Simulation Foundations, Methods and Applications

Masoud Fakhimi
Navonil Mustafee *Editors*

Hybrid Modeling and Simulation

Conceptualizations, Methods and Applications

Springer

Simulation Foundations, Methods and Applications

The modeling and simulation community extends over a range of diverse disciplines and this landscape continues to expand at an impressive rate. Over recent years, modeling and simulation has matured to become its own discipline, while continuing to provide support to other disciplines. As such, modeling and simulation provides the necessary conceptual insights as well as computational support which has an established record of significantly enhancing the understanding of dynamic system behavior and improving the system design process, as well as providing the foundations for computational sciences and practical applications, from cyber-physical systems to healthcare. Hybrid methods and combinations with artificial intelligence and machine learning open new possibilities as well. The ever-increasing availability of computational power and the availability of quantum computers make applications feasible that were previously beyond consideration. Simulation is pushing back the boundaries of what it can be applied to and what can be solved in practice. Its relevance and applicability are unconstrained by discipline boundaries.

Simulation Foundations, Methods and Applications hosts high-quality contributions that address the various facets of the modeling and simulation enterprise. These range from fundamental concepts that are strengthening the foundation of the discipline to the exploration of advances and emerging developments in the expanding landscape of application areas. The underlying intent is to facilitate and promote the sharing of creative ideas across discipline boundaries.

As every simulation is rooted in a model, which results from simplifying and abstracting the reference of interest to best answer research questions or support the application domain of interest, we understand the model development phase as a prerequisite for any simulation application. There is an expectation that modeling issues will be appropriately addressed in each presentation. Incorporation of case studies and simulation results will be strongly encouraged.

Titles of this series can span a variety of product types, including but not exclusively, textbooks, expository monographs, contributed volumes, research monographs, professional texts, guidebooks, and other references.

These books will appeal to senior undergraduate and graduate students, and researchers in any of a host of disciplines where modeling and simulation has become (or is becoming) an important problem-solving tool. Some titles will also directly appeal to modeling and simulation professionals and practitioners.

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"To my parents, Ali and Soheila, whose love and wisdom light my way, and to all my mentors, whose insights into modelling and simulation carved paths in my mind."

Masoud Fakhimi

"To the authors who submitted to the 'Hybrid Simulation' track of the Winter Simulation Conference between 2014 and 2018, to my colleagues and friends who supported the development of the track during its infancy, and above all, to my children, Neelabh and Rudraneel, who make less time seem like more time."

Navonil Mustafee

Foreword

"All Creatures Great and Small" is a BBC television series with iterations since the late 1970s. The series, which takes place in the rolling hills and valleys of the English county of Yorkshire, has one episode that cause me to reflect on this excellent book edited by Masoud Fakhimi and Navonil Mustafee. In this memorable episode, the veterinarian is talking to a boy about his dog. The dog is on the mend, and the veterinarian says that the dog is likely to recover well because it has genetics consistent with "hybrid vigor."

Hybrid vigor is known more scientifically as *heterosis*. Heterosis is defined as improved biological function resulting from hybrid genetics of the parents. Could hybrid modeling and simulation similarly confer a positive outlook? It gets one thinking, but alas, the sorts of models that we create in areas such as operations research are not biological. We may engage in hybrid solutions because it matches our professional interests and inclinations.

What does hybrid modeling and simulation mean? We first acknowledge a difference between modeling and simulation. A model is a language artifact, whose computer-based execution defines a simulation. The model is designed and then the computer executes the model. Because the model is hybrid, the execution can also be hybrid in a computational sense. Continuous models are mathematical and require numerical integration. The models look different when compared with discrete-event models, and the method of solving the models is also different.

Chapter authors take two or more types of approaches. For instance, discrete-event simulation (DES), system dynamics (SD), and agent-based modeling (ABM) are frequently employed. DES, SD, and ABM are different ways of thinking about a problem space. This variety yields a hybrid solution. Decades ago, there were references to Ashby's *Law of Requisite Variety* in cybernetics. This law captures the complexity of state space. This complexity has an analog with information theory: more information in complex systems with many states and events. In the 1980s, simulation researchers constructed the area of "combined modeling." Combined models connect discrete events with continuous time and space modeling, as in fluid dynamics. The collection of scholarly expositions in this new book builds upon early work in cybernetics, systems theory, and combined discrete event/continuous systems.

Even though DES, SD, and ABM reflect our current thinking about hybridity, we need to expand this collection by using other types of models that we routinely use in defining and explaining systems. Information and data modeling are performed. A database and its data are defined with a schema. The knowledge that includes the models extends to semantic web terminology, with the phrase *ontology*. This is also modeling. Packages that we use to aid in our simulation, such as AnyLogic, include 2D and 3D computer-aided design (CAD) models. Even though semantic web models and CAD models do not involve the forward march in time, these models also need to be considered as part of a hybrid modeling system.

To return to our musing on hybrid vigor, our community is left wondering whether there is something akin to biological performance. Can a hybrid model be better or more inclusive than a model that is not hybrid? Is there an argument to be made for hybridity in modeling and simulation? Let's explore two arguments. Both arguments stem from qualities of people.

