Uncertainty and Operations Research

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Hesitant Fuzzy and Probabilistic Information Fusion

Theory and Applications

Springer

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Decision analysis based on uncertain data is natural in many real-world applications, and sometimes such an analysis is inevitable. In the past years, researchers have proposed many efficient operations research models and methods, which have been widely applied to real-life problems, such as finance, management, manufacturing, supply chain, transportation, among others. This book series aims to provide a global forum for advancing the analysis, understanding, development, and practice of uncertainty theory and operations research for solving economic, engineering, management, and social problems.

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Preface

The influx of massive information provides multiple perspectives, multi-level reference criteria, and diverse reference data for scientific decision-making. Meanwhile, it also amplifies uncertainty in decision-making processes. In traditional decision-making problems, people are accustomed to using single and accurate numbers to depict and analyze characteristics. Nowadays, people are inundated with information from various channels which makes it difficult to discern the effectiveness of decision-making information. Furthermore, people are more likely to become indecisive, and suffer from the trouble caused by the integration of massive amounts of information, which have brought unprecedented obstacles to the advancement of decision-making science.

Probability theory and fuzzy set theory that show different aspects of uncertainty in real life, respectively, provide theoretical support for solving the decision-making problems under the condition of bounded rationality. As the extension of fuzzy set theory, hesitant fuzzy sets have attracted wide attention in the decision-making field and are often used to solve various practical problems because of their outstanding performances in describing the indecisive psychology of decision-makers. Probability theory which has significant advantages in describing, predicting, and inferring objective laws has been widely applied in decision-making theory.

Based on the probability theory and approaches, this book has carried out in-depth research and exploration on the solution and application of several types of realistic problems in hesitant fuzzy multi-attribute decision-making. The main work of this book is summarized as follows:

(1) A hesitant fuzzy multi-attribute decision-making method based on probability and opinion dynamics is proposed. Firstly, in order to simulate the evolution law of hesitant fuzzy opinions, the multiplication rule between the real matrix and the hesitant fuzzy matrix is defined and related operation properties are studied which lays the foundation for the application and expansion of hesitant fuzzy set theory. Secondly, the hesitant fuzzy DeGroot opinion dynamics model which can be used to predict the opinion of a group is proposed. Three

kinds of opinion transformation matrices with the consideration of the similarity degree, self-confidence degree, and authority degree are constructed and the consensus condition for the model is discussed as well. Finally, the multi-attribute decision-making method based on the hesitant fuzzy DeGroot opinion dynamics model is applied to the emergency response decision-making of public health emergencies. The experimental results show the effectiveness, feasibility, and practicality of the method.

(2) The dual hesitant fuzzy multi-attribute decision-making method based on probability distribution is provided, and then it is extended to the hesitant fuzzy environment. Firstly, in order to reduce the negative impact of cognitive bias on decision-making, this book studies the distribution characteristics of statistical data for decision-making, analyzes the possible sources of bias in real number environment. Secondly, inverse proportional function, linear function, and the normal distribution function are utilized to establish decision-making models in the dual hesitant fuzzy environment. Because hesitant fuzzy set can be regarded as a special case of dual hesitant fuzzy set, these models are also suitable for solving hesitant fuzzy decision problems. Finally, the validity and rationality of these models have been tested in the decision-making case for recruitment interview of product manager.

(3) Multi-attribute decision-making methods based on probabilistic hesitant fuzzy entropy are introduced. Firstly, based on the classic fuzzy entropy, the axiomatic definition and specific calculation method for probabilistic hesitant fuzzy entropy based on membership degree are given. Secondly, based on the detailed analysis of the rationality of distance-based hesitant fuzzy entropy, the distance-based entropies for probabilistic hesitant fuzzy elements which are inversely proportional to the distance measures among the elements and the fuzziest element are proposed. Then, multi-attribute decision-making methods based on probabilistic hesitant fuzzy entropy are introduced. Finally, the validity of the method is verified in the Belt and Road venture capital case and it was compared with the entropy-based decision-making methods.

(4) Integration methods for continuous hesitant fuzzy information in group decision-making are proposed. Since the existing integrated methods of hesitant fuzzy information have become too complicated to meet the needs of increasingly complex practical decision-making problems, we initially combine the related knowledge of probability theory to introduce the concept of continuous hesitant fuzzy element. Following this, the concept of uniform hesitant fuzzy element is given, and discrete (probabilistic) hesitant fuzzy information is transferred to continuous one, benefitted from the connection between uniform hesitant fuzzy elements and continuous hesitant fuzzy elements with uniform distribution. Subsequently, integration methods of continuous hesitant fuzzy elements based on mathematical derivation are developed, which lays a theoretical foundation for the continuity of hesitant fuzzy information. Additionally, facing the problem that the method of mathematical derivation is too tedious, based on computer simulation, we propose another integration method of continuous hesitant fuzzy elements, which is more concise and easier to apply. Finally,

an example of the evaluation of water resources emergency management plans is given to apply the above method to solve practical decision-making problems.

