

Lecture Notes on Data Engineering
and Communications Technologies 207

Bing-Yuan Cao
Shu-Feng Wang
Seyed Hadi Nasser
Yu-Bin Zhong *Editors*



Intelligent Systems and Computing

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Lecture Notes on Data Engineering and Communications Technologies

207

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Congratulations

Abstract. When this meeting was held during the world COVID-19 epidemic, people increasingly felt the importance of science and knowledge and the value of scientists. The author, on behalf of the Iranian Operations Research Society, warmly congratulates the convening of this conference.

Keywords: COVID-19; four sessions and one celebration; congratulates

Ladies and gentlemen:

Good morning! Today we have four meetings as follows, the 10th International Conference on Fuzzy Information and Engineering, the 2nd International Conference of Operations Research and Management, the 6th Annual Meeting of Guangdong Provincial Operations Research Society, the 3rd Annual Meeting of Guangdong, Hong Kong and Macau Operations Research Society, and the 10th Anniversary Celebration of the Guangdong Operations Research Society which are held with the efforts of dear colleagues, institutions, universities, and scientists in China.

I would like to express my warm congratulations on the “four sessions and one celebration!”

As we know, in recent years, the world’s conditions have progressed with the spread of the Corona epidemic virus in such a way that it can be said, except for the scientific approach and the help of scientists, the survival of mankind was in danger.

Therefore, the importance of science and knowledge and the value of scientists became more and more felt. Such events will undoubtedly play a basic and key role in the present and in building a better future.

On behalf of the Iranian Operations Research Association, I would like to express my gratitude to all the planners, secretaries of the scientific and executive committees, associations, institutions, and universities that cooperate and support, and especially to my dear, valuable, and hard-working friend and colleague Professor Cao. I am hoping to hold an in-person event with the health and happiness of all friends and colleagues.

Sincerely yours

*Elected as President of the Iranian Operations Research Society in May 2023.

December 2022

Seyed Hadi Nasser
Vice-president of Iranian
Operations Research Society,
Head of Research Center
of Optimization and Logistics

Preface

This international conference was held online on December 28, 2022, jointly organized by associations, including the 10th International Fuzzy Information and Engineering Association (10th ICFIE preparation), the 2nd International Operations Research and Management Association (2nd ICORG preparation), the 6th Guangdong Provincial Operations Research Society (6th GDORS), the 3rd China Guangdong–Hong Kong–Macao Operations Research Society (3rd CGHMORS), and the 10th Anniversary Celebration for the Founding of the Guangdong Provincial Operations Research Society (CTAGDORS) (also known as the “Four Meetings and One Celebration”). Representatives attended the meeting from countries and regions such as China, Canada, France, the UK, Iran, Pakistan, Malaysia, and Hong Kong.

The 12th Academic Annual Meeting of the Fuzzy Information and Engineering Branch of the Chinese Operations Research Society (FIEBORSC) was held online on September 17, 2022, with representatives and research institutions attending the meeting from 83 higher education institutions and from all over the country.

More than 500 representatives attended the two conferences online. In order to document this great historical event, a collection of Lecture Notes on Data Engineering and Communications Technologies papers was published by the world-renowned publisher Springer Nature.

The opening ceremony of the “Four Meetings and One Celebration” International Conference was presided over by Associate Professor Wang Peihua, