete guide to the study of the dynamic behavior of electromagnetic waves in non-homogeneous media. It goes from theoretical background, passes through the numerical methods for solving this type of problem, and concludes with the presentation of some ad-hoc computer applications conceived to solve them. A lot of results are finally presented, referring especially to the case of shielded microstrip transmission lines.

Senior member, IEEE, University
Politehnica of Bucharest
Bucharest, Romania

George Lojewski

# Preface

This book aims to provide valuable insights into the challenges of electromagnetic field behavior in the microstrip line. It builds on a similar approach from a publication some years ago [1]. Furthermore, the book features a set of interactive program applications that are designed to help study the dynamic behavior of the electromagnetic field in the shielded microstrip line and other microwave circuits.

This work is a summary of my experience in the microwave field throughout my professional career. I started working on projects and practical achievements in my first years as a student when I came across the first edition of Gonzales' work [2], which challenged me to design microwave amplifiers using microstrip lines and $\boldsymbol{S}$ parameters [3–5].

The importance of developing rigorous mathematical models emerges from the fact that the optimal design of microwave devices using microstrip lines requires the possession of solid information regarding the propagation characteristics of the electromagnetic field and the configuration of all existing wave modes of the line.

In a series of specialized works [6–12], the analysis and calculation of the parameters of the microstrip lines is carried out under the assumption of the quasi-static approximation, which assumes that the fundamental mode of the propagation wave can be approximated with the transverse electromagnetic mode (TEM). Such an approach allows the readers to obtain satisfactory results solely for the values of the longest wavelengths in the microwave frequency range when the wavelength considerably exceeds the transverse dimensions of the line. The design based on the quasi-static approximation can be accepted when the operating frequency of the microwave devices is less than 3 GHz, and the substrate has a low relative dielectric permittivity (typically less than 6).

Recent achievements in the microwave field require, however, the operation of microstrip lines at much higher frequencies, reaching hundreds of GHz, as well as the use of substrates with high relative permittivity [1, 3–5, 7, 13–35]. Along with the increase of the operation frequency, as the displacement in the field of centimeters and millimeters waves, the quasi-static analysis of the microstrip line produces increasing errors. This phenomenon is the consequence of the microstrip line's dispersive character (parameters vary according to frequency) and the existence of higher-order wave modes in the line. Since the microstrip line is an inhomogeneous structure, containing two dielectric media (air and other dielectric) with different properties, the propagation mode is a hybrid one and cannot be associated with the TEM propagation mode.

The study of the behavior of the electromagnetic field in the shielded microstrip line presumes the objectives below, presented in detail in the content of the book, are met:

1. To deal with the real nature of the hybrid propagation modes, respectively to determine the components of the electromagnetic field, corresponding to the fundamental (energetically dominant) hybrid propagation mode, to determine the higher-order hybrid propagation modes, and to allow obtaining the information about the dispersion characteristics of the line parameters.
2. To consider the microstrip line placed inside a metal box and thus allow taking into account the conditions generated by electrical shielding effects.
3. To consider, for practical reasons, the fact that the dimensions of the shielding box must be much larger, compared to the thickness of the dielectric medium in the substrate and the width of the metal strip placed between the dielectric media, respectively air and dielectric substrate.
4. To use a sufficiently general method to allow obtaining some general solutions, which can be extended to microstrip structures with physical-geometric inhomogeneities of the conductors of more complex lines, due to multiple changes in their sizes, which are specific, in the case of resonators, filters, couplers, or slotline configurations and coplanar waveguides [11].
5. To use correct approximations, so that the accuracy of the calculations is limited only by the computing power and the software used; the approximations accepted in the specialized literature consider that the dielectric media in the microstrip structures are lossless and regarding the infinite conductivity of the conductor.

The difficulties involved in developing the study of the electromagnetic field in the microstrip line consist, mainly, in simultaneously meeting the objectives established above.

Considering the numerical approximation techniques, used to solve equations with partial derivatives, the works published in the specialized literature [14–18, 36–43], dealing with the analysis of hybrid propagation modes and the dispersion properties of microstrip lines, can be divided into two groups located, in a manner of speaking, at two extremes of the known spectrum of applicability to the problems of electromagnetism.