The first argument is based on education. In education, we observe that every person is different: Different backgrounds and interests. And yet, our school systems are based on mass production through uniform lessons and subject area disciplines. More ideally, we need differentiated learning where there is a tutor who can converse in a wide range of subjects. I was fortunate to explore this idea with my host, Nav, in Exeter during the summer of 2018 with the benefit of a Leverhulme Trust fellowship. We engaged with the Royal Albert Memorial Museum (RAMM) to see how art could be leveraged to understand computer science and scientific modeling. This engagement bridged art, mathematics, and science. The underlying hypothesis is that all objects, including those in museums, have multiple subject area-based interpretations: from art to science. These interpretations have the potential to bring people and subjects together. Curiously, recent research in AI large language models (LLMs) has yielded technologies to assist in this hybrid learning. Khan Academy has been in the forefront with a new type of chatbot tutor.

The second argument is based on management science. This is where I remember parts of my career. When I started my industrial life, I was a small cog in a huge machine that made ships and submarines within the Newport News Shipbuilding company. I then went on to assist in researching CAD models for aircraft at NASA Langley Research Center. Ships and airplanes are very complex, or some would say that they are very complicated. The complications manifest themselves in the assortment of disciplines needed to model and simulate. A variety of people are needed to make the products. The evolving complexity reflects the parts and the people. Large-scale models indicate more disciplines being involved. And this is why this book, and the field of hybrid modeling and simulation, are required. It isn't just that hybridity is "nice to have." It is a "must have" type of research. As we grow our models and simulations, this comes about with increased complexity. This complexity in turn requires more diverse workers, who come with different disciplines—various ways of seeing the world.

I close this short essay with experience I've gained since working for the State of Florida in the modeling and simulation of catastrophic wind and flood risk since the mid-1990s. To understand these risks, one needs the combined, hybrid expertise of different types of science and engineering. The actuarial profession is central to gauging financial risk. This combination includes the ability to deal with people who collaborate across different sectors. Different fields and people mean more complexity leading to a large-scale enterprise. Each player will come knowing different types of models. A hybrid approach is the only possible outcome. This is why this book is a "must read." As we move beyond building small-scale models for ourselves or in our own small labs, we necessarily embrace the hybrid. We are ready to segue to our own version of all models great and small.

Richardson, USA Paul Fishwick

Preface

Hybrid Modeling and Simulation: Conceptualizations, Methods, and Applications aims to advance our knowledge of mixing methods from the field of modeling and simulation (M&S) and other scientific disciplines. Numerous textbooks and reference bodies of work provide an excellent foundation for discrete and continuous simulation methodologies. However, this is the first book that presents an integrative body of work on hybrid M&S.

Our field has distinct research communities where techniques like agent-based simulation (ABS), system dynamics (SD), and discrete-event simulation (DES) have evolved through decades of research and practice. As these sub-fields have continued to develop, conventional simulation approaches, i.e., using either ABS, DES, or SD, have transitioned to hybrid simulation (e.g., DES+ABS, ABS+SD). The hybrid approach becomes necessary when systems get increasingly complex and individual methods cannot adequately capture their intricacies. Combining methods leverages the strengths of techniques and presents the opportunity to develop a better representation of the system compared to using a single approach.

This book has extended the discussion on model hybridisation by considering methods and approaches developed outside our field. This is referred to as Hybrid Modeling. Thus, the term *Hybrid Modeling and Simulation* in the book's title calls attention to both *Hybrid Simulation* (models predominantly developed within our field) and *Hybrid Modeling* (models that intersect with approaches developed in broader scientific disciplines).

Unlike hybrid simulation, the term hybrid modeling is open to multiple interpretations. As the book is primarily targeted at the M&S community, we define a hybrid model as including at least one simulation technique that is combined with research approaches from a wider array of disciplines.

Further, our definition of hybrid consists of the conjoint application of cross-disciplinary techniques in one or more stages of a simulation study (Chapter 1 provides an integrative taxonomy of hybrid simulation and hybrid modelling). For a model to be classed as a "hybrid model" (as per the definition above), it should include at least one core simulation technique, e.g., DES (it can also be a hybrid simulation), and additionally should have deployed knowledge artefacts from other scientific disciplines in one or more stages of a simulation study (e.g., conceptual modelling stage, validation and verification, experimentation). These artefacts could be disciplinary modelling methods and research paradigms, new ways of framing and answering research questions such as hypothesis testing, use of methodological approaches from hard sciences through formulation of theories and controlled experimentation, new ways of collecting data by adopting ethnographic and other social science methods, novel analysis of primary/secondary data using approaches such as structural equation modelling, and deployment of standards and best practices that have stood the test of time in other disciplines. In the current literature, examples of hybrid models are mostly restricted to those developed in conjunction with Operations Research methods (including various analytics/ML models) and/or applied computing approaches. Examples include hybrid models that combine ABS with machine learning, those that have used DES with distributed computing and studies combining forecasting models with computer simulation.

A Call to the Community!

Our call for book chapters specifically sought contributions to both hybrid simulation and hybrid modeling. Thus, the first objective was to present work that contributed to hybrid simulation conceptualisations, work that applied existing modelling formalisms to hybrid simulation, identified frameworks for mixing methods and developed exemplar hybrid applications. The second objective was to include a collection of chapters that extend this current state-of-the-art in mixing methods and which deployed simulation alongside research approaches from outside M&S; the latter an example of leveraging cross-disciplinary strengths through the use of hybrid models as enablers of multidisciplinary, interdisciplinary, and transdisciplinary research.

Consistent with the intense interest in the M&S community in mixing simulation methods, most of the chapters received were on hybrid simulation. Thus, in relation to objective two, the book includes only a few chapters on hybrid models. Irrespective, we hope our book will broaden the discussion on what hybridity means in M&S and its different facets!