(5) The probability-based hesitant fuzzy assessment model is applied for the political risk assessment of complicated investment in the context of the Belt and Road. Firstly, the complexity of the political risk assessment is analyzed and the assessment data are collected in the hesitant fuzzy form to show the uncertain information in the problem. Secondly, the corresponding assessment index system with the consideration of the impact of the epidemic is constructed. Then, the prospect theory and the decision-making methods in Chaps. 2 and 3 are combined to build a complete investment political risk assessment model in the context of Belt and Road. Finally, the stability and reliability of the model are analyzed and discussed.

Generally, this book is primarily dedicated to addressing issues with uncertainty by fusing probability and hesitant fuzzy theory, and has made attempts in the following areas: depicting the transmission law of hesitant fuzzy information using probabilistic knowledge, digging the distributed characteristics of both the dual hesitant fuzzy information and the hesitant fuzzy information, investigating the entropy measures of the probabilistic hesitant fuzzy information and adopting the probabilistic distribution to simplify the integration of hesitant fuzzy information, etc. It can be used as reference for postgraduate and senior undergraduate students in fuzzy mathematics, operations research, information science, management science, etc.

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Chapter 1
Introduction

The connotation of uncertainty in matters are diverse. It may be random uncertainty or fuzzy uncertainty, and most of the time, multiple uncertainties coexist. Based on this fact, the research in this book focuses on the multi-attribute decision-making problems that involve multiple types of uncertainty. Therefore, our introduction starts with the cognitive origins and description of uncertainty. Then before presenting the research focuses of this book, related literature reviews of the existing research are given.

1.1 Background

In this section, the cognitive processes of uncertainty and the two ways in which uncertainty information is expressed are briefly reviewed. Additionally, a statement explaining the necessity of the research in this book is provided.

1.1.1 Cognition of Uncertainty

Since the dawn of independent thought in human beings, the philosophical exploration of the nature of the world has been ongoing without interruption. Throughout history, the certainty which can be represented by regularity, inevitability and unity can eliminate the confusion and fear caused by uncertainty has always been the goal of people to pursuit. The thinking about certainty can be traced back as far as ancient Greece. Thales, who is hailed as the "ancestor of science and philosophy" by Westerners (Skirbekk and Gilje 2001), believed that everything is made of water. This simple idea of certainty aimed to reveal the unity of all things. Then Plato put forward the concept of an absolute ideal world in his work "The Republic". He believed that

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the ideal world is eternal and unchanging, and it is definite, while the phenomenal world is constantly changing and merely a reflection of the ideal world (Skirbekk and Gilje 2001). Since the Renaissance, continental rationalism, initiated by Descartes, has claimed that scientific knowledge is unambiguous and clear. In modern times, Hegel's objective idealism attributes the original cause and innermost essence of all things to the "absolute spirit" (Skirbekk and Gilje 2001). Marx's dialectical materialism believes that the world is unified and the world is unified by matter (Engels 1954). Although the two viewpoints are diametrically opposed, they both represent key claims of determinism.

Being the other side of certainty, the thought of uncertainty encompasses randomness, ambiguity, roughness, unpredictability, etc. It originated with the ancient Greek philosopher Anaximander, who believed that the origin of the world was uncertain (Thilly 1914). In addition, there are several other expressions of uncertainty proposed by philosophers. Gorgias argued that non-being does not exist (Skirbekk and Gilje 2001). Heraclitus claimed that you cannot step into the same river twice (Thilly 1914). Piron, a representative of skepticism, stated that we can't know anything, and Wittgenstein referred to the concept of the "unspeakable" (Skirbekk and Gilje 2001).

On the surface, certainty and uncertainty seem like opposing concepts, but in reality, they are not completely mutually exclusive and can even transition into one another at times. One of the most representative examples is that in sixteenth-eighteenth century Europe, materialist empiricism, which represented certainty, eventually evolved into idealist agnosticism, which represented uncertainty. Initially, Bacon, as the precursor of British materialist dialectics and experimental natural science, believed that the acquisition and verification of knowledge requires experience and that experience must be deterministic (Thilly 1914). Later, after being criticized and influenced by Descartes, Leibniz and others, who represented rationalism on the European continent, it developed into Hume's agnosticism in the seventeenth century, which holds that the existence of the external world is unknowable and everything in the world can be attributed to subjective phenomena or experience (Thilly 1914).

The development of philosophy and science has always been inextricably linked. Over time, humanity's perception of scientific knowledge has gradually evolved from certainty to the acceptance and development of uncertainty. In the early stages, under limited cognitive conditions, the position of certainty in scientific research was unshakable. It is generally believed that even complex scientific phenomena adhere to a simple and immutable inner essence, which is definite. Especially in the sixteenth century, the classical mechanical system established by Newton perfectly explained the relationship between force and motion in the macroscopic world. In his Mathematical Principles of Natural Philosophy, he described a certain world governed by natural laws (Newton 2016). This makes people more convinced that they can achieve a precise and definitive understanding of things as long as they continuously enhance their cognitive abilities. The famous astronomer Laplace made a classical and bold statement regarding the concept of certainty: If the initial conditions are known, the future state of an object can be predicted. In his expression, all objects