The first group concerns numerical methods preceded by significant analytical processing, while the second group is characterized by an extremely rudimentary analytical processing, and thus the entire burden is transferred upon the computational procedures available on the market.

Among the approaches that use detailed analytical processing, those of R. Mittra and T. Itoh [37] stand out, due to the first application of these techniques to microstrip structures, which, by modifying the conventional method (which involves solving the Dirichlet and Neuman problems from the analyzed domain), aim to determine the propagation modes in the microstrip line with the support of integral equations and use, in this sense, a series of functions with very fast convergence. The work of G. I. Zysman and D. Varon [40], who approached the electrodynamic problem of microstrip lines with the help of the system of integral equations, transformed into a matrix equation, is on the same line, but, unfortunately, the authors of articles [37] and [40] do not provide details on how to solve systems of integral equations. The method most often used in electrodynamics problems is the Fourier method, in which the solutions of the differential equations of the electromagnetic field are determined in the form of a series of functions adapted to the microstrip structure, and partial sums of the series are used to approximate the solutions. G. I. Veselov, together with his team [18], presented in their work the results of the analysis of microstrip electrodynamic structures, without revealing, however, in any of the works that followed [18], how could be obtained the infinitely homogeneous system of equations and the manner of solving it.

From the second mentioned group, which aims at numerical techniques often used in solving dynamic systems, the approaches of P. Daly [39], which uses the Finite Element Method, and those of J. S. Hornsby and A. Gopinath [36] are noteworthy; they use the Finite Difference Method and aim to meet all the above-mentioned objectives and insist less on objectives 3 and 4.

Divided into eight chapters, the work approaches in a logical sequence, with the appropriate details, the rigorous study of the electromagnetic field in the microstrip line and microwave circuits, and it concludes by providing examples of applications dedicated to the calculation of microwave circuit parameters.

The chapters composing the book deal with the following topics:

Chapter 1—"Study of the Electromagnetic Field from the Shielded Microstrip Line Using the Electrodynamic Method"—presents the steps necessary to determine the configuration of the hybrid propagation modes from the shielded microstrip transmission line and the method of adapting the selected mathematical model to more complicated microstrip structures, selecting in this regard the coupled microstrip lines. The end of the chapter summarizes the results of the analyses and simulations carried out using the Matlab development environment and the conclusions derived therefrom.

Chapter 2—"Study of the Electromagnetic Field from the Shielded Microstrip Line Using the Finite Difference Method"—presents the analysis of the electromagnetic field using a high-performance numerical method, which was successfully used to solve the most complex problems of electrodynamics, and which allows the approximation of the equations Helmholtz in a finite number of points in the analyzed domain. The final section of the chapter presents the graphs of the components of the electromagnetic field, and through some eloquent determinations, the limits of the formulas used by the quasi-static method at higher microwave frequencies.

Chapter 3—"Parameters of the Shielded Microstrip Line"—reviews and determines the ways of calculating the main parameters of the line, first, with the help of the quasi-static approximation and, later, with the help of the electrodynamic analysis of the electromagnetic field. At the same time, the specific magnitudes of the field propagation in the shielded microstrip line are also defined.

Chapter 4—“Circuit Elements of the Microwave Range”—is dedicated to the presentation of several circuit elements found in the configuration of microwave circuits (inductors, capacitors, resistors, resonators, transitions and excitation devices of transmission lines, directional couplers, dividers, and power adders), without targeting their exhaustion.

Chapter 5—“Study of Microwave Circuits Using Scattering Parameters”—aims to offer a valid method in the field of microwave, which eliminates the difficulties involved in the analysis of multi-port with the help of impedance and admittance parameters. In the final part of the chapter, the flow diagram method is presented, which allows the calculation of the gain of a two-port, an essential parameter in the case of the analysis of active microwave circuits.

Chapter 6—“Microwave Transistor Amplifier”—presents how the stability of an active microwave structure is analyzed, the calculation algorithm of narrowband microwave transistor amplifiers by the graph-analytical method, and other aspects related to the specificity the field addressed (design of match circuits, connection schemes, considerations for the practical implementation of amplifiers).

In Chap. 7—“Study of Inhomogeneities of Microwave Circuits,” the foundations are laid for a method of calculating the structure of the electromagnetic field, in which the diversity and complexity of the wave modes and the multitude of inhomogeneities of the transmission line are considered which the configuration of a microwave circuit requires.

Chapter 8—“Interactive Software Program”—issues a challenge and at the same time an invitation addressed especially to students to extend the suite of interactive applications generically named “Microwave Solutions” for the calculation of certain parameters of electromagnetic field distributions specific to the transistor amplifier. All methods were implemented using the Matlab-integrated development environment. The chapter presents, only by way of example, some concrete applications to illustrate the way of using the program package, and their conception in the form of a modular implementation facilitates the future integration into the package of other components, circuits, and microwave applications.

The presentation of the appendixes, which clarify some aspects of the first two chapters, and the selective References complete the applied approach of the solutions proposed to the readers.

I would like to express my appreciation to Springer Nature for publishing this work. Once again, this team has demonstrated their efficiency and professionalism.

Words cannot express my respect and recognition to Prof. Piet van Genderen and Prof. George Lojewski for taking their time to review my work and providing the foreword.

My deepest gratitude also goes to the Ursu spouses, respectively the late mathematician Felicia Ursu and the Ph.D. mathematician Ioan Ursu, for illuminating the

challenges of mathematics, understanding, and validating the stimulating approaches that the microwave field immutably imposes.

My family deserves my heartfelt gratefulness for their unwavering understanding and support.

Bucharest, Romania Ştefan Cantaragiu

## References

1. Cantaragiu, Ş. (2000). *Microwave circuits – Numerical calculation methods*. All Educational Publisher. ISBN 973-684-165-0.
2. Gonzales, G. (1984). *Microwave transistor amplifier, analysis and design* (1st ed.). Prentice Hall. ISBN-10: 0132543354, Pearson Publisher, 1996 (second edition), ISBN-13: 978-0132543354.
3. Cantaragiu, Ş. (1987). *Microwave module in microstrip technology for S-bandwidth receiver*. National Contest of scientific and technical innovation, Electronics and microelectronics section, Iasi, November 26–28, 1987.
4. Niculescu, T., Cantaragiu, Ş., Alexandrescu, G., & Petre, N. (1992). *Phased arrays – Design elements* (Vol. 1). Military Technical Academy Publisher.
5. Cantaragiu, Ş. (1994). *Electromagnetic field calculations of microstrip lines using finite difference method*. Technische Mitteilung TM-S-EA 94.08.02-2, Oerlikon Contraves, Zurich.
6. Schneider, M. V. (1969). Microstrip lines for integrated circuits. *Bell System Technical Journal, 48*, 1421–1444.
7. Yamashita, E., & Mitra, R. (1968). Variational method for the analysis of microstrip lines. *IEEE Transactions on Microwave Theory and Techniques, 16*, 251–256.
8. Judd, S. V., Whiteley, I., Clowes, R. J., & Rickard, D. C. (1970). An analytical method for calculating microstrip transmission line parameters. *IEEE TMTT, MTT-18*, 78–87.
9. Stinehelfer, H. (1968). An accurate calculation of uniform microstrip transmission lines. *IEEE TMTT, MTT-16*, 439–444.
10. Wheeler, H. A. (1965). Transmission – Line properties of parallel strips separated by a dielectric sheet. *IEEE TMTT, MTT-13*, 172–185.
11. Sobol, H. (1971). Applications of integrated circuits technology to microwave frequencies. *Proceedings of the IEEE, 59*, 1200–1211.
12. Sylvester, P. (1968). TEM wave properties of microstrip transmission lines. *Proceedings of IEEE (London), 115*, 43–48.
13. Kwolters, C., & Clar, P. L. (1967). Microstrip transmission lines on high dielectric substrates for hybrid microwave integrated circuits. In *1967 G-MTT Symposium Digest* (pp. 129–131).