We hope our endeavour will be a call to the community to conduct further research on mixing disciplinary approaches with M&S. This would pave the way for an increasing number of studies on cross-disciplinary hybrid models, which, in the future, (we hope) will follow the same trajectory of growth as we witness for hybrid simulation! Indeed, the challenges that humanity is faced with today, from climate change and the need for climate-resilient regions and infrastructures to the need for sustaining economic growth whilst achieving net zero emission targets, will require the marriage of methods which were theorised, developed, refined and perfected in scientific disciplines, many of which had traditionally existed in isolation. This book is a step towards bridging this divide!

The book includes a foreword by Paul Fishwick, Chair Emeritus of Arts, Humanities, and Technology at The University of Texas at Dallas, whose work connects Arts and Humanities with Engineering, Mathematics, and Computer Science.

Chapter Summaries

We organised the chapters into three themes: conceptualisations and frameworks; formalisms and methods; applications.

The first four chapters focus on the conceptual aspects of Hybrid M&S, offering readers a foundational understanding of hybrid modeling and hybrid simulation and the use of conceptual modelling approaches. The first chapter is authored by the book's editors. In Chap. 1, Navonil Mustafee and Masoud Fakhimi outline the significance and core concepts of hybrid M&S, constructing a narrative around its research and developing an integrative taxonomy for both hybrid simulation and hybrid modelling. Chapter 2, by William Jones, Kathy Kotiadis, Jesse R. O'Hanley, and Stewart Robinson, introduces a novel method for conceptual modelling in hybrid simulations, emphasising its role in enhancing communication and software development. In Chap. 3, Richard A. Williams presents a semi-systematic literature review on hybrid conceptual modelling within organisations, focusing on social and socio-technical systems. Lastly, Chap. 4 by Andreas Tolk, Jennifer A. Richkus, and Yahya Shaikh underscores the importance of participatory modelling to include diverse community insights, presenting a conceptual framework and open research questions in the field.

Chapters 5 through 10 delve into the methods theme and cover hybrid methodologies and formalisms. Chapter 5 by Fernando J. Barros focuses on model product lines (MPLs) within the HyFlow++ framework, presenting the πHyFlow++ implementation for hybrid simulation. In Chap. 6, Saptaparna Nath and Gabriel A. Wainer use CELL-DEVS modelling to study the influence of social media influencers on follower engagement, employing the Cell-DEVS formalism through the Cell-DEVS Cadmium simulation environment. Chapter 7 by Niclas Feldkamp is on integrating machine learning with simulation. It takes a hybrid modelling perspective, providing guidelines and use case examples for their combined application. Chapter 8, authored by Najiya Fatma, Pranav Shankar Girisha, and Varun Ramamohana, presents a hybrid modelling approach using simulation and machine learning for real-time delay prediction in complex queuing systems; a case study on kidney transplantation waitlists is presented. In Chap. 9, Susan Howick, Itamar Megiddo, Le Khanh Ngan Nguyen, Bernd Wurth and Rossen Kazakov, explore the integration of SD and ABM, offering methodological insights and practical considerations for developing SD-ABM hybrid simulations. Finally, Chap. 10 by Alison Harper, Thomas Monks, and Sean Manzi proposes a hybrid method for enhancing the usability and sharing of simulation models through containerisation with continuous integration, demonstrating this with a Python-based orthopaedic elective recovery planning model.

Chapters 11 through 13 relate to the third theme on applications and case studies in hybrid modelling and simulation, offering in-sights into real-world implementations. Chapter 11, by Anastasia Anagnostou and Simon J. E. Taylor, discusses the application of hybrid simulation in healthcare, particularly in emergency medical services and pandemic crisis management, highlighting its role in holistic analysis and management. In Chap. 12, Vishnunarayan Girishan Prabhu and Kevin M Taaffe present a hybrid modelling approach using machine learning to optimise healthcare operations. They illustrate this through a case study in an emergency department, integrating forecasting, hybrid simulation, and mixed integer linear programming to enhance physician shift scheduling and patient safety. Finally, Chap. 13 by Kavitha Balaiyan, R. K. Amit, Amit Agarwal, and T. V. Krishna Mohan addresses demand forecasting challenges in airline revenue management. The authors propose a sequential two-stage hybrid modeling approach—a simulation-based heuristic algorithm for parameter estimation in joint-forecasting models, using actual airline data in the Airline Planning and Operations Simulator.

As outlined in the chapter summaries, *Hybrid Modeling and Simulation: Conceptualizations, Methods, and Applications* unravels the complexities of hybrid M&S, highlighting its application in multiple areas. The book is designed to foster a deeper comprehension of how various modelling methods can be combined to enhance decision-making and problem-solving in complex environments. The chapter contributions present a call to embrace the complexity of the world around us and to seek out the synergies of hybrid approaches for a deeper understanding of the problem space and engender improved decision-making. For anyone intent on mastering the art of hybrid M&S, this book is an essential companion.