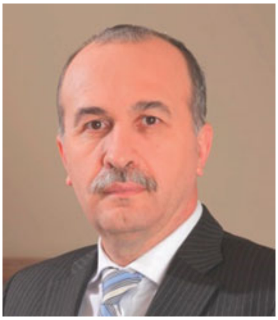
14. Akhtarzad, S., Rowbotham, R., & Johnes, P. (1975). The design of coupled microstrip lines. *IEEE Transactions on Microwave Theory and Techniques TMTT, 33*(6), 486.
15. Patrick, S., & Webb, K. (1992). A variational vector finite difference analysis for dielectric waveguides. *IEEE Transactions on Microwave Theory and Techniques, 40*, 692–698.
16. Huang, W.-X., & Itoh, T. (1988). Complex modes in lossless shielded microstrip lines. *IEEE TMTT, 36*, 163–165.
17. Pozar, D. (2011). *Microwave engineering* (4th ed.). Wiley, ISBN-10: 0470631554, ISBN-13: 978-0470631553.
18. Veselov, G. I. (Coordinator). (1988). *Microelectronîe ustroistva SVCi* (Microelectronic devices), Moskva. ISBN 5-06-001170-4.
19. Lojewski, G. (1996). *Transmission lines for high frequencies*. Technique Publisher.
20. Niculescu, T. (1994). *Propagation of radio waves and antenna-feeder installations*. Military Technical Academy Publisher.
21. Niculescu, T., & Broadband, T. V. (1988). *Antennas*. Military Publisher.
22. Palade, T. (1997). *Microwave techniques.* Genesis Publisher. ISBN 973-98204-3-3.
23. Baican, R. (1998). *Microwave integrated circuits*. Promedia Plus Publisher. ISBN 973-97377-6-5.
24. Gavriloaia, G. (2001). *Numerical analysis of the microwave field.* Teora Publisher. ISBN 973-20-0686-2.
25. Tomescu, A., Tomescu, I. B. L., & Tomescu, F. M. G. (2002). *Numerical modeling of the electromagnetic field*. Matrix Rom Publisher.
26. Lojewski, G. (2005). *Microwave devices and circuits*. Technique Publisher. ISBN 973-31-2263-7.
27. Crisan, N. (2008). *Antennas and circuits for microwaves*. Risoprint Publisher. ISBN 978-973-751-867-5.
28. Crisan, N., Palade, T., Cremene, L., & Puschita, E. (2008). *Microwaves – Applications*. UTPRESS Publisher. ISBN 978-973-662-377-6.
29. Iordăchescu, G.-A. (2018). *Microwaves: Theory and applications*. Publisher of the University of Pitesti. ISBN 978-606-560-595-4.
30. Cantaragiu, Ş. (1999). Analysis of shielded microstrip lines by finite-difference method. In *IEEE Xplore, 6th IEEE International Conference on Electronics, Circuits and Systems* (Cat. No. 99EX357) (pp. 565–567). IEEE Conference Publication.
31. Cantaragiu, Ş., & Szilagyi, A. (2002). Active combined radar/laser protection system. NATO, SET Panel Symposium, Prague, Czech Republic, 22–23 April 2002.
32. Coman, C. I., Lager, I. E., Ligthart, L. P., Cantaragiu, Ş., & Szilagyi, A. (2003). *The effect of mutual coupling in smart antenna for electronic warfare applications*. NATO IST/SET Symposium, Chester, United Kingdom, 7–8 April 2003.
33. Ioana, C., Candel, I., Cantaragiu, Ş., & Şerbănescu, A. (2010). Monitoring transient phenomena in power networks: The keypoint of energetic distribution

security. *Annals – Series on Science and Technology of Information of Academy of Romanian Scientists*, ISSN 987-606-521-022-6, *1*(1).