London, UK — Masoud Fakhimi
Exeter, UK — Navonil Mustafee

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Part I
Conceptualisations and Frameworks

Chapter 1
Towards an Integrative Taxonomical Framework for Hybrid Simulation and Hybrid Modelling

Navonil Mustafee and Masoud Fakhimi

Abstract The modelling and simulation (M&S) literature identifies two forms of hybrid studies—hybrid simulation (HS) and hybrid model (HM). While HS is the combined application of simulation techniques such as discrete-event simulation, agent-based simulation and system dynamics, HM has a cross-disciplinary focus; the objective is to apply research paradigms and approaches, conceptualisations and frameworks, methods, tools and techniques from disciplines such as computer science, engineering, OR and economics in one or more stages of an M&S study. The growing volume of literature on HS evidences the shift from conventional (one-technique) models to HS. It is expected that HM will follow the same trajectory. However, further studies are essential in contextualising hybrid M&S research and identifying opportunities for hybridisation. Towards this, three related themes of research are explored: (a) conceptualisation of HM in the context of the lifecycle of an M&S study, (b) a classification scheme, and (c) mapping existing literature in HS and HM. The themes are based on a series of authors' papers published in the Journal of the Operational Research Society and the Winter Simulation Conference. The chapter concludes with an integrative taxonomical framework that identifies the current advances and opportunities for future research in hybrid M&S.

Keywords Hybrid simulation · Hybrid modelling · Operational research · Taxonomy · Conceptual modelling

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1.1 Introduction

M&S techniques such as discrete-event simulation (DES), agent-based simulation (ABS) and system dynamics (SD) enable the development of computational models that aid decision-making. They are among the most frequently used techniques in operations research/management science (OR/MS) [28]. There are numerous examples of their application in manufacturing and business [18], supply chain [37, 50], healthcare [5, 20, 42] and other domains. Many studies now apply multiple M&S techniques to develop hybrid simulations (HS) that combine DES, ABS and SD [6]. HS allows for a better representation of the system being modelled, enables opportunities for analysis at different levels of resolution and provides greater insights into the evolving dynamics of the system.

A parallel but related theme of research is the development of hybrid models (HM) that combine conventional simulations (i.e. single-technique DES, ABS and SD models) or HS, with research approaches, methods and techniques from disciplines such as computer science/applied computing, economics, engineering, information and communications technology, operations research, software engineering and social sciences. Similar to HS, the objective here is to explore the synergies of combining discipline-specific methods in developing M&S studies, studies that go beyond what would otherwise be possible if only approaches developed within our discipline were being used. However, unlike HS, the focus is not limited to combining only simulation techniques, but rather to integrating simulations with theories, frameworks, methods and established research approaches that have been tried and tested and have existed as extant knowledge within distinct academic disciplines [31, 51]. These knowledge artefacts can be seen as the body of knowledge (BOK) that is used by a discipline to guide practice [39], and absorbing them into M&S studies presents the opportunity to complement (rather than supplement) the techniques traditionally used within our field. In social sciences, for example, scientists increasingly use computational experiments through social simulations to explore and test hypotheses concerning aspects of collective action and group dynamics [47].

The book chapter is based on a series of four papers that the authors have published in the *Journal of the Operational Research Society* and the *Winter Simulation Conference* [29–31, 43]. The papers are either conceptual or a mix of conceptual and review papers. The objective of the book chapter is to construct a narrative for HM based on three related themes of hybrid research: (a) conceptualisation of HM in the context of the distinct phases of the lifecycle of an M&S study, which identifies the opportunities for the combined application of cross-disciplinary approaches; (b) a classification scheme for HS and HM; (c) categorisation of the existing work using the classification scheme. These discussions lead to the development of an integrative taxonomical framework for HS and HM. The taxonomy is based on the original HS and HM classification presented by [31] and its extension by [30]. The integrative taxonomical framework is organised in multiple levels of hierarchy.

1.2 Stages of a Simulation Study and Opportunities for Hybrid Modelling

A M&S study begins with the investigation of a real-world problem or a consideration for a future system. A conceptual model is developed and validated, followed by the implementation of a computer model. In the verification stage, the model is checked to ensure that it is a good representation of the conceptual model and is free from errors. Scenarios for experimentation are developed, followed by an analysis of the results of the simulations for possible implementation of the results of the simulation study. These M&S study stages enable us to systematically explore complementary techniques for problem understanding and conceptualisation, model implementation, validation and verification, scenario development and experimentation, results analysis and implementation of the results. Based on the discipline-specific methods and what it has to offer, this added value gained could be mapped to various stages of an M&S study [31]:

- *Problem formulation/Conceptual modelling*: A systems engineering modelling language called systems modelling language (SysML) was proposed by [11] to capture the inception stage of a healthcare service development lifecycle. Soft OR/Problem structuring methods have been widely used to aid the development of conceptual models for DES. Examples include the use of soft systems methodology (SSM) by [21, 23], group model building by Bérard [4] and qualitative systems dynamics (QSD) by Powell and Mustafee [43].
- *Input/Output data analysis*: The use of Hard OR methods with M&S is reported by Mustafee and Bischoff [27], who combined load plan construction heuristics (cutting and packing optimisation) and ABS, with the output of the optimisation algorithm serving as the input for the simulation, and Harper et al. [16] and Harper and Mustafee [15] who used forecasting with DES to model endoscopy services.
- *Model Formalism*: Model-driven engineering and domain-specific modelling languages [58], modelling formalisms such as based on discrete-event system specification (DEVS), e.g. dynamic structure discret-event system specification (DSDEVS) [3] and meta-modelling using UML [52] are examples of deploying complementary methods from fields such as software engineering in the implementation stage of a simulation study.
- *Model Development/Implementation*: Simulation techniques such as ABS, DES and SD can be combined to implement an HS study.
- *Experimentation*: Studies that have used approaches from computer science/applied computing in the experimentation phase of an M&S study include works by Lendermann et al. [24], who use parallel and distributed simulation for high-fidelity supply chain optimisation, Park and Fishwick [40] who present a graphics processing unit (GPU)-based framework supporting fast DES, and Mustafee and Taylor [33] who developed the "WinGrid" desktop grids to execute simulations over distributed resources.

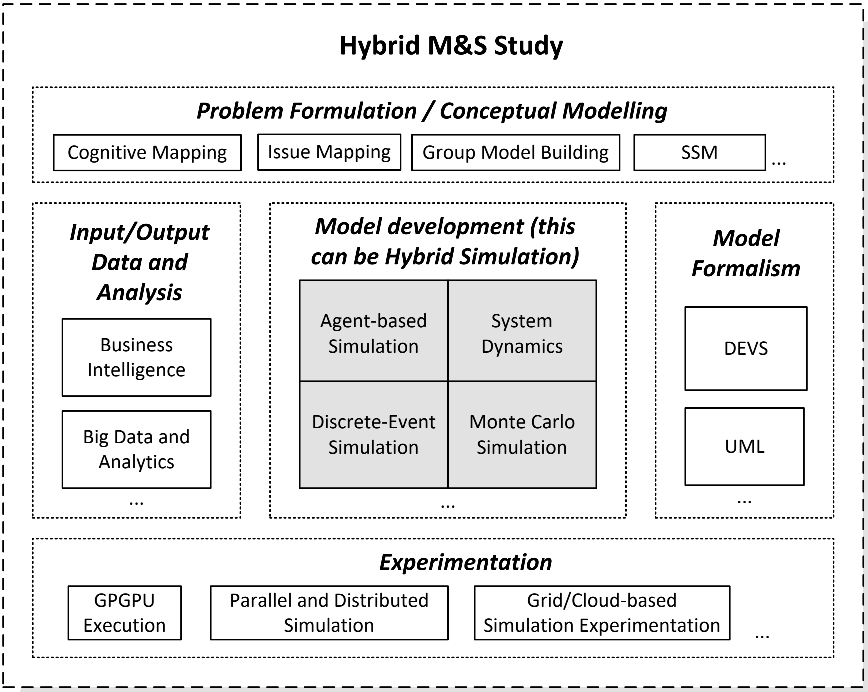

Fig. 1.1 Hybrid modelling extends hybrid simulation by deploying broader disciplinary methods and approaches to different stages of a simulation study; "…" denotes the presence of other methods [43]

Figure 1.1 presents the conceptualisation of an HM study. It presents some examples of disciplinary methods that have been used in various stages of an M&S study (white rectangles) and simulation techniques, including HS, that have been used in the model development/implementation stage (grey boxes). The three dots (…) denote the presence of other methods.

1.3 The Unified Conceptual Representation of Hybrid Simulation and Hybrid Modelling Through a Classification Scheme

The discussions on HS and HM, including Fig. 1.1, were first presented in [43]. It clarified that approaches from the wider OR (including Soft OR) and disciplines such as software engineering and applied computing, when used with M&S techniques, enable opportunities to realise synergies and engender improved insights. Mustafee and Powell [31] developed a unifying conceptual framework and a classification scheme to clarify the hybrid terminologies and to enable the exploration of complementarity between HS and HM. Although the framework furthered the discussion on hybrid from HS to HM, a critique is that the definition of HM mostly considered methods from the broader OR discipline (e.g. analytical modelling, game-theoretic modelling, forecasting). Mustafee et al. [30] added to the original classification by explicitly referring to cross-disciplinary methods and research approaches and the opportunities to combine them with M&S techniques. The extension is consistent with the intention of the authors of the original work who note that "*A classification scheme also has the benefit of being extensible, thus allowing the accommodation of new types of hybrid models that may be realised in future*" [43]. The work by Mustafee et al. [30] thus transitioned the original OR-focused classification of HM in Mustafee and Powell [43] to a classification scheme with disciplinary intersections with M&S, which was how the authors (*ibid.*) originally conceptualised the term HM.

In developing the original classification scheme, Mustafee and Powell [31] used the definitions of paradigm, methodology, technique and tool from Mingers and Brocklesby [25] and adapted them for the hybrid context. They identified five types of hybrid models, namely, Types A, B, C, D and D.1 (Type D.1 is a sub-type of Type D). Mustafee et al. [30] extended this original scheme to Define Type E hybrid models. The authors acknowledged that the definition could be open to multiple interpretations. However, the aim was to provide consistency in using the terms rather than seek a consensus.

The definitional parameters are presented next, followed by a discussion on the various model types defined in the classification scheme.

- **Paradigm:** Paradigms are a "*very general set of philosophical assumptions that define the nature of possible research and interventions*" [25]. Qualitative (interpretive, subjective, soft) and quantitative (positivist, objective, hard) paradigms are well understood. Computer simulations are computational models used for experimentation; they belong to the quantitative paradigm. Models may also be qualitative. As the classification was developed from the standpoint of M&S and models for decision-making, the authors restricted the discussion on paradigms to those relevant to OR.
- **Methodology:** Methodologies develop within a paradigm and usually embody philosophical assumptions [25]. In Mustafee and Powell [31], the authors mainly distinguish between discrete and continuous methodologies in the quantitative paradigm. In the qualitative paradigm, methodologies include soft systems methodology (SSM) and qualitative system dynamics (QSD).
- **Technique:** Techniques exist within the context of methodologies and have well-defined purposes [25]. Mustafee and Powell [31] distinguish between discrete techniques, such as DES and ABS, and continuous techniques, such as stock and flow models developed in SD and numerical computational fluid dynamics (CFD) models.
- **Tool:** Based on Mingers and Brocklesby's [25] definition of tools as an artefact that performs a particular technique, Mustafee and Powell [31] considered

tools as M&S packages and software libraries. Tools are not considered in this classification.