34. Cantaragiu Ş. (2023). *Microunde Soluții Numerice – Micro-ondes Solutions Numeriques* (pp. 1–400). Publisher of Academy of Romanian Scientists. ISBN 978-630-6518-03-6 and Technique Publisher, https://doi.org/10.56082/9786306518203, ISBN 978-973-31-2410-8, Bucharest.
35. Cantaragiu, Ş. The study of the electromagnetic field from the shielded microstrip line using the electrodynamic method. *Annals – Series on Science and Technology of Information of Academy of Romanian Scientists*. https://doi.org/10.56082/annalsarsciinfo.2023.1-2.36. ISSN online 2066-8562, Bucharest, November 25, 2023.
36. Hornsby, J. S., & Gopinath, A. (1969). Numerical analysis of a dielectric loaded waveguide with a microstrip line finite difference methods. *IEEE TMTT, 17*, 684–691.
37. Mitra, R., & Itoh, T. (1971). A new technique for the analysis of the dispersion characteristics of microstrip lines. *IEEE TMTT, MTT- 19*, 47–57.
38. Denlinger, E. J. (1971). A frequency dependent solution for microstrip transmission lines. *IEEE TMTT, MTT-19*, 30–39.
39. Daly, P. (1971). Hybrid-mode analysis of microstrip by finite element methods. *IEEE, MTT- 19*, 19–25.
40. Zysman, G. I., & Varon, D. (1969). Wave propagation in microstrip transmission lines. In *1969 G-MTT Symposium Digest* (pp. 3–9).
41. Wolters, K. C., & Clar, P. L. (1968). Analysis and experimental evaluation of distributed overlay structured in Microwave integrated circuits. In *G-MTT Symposium Digest* (pp. 123–130).
42. Corr, D. G., & Davies, J. B. (1972). Computer analysis of the fundamental and higher order modes in single and coupled microstrip. *IEEE Transactions on Microwave Theory and Techniques, MTT- 20*, 669–677.
43. Hayata, K., Koshiba, M., Eguchi, M., & Suzuki, M. (1985). Vectorial finite element method without spurious solutions for dielectric waveguiding problems using transversal magnetic field component. *IEEE, MTT- 34*, 1120–1124.

# Contents

# About the Author



**Ștefan Cantaragiu, PhD** in Engineering Science and Senior Researcher, is corresponding member of the Academy of Romanian Scientists, Science and Information Technology section.

He is a graduate of the Military Technical Academy in Bucharest, where he studied electronic and electrotechnical engineering with a specialization in air defense systems.

Throughout his career, Dr. Ștefan Cantaragiu has coordinated and led numerous defense research and technological programs and projects, as Managing Director of Military Equipment and Technology Research Agency in Romania.

He has also held various leadership positions in the private corporate industry, such as Chief Technology Officer, Director of Research and Development Programs, and Chief Executive Officer.

He represented Romania as member of Research Technology Board in former NATO Research and Technology Organization.

Dr. Ștefan Cantaragiu is the author of numerous scientific works in several fields like microwave circuits, antennas, radars, guidance systems, e-defense, command-control systems, and communications.

# Chapter 1
# The Study of the Electromagnetic Field from Shielded Microstrip Line Using Electrodynamic Method

## 1.1 The Fundamental Equations of Electromagnetism

The problem formulation that requires analysis aims to meet all the objectives outlined in the book's Preface Chapter. It involves two main stages, respectively transitioning from the real object to the physical model, while the second stage requires the mathematical formalization of the physical model that was adopted. The system of linear equations, selected as a mathematical model, helps analyze the electromagnetic field behavior in a shielded microstrip line, contributing to the electrodynamic structure analysis.

The rigorous configuration of the electromagnetic field and calculation of the shielded microstrip line parameters and their dispersion characteristics could be established using the electrodynamic analysis.

Where does the necessity and importance of the electrodynamic analysis of the microstrip structure phenomena come from?

An attempt to answer this question is presented next, but other relevant arguments in favor of using this approach will be revealed further in the content of this chapter.