The unified conceptual representation of hybrid models and their classification places the HS and HM literature in context (Fig. 1.2). The classification consists of three forms of HS (Type A, B, C) and three forms of HM (Type D, D.1 and E), respectively; hybrid study serves as an umbrella term for both HS and HM models/studies. The functional definitions of Type A-E are presented next.

- **Type A—Multi-methodology hybrid simulation**: In the quantitative paradigm, Mustafee and Powell [31] distinguish between discrete methodologies, where the system state changes in discrete timesteps, and those which are continuous, where the system state changes continuously based on underlying differential equations. They state that a *multi-methodology HS* comprises simulation techniques with discrete and continuous elements, e.g. SD-DES and SD-ABS.
- **Type B—Multi-technique hybrid simulation**: Mustafee and Powell [31] state that a *multi-technique HS* uses two or more techniques under the same methodology. An example could be a CFD-SD study that uses CFD to model traffic flow and SD to investigate strategic policy related to urban transportation. The combined application of ABS-DES is also Type B HS since the integrated

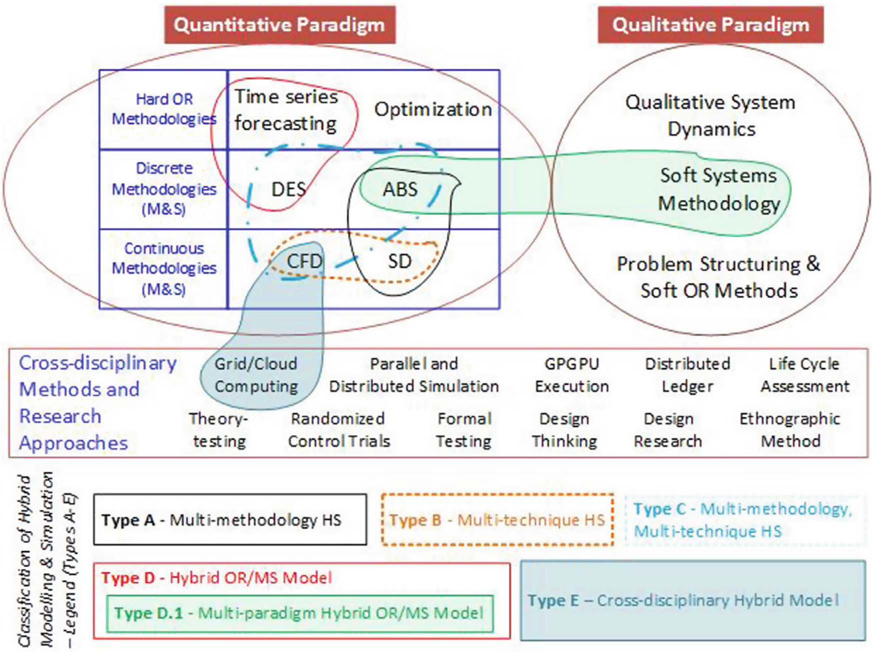

Fig. 1.2 Unified conceptual representation of hybrid M&S with a classification of distinct HS and HM model types [30]. Note that the techniques identified in the figure are not exhaustive; for example, there are numerous OR methods that can be included under Hard OR methodologies

approach combines emergence modelling using ABS with a technique based on queuing theory (DES).

- **Type C—Multi-methodology, multi-technique hybrid simulation**: Multi-methodology, multi-technique HS consist of at least three techniques classified under discrete and continuous methodologies, with the combined mix of techniques spanning both methodologies. Studies that combine the SD technique (continuous) with DES and ABS (both of which are discrete methodologies) fit this definition.
- **Type D—Hybrid operations research/management science (OR/MS) model:** These are HMs that combine M&S with techniques used in the field of OR/MS. An example of this is the combined application of analytical modelling with simulation. The reader is referred to Mustafee and Katsaliaki [28] for OR/MS techniques that are most widely used in the literature.
- **Type D.1—Multi-paradigm hybrid OR/MS model:** Computer simulations are computational models aligned to the quantitative paradigm. However, several studies have identified the use of qualitative/Soft OR approaches in the conceptual modelling phase of an M&S study, e.g. the use of SSM [21, 23]. Type D.1 models are thus a sub-type of Type D and, like the latter, focus on identifying synergies between M&S techniques and methods employed in the wider OR/MS discipline. Mustafee and Powell [31] state that Type D.1 models intersect paradigms! Finally, Type D and D.1 are referred to as HM (rather than HS) since the simulation is only one element of the hybrid model; the other element is from either Soft or Hard OR.
- **Type E—Cross-disciplinary hybrid models:** Mustafee et al. [30] define the Type E *model as one that* combines simulation (SD, DES, ABS, or an HS) with cross-disciplinary techniques from fields such as arts and humanities, economics, computer science/applied computing and systems engineering. *Type E* models go beyond the use of simulation with OR/MS in multiple stages of an M&S study (as is the case with *Model Type D, D.1*). As the realisation of the Type E model generally requires modelling expertise that goes beyond only M&S and OR/MS, it is important that researchers in our field engage with scholars from other scientific disciplines. For example, cloud-based execution of CFD simulations uses theoretical constructs to model agent relationships (e.g. theory of planned behaviour). The cross-disciplinary HMs can thus be considered as enablers of multi-, inter- and transdisciplinary research [51]. In the remainder of the chapter, we use the term HM to refer to both hybrid OR/MS models (Type D, D.1) and cross-disciplinary hybrid models (Type E), unless we need to specifically refer to either type, and in which case we will revert to the original type definitions.

1.4 Mapping Existing Literature to the Classification Scheme for HS and HM

The section builds on the HS-HM classification (Fig. 1.2) by presenting examples of existing hybrid studies and mapping them to distinct model types (i.e. Model Types A, B, C, D, D.1 and E). Mustafee et al. [29] note that although the hybrid classification can help develop a frame for mixing simulation with cross-disciplinary methods in multiple M&S study stages, researchers may continue to experience a gap in translating their conceptual understanding into the development of an empirical HM study. A reflection of the literature on existing hybrid studies (including those that may not have used the term HM but were, in essence, mixing simulation with a wider plethora of non-M&S techniques) and mapping them to the HS-HM classification scheme may help address the gap. Towards this, we use examples presented in Mustafee et al. [29, 30].

1.4.1 Type A Multi-methodology Hybrid Simulation

In OR/MS, Type A HS typically focuses on the combined use of DES or ABS (both discrete methods) with SD as the continuous method. In engineering, computational fluid dynamics (CFD) is widely used to model fluid flow and is a continuous simulation method. Thus, Type A models can be characterised as those that combine techniques from both discrete and continuous methodologies. In the classification scheme, these combinations of techniques are identified as sub-types of the Type A model. Table 1.1 lists examples of studies that could be mapped to specific sub-types. The classification is extensible, and further sub-types may be identified.

1.4.2 Type B Multi-technique Hybrid Simulation

Type B HS employ two or more techniques from either the discrete or the continuous methodology, for example, an HS using DES-ABS. However, there is debate about whether combining two discrete techniques qualifies as a hybrid. In our classification, the combined application of DES-ABM is defined as a sub-type of Type B HS since there are fundamental differences in the execution of the simulation logic, which makes them agreeable to model systems that benefit from adopting both a queuing and an emergence-based approach. Similarly, an HS that combines SD-CFD is identified as a sub-type of Type B HS. Table 1.2 presents some examples. Further sub-types of the Type B model can be identified from the literature.

Table 1.1 Examples of Type A—Multi-methodology HS

Type A model sub-type	Description with emphasis on the use of M&S methods and application area	References
ABS-SD	The authors develop an integrated ABS-SD model to understand behavioural diversities associated with multi-type labourers in multinational projects, revealing the associated impacts and improving project management. ABS was used to model the behaviour of the labourers and estimate their performance, with the SD model using this data to summarise these individual performances and evaluate the deviation in the timelines of the project (**Construction Planning**)	[55]
ABS-SD	An HS was developed to estimate the market share evolution of electric vehicles. Agent-based discrete choice models of consumer choice and awareness were combined with macro-level SD elements that model the interdependencies between consumer choice, technology evolution and available infrastructure for electric vehicles (**Transportation**)	[22]
DES-SD	The authors develop an HS to analyse “schedule risk” in infrastructural projects. DES modelled the construction processes, resource usage and other micro variables, with SD representing the feedback associated with work allocation, rework, etc. and provided a systems perspective (**Construction Planning**)	[56]
DES-SD	The authors investigate total productive maintenance using SD-DES HS. The problem being modelled involved both maintenance scheduling (DES) and considerations for human factors such as attitude (SD) (**Maintenance**)	[36]
ABS-CFD	To demonstrate the feasibility of a hybrid approach for evacuation planning, the authors model the hypothetical case of toxic aerosol release in downtown Los Angeles (using CFD), and simulate the response of a large spatially distributed agent population (ABS) (**Evacuation Planning**)	[12]
DES-CFD	The authors propose an HS consisting of a DES that models the flow of materials through a production line (manufacturing system simulation) with a CFD simulation of a compressed air system. This enables the combined evaluation of the aforementioned systems, with the overall objective of optimising energy consumption per unit of production (**Manufacturing**)	[35]

1.4.3 Type C Multi-methodology, Multi-technique Hybrid Simulation

Type C HS has elements of both Type A (multi-methodology) and Type B (multi-technique). In our classification, Type C multi-methodology, multi-technique HS represent the combined application of three or more simulation techniques, of which at least two techniques employ either continuous or discrete methodologies. Table 1.3 lists two sub-types of Type C, namely DES-ABM-SD and DES-ABM-CFD, however,