The behavior of dynamic systems is described by a mathematical law expressed in the form of a general linear system of differential equations:

$$\begin{aligned}
\dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\
\dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\dot{x}_n &= a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n,
\end{aligned}$$

which characterize any material motion, from the vibrations of spring to the motion of a space shuttle. Consequently, this system of equations is used to solve vibration problems. Maxwell's differential equations are used to determine the electromagnetic field in microstrip lines due to the undulatory nature of microwave.

Ş. Cantaragiu, *Microwave Numerical Solutions*,
https://doi.org/10.1007/978-3-031-61209-1_1

It is necessary to develop a mathematical model that is based on the analysis of the electromagnetic field in the shielded microstrip transmission line. This is because the physical model of the line contains certain configurations that are significantly different, including dielectric and conductive media with distinct electrodynamic properties, as well as the singularities represented by the edges of the metallic microstrip placed between the dielectric media (air and dielectric substrate).

There are two coherent aspects regarding the resolution of discontinuities, as follow:

- From a physical point of view, the edges are not geometrically perfect and show "rounding."
- From a mathematical point of view, the approximation methods come precisely to correspond to these geometric "imperfections."

Discontinuities in the solutions of Maxwell's differential equations can cause singularities. This leads to infinite values of the energy of the electromagnetic field in finite space near edges of microstrip. To resolve these issues, electrodynamic phenomena are analyzed using approximation, convergence, and optimization methods.

* * *

The electromagnetic field distribution in a shielded symmetric microstrip line is determined using Maxwell's equations, which are well-known in the specialized literature. Figure 1.1 shows the cross-sectional configuration of the shielded microstrip line in the $x0y$ plane.

The electromagnetic study of a perfect dielectric medium (linear, homogeneous, and isotropic) leads to the determination of an electromagnetic field, consisting of the electric field vector $\vec{E}$ and the magnetic field vector $\vec{H}$, functions of point and time, whose propagation is studied in harmonic mode, that is [1]:

$$\vec{\boldsymbol{E}} = \vec{E}(x, y, z)\mathrm{e}^{j\omega t},$$

$$\vec{\boldsymbol{H}} = \vec{H}(x, y, z)\mathrm{e}^{j\omega t},$$

where $\omega$ is the angular frequency. The field distribution law is not a function of $z$ in the cross section of the shielded microstrip line.

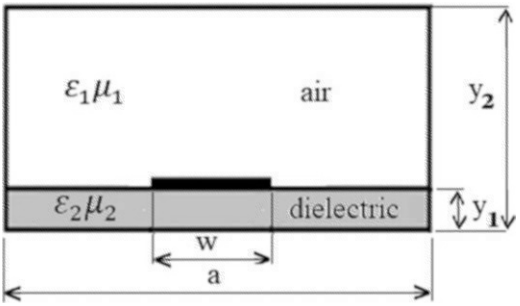


**Fig. 1.1** Cross section of shielded symmetrical microstrip transmission line

Instead, the propagation along the microstrip line is a function of $z$ and takes place in the form of a progressive wave:

$$f(z) = \mathrm{e}^{-\gamma z} \tag{1.1}$$

The expressions $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{H}}$ represent, at the same time, vectors and complex amplitudes of the electric and magnetic field. These fields, together with the electric flux density $\vec{D}$ and magnetic flux density $\vec{B}$ vectors, check Maxwell's evolution equations, respectively:

- The law of induction:

$$\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \tag{1.2a}$$

- The law of Ampere:

$$\nabla \times \vec{H} = \vec{J}_c + \vec{J}_d \tag{1.2b}$$

and the state equations, respectively:

- The law of Gauss for electric field:

$$\nabla \cdot \vec{D} = \rho_v \tag{1.2c}$$

- The law of Gauss for magnetic field,

$$\nabla \cdot \vec{B} = 0 \tag{1.2d}$$

where:

- $\vec{J}_c$ is the conduction current density vector.
- $\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$ is the displacement current density vector.
- $\rho_v$ is the complex amplitude of the volume charge density (scalar quantity).