Table 1.2 Examples of Type B—Multi-technique HS

Type B model sub-types	Description with emphasis on the use of M&S methods and application area	References
SD-CFD	The authors developed a SD model to simulate vehicle movements with different traffic volumes and a CFD model to simulate the dispersion of pollutants. The objective of the study was to investigate the effects of traffic volume and toll collection methods on the dispersion of pollutants at a toll plaza **(Transportation)**	[17]
DES-ABS	The authors present a case study based on the London Emergency Medical Service where the DES and ABS elements model the hospital processes and first responders/ambulances, respectively **(Healthcare)**	[2]
DES-ABS	The authors implement a Type B hybrid ABS-DES model for the planning of capacity and patient flow in a post-term pregnancy outpatient clinic. The DES models the processes through the clinic, and the ABM models pregnant women as agents **(Healthcare)**	[53]

other sub-types can be identified in the literature. As the classification of hybrid M&S is extensible, it deviates from the definition of HS presented in Brailsford et al. [6] and is restricted to the use of particular combinations of DES, ABM and SD. As the classification presented in Mustafee et al. [30] has cross-disciplinary elements, such extension was necessary in order to incorporate a wider array of simulation techniques, for example, computational fluid dynamics (CFD) is a numerical simulation technique widely used in engineering. Table 1.3 presents examples of DES-ABM-CFD and DES-ABM-SD sub-types. Similar to Type A and Type B, other sub-types of Type C HS may be defined in future.

1.4.4 Type D Hybrid OR/MS Models

Type D HM combines M&S techniques with Hard OR approaches such as forecasting, analytical modelling, mathematical programming and optimisation, metaheuristics, game theory, graph theory, inventory models, multiple-criteria decision-making (MCDM), data envelopment analysis (DEA), process mining and machine learning. The classification can, therefore, include numerous sub-types of Type D models, each identifying a particular combination of M&S and Hard OR methods. Table 1.4 lists examples of a few sub-types of Type D OR/MS HMs.

Table 1.3 Examples of Type C—Multi-methodology, multi-technique HS

Type C model sub-types	Description with emphasis on the use of M&S methods and application area	References
DES-ABS-CFD	The authors developed an HS for evaluating countermeasures for chemical gas emergencies. The gas flow dynamics are modelled in CFD, human movement in ABS and an evacuation model in DEVS (**Evacuation Planning**)	[46]
DES-ABS-SD	The authors combined two discrete methods (DES and ABS) and one continuous method (SD) and applied them to a case study on earthmoving operations. The DES models the process flow of the earthmoving operation; the trucks and drivers are modelled as agents; and SD was used to model agents' physiological processes and decision behaviours (**Construction Planning**)	[13]
DES-ABS-SD	The authors developed an integrated DES-ABS-SD model to complement the standard lifecycle assessment (LCA) methodology. They validated the model using a case study of drink products (e.g. bottled water). SD was used to model the lifecycle of each beverage (e.g. bottled water production and recycled bottles), distribution and energy use; customer behaviour was modelled in ABS. Although the authors claim to have used two discrete methods, the hybrid model has no inherent queuing structures (**Environment**)	[54]
DES-ABS-SD	The authors developed a Type C HS for the assessment of innovative healthcare technologies, namely to evaluate mobile stroke units and prostate cancer screening. DES was used to represent hospital processes, and agents were generated from the SD component of the hybrid model (**Healthcare**)	[7]
DES-ABS-SD	The authors developed an HS for energy efficiency analysis, using SD to model the energy demand of production processes and DES/ABS to map the material flows and logistic processes applied to the mechanical processing of die-cast parts. DES provided meso-level workflow perspective, and ABS modelled micro-level active processes (**Manufacturing**)	[45]

1.4.5 Type D.1 Multi-paradigm Hybrid OR/MS Models

Type D.1 is a multi-paradigm HM (refer to the definition of paradigm in Sect. 1.3) that combines computer simulation with Soft OR techniques such as soft systems methodology (SSM), qualitative system dynamics (QSD) and cognitive mapping. Type D.1 bridges the qualitative and quantitative paradigm and should not be seen merely as a sub-set of the Type D model. Table 1.5 presents examples of Type D.1 models.

Table 1.4 Examples of Type D Hybrid OR/MS Models employing Hard OR methods

Type D model sub-types	Description with emphasis on the use of M&S methods and application area	References
Forecasting with DES	The authors used demographic projections and regression analysis to forecast demand for diagnostic services and used this as inputs into a DES to support long-term capacity planning (**Healthcare**)	[15]
Optimal packing problem with ABS	The authors developed an HM to analyse trade-offs between loading efficiency (using container Loading optimisation algorithms) and various important considerations in relation to the cargo, such as its stability, fragility or possible cross-contamination between different types of items over time (ABS) (**Transportation**)	[27]
Optimal coverage problem with ABS	The authors combine ABS and optimisation model to find the location of wireless sensors that maximises security coverage. The use of ABS is innovative as it allows them to evaluate scenarios in which intruders are intelligent, i.e. they can learn from others (**Security**)	[19]
Process mining with DES	The authors integrated process mining in the conceptual modelling phase of an M&S study to support the development of DES models (**Healthcare**)	[1]
Machine learning with DES	The authors investigated an HM approach that integrates simulation modelling with Machine Learning in an attempt to improve the validity of the simulation model outputs (**Healthcare**)	[10]

1.4.6 Type E Cross-Disciplinary Hybrid Models

Distinct from Model Types D and Type D.1, which mainly focus on using simulation with broader OR/MS methods, Type E HM necessitates cross-disciplinary engagement. From the perspective of our research community, exploration of the extant knowledge in disciplines such as engineering and computer science, data science, arts and humanities, medicine and health sciences allow us to identify established research philosophies, methods, techniques and tools, and which could be deployed in conjunction with computer simulation in one or more stages of a M&S study [30]. Table 1.6 presents examples of Type E models.