The conservation equation, also called the continuity equation [1], ensures the connection between $\vec{J}$ and $\rho_v$, and is written in the form

$$\nabla \cdot \vec{J} + \frac{\partial \rho_v}{\partial t} = 0 \tag{1.3}$$

It is noticed that the two laws of Gauss are immediate consequences of Eqs. (1.2a), (1.2b) and (1.3). Perfect dielectric and perfect magnetic media verify the relationships

$$\vec{D} = \varepsilon \vec{E}, \tag{1.4a}$$

$$\vec{B} = \mu \vec{H}, \tag{1.4b}$$

where $\varepsilon$ is the dielectric permittivity of the medium and $\mu$ represents its magnetic permeability.

Conducting media verify Ohm's law, respectively:

$$\vec{J}_c = \sigma \vec{E}, \tag{1.5}$$

where $\sigma$ is the conductivity of the medium.

In vacuum, its permittivity and permeability are always constant and have the values

$$\varepsilon_0 = \frac{1}{36\pi}\, 10^{-9} \frac{F}{m},$$

$$\mu_0 = 4\,\pi 10^{-7} \frac{H}{m},$$

and

$$\varepsilon_0 \mu_0 c_0^2 = 1,$$

where $c_0 = 3 \times 10^8$ m/s and is the speed of light in a vacuum.

If the curl operator is applied to Maxwell's first equation of evolution, (1.2a), the following relation is obtained:

$$\nabla \times \nabla \times \vec{E} = -\nabla \times \frac{\partial \vec{B}}{\partial t},$$

in which, if relation (1.4b) and the homogeneity property of the linear differential operator are considered, the following relation is obtained:

$$\nabla \times \nabla \times \vec{E} = -\frac{\partial}{\partial t} \mu \nabla \times \vec{H} \tag{1.6}$$

Then, relation (1.2b) is introduced in the expression (1.6), in which it was considered that the displacement current density vector, $\vec{J}_d$, is much higher than the conduction current density vector, $\vec{J}_c$, in the dielectric media [1], and finally, the following is obtained:

$$\nabla \times \nabla \times \vec{E} = -\varepsilon\mu \frac{\partial^2 E}{\partial t^2}. \tag{1.7}$$

In relation (1.7), the double curl operator formula is used and an equation corresponding to the electric field vector is obtained, respectively:

$$\varepsilon^{(\delta)}\mu^{(\delta)} \frac{\partial^2 \vec{E}}{\partial t^2} - \Delta\vec{E} = 0, \tag{1.8a}$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian expressed in Cartesian coordinates. The index $\delta$, introduced to differentiate the two domains in Fig. 1.1, has the value 1, when Eq. (1.8a) describes the behavior of the electric field in air, and the value 2, when the equation describes the behavior of the field in the dielectric medium placed under the metal strip.

Analogously, the equation corresponding to the magnetic field vector is obtained, respectively:

$$\varepsilon^{(\delta)}\mu^{(\delta)} \frac{\partial^2 \vec{H}}{\partial t^2} - \Delta\vec{H} = 0 \tag{1.8b}$$

The wave propagation speed through the transmission line is calculated with the relation

$$c = \frac{1}{\sqrt{\varepsilon^{(\delta)}\mu^{(\delta)}}} = \frac{1}{\sqrt{\varepsilon_0\mu_0\varepsilon_{r\delta}\mu_{r\delta}}},$$

where $\varepsilon_{r\delta}$ and $\mu_{r\delta}$ represent the relative permittivity and the relative permeability of the media, respectively.

Considering Eqs. (1.2a) and (1.2b) and relations (1.3), (1.4a), (1.4b), and (1.5), the following is obtained:

$$\nabla \times \vec{E} + i\omega\mu^{(\delta)}\vec{H} = 0, \tag{1.9a}$$

$$\nabla \times \vec{H} - i\omega\varepsilon^{(\delta)}\vec{E} = \vec{J}, \tag{1.9b}$$

$$\nabla \cdot \vec{J} - i\omega\rho = 0, \tag{1.9c}$$

$$\vec{J} = \sigma\vec{E}, \tag{1.9d}$$

and Eqs. (1.8a) and (1.8b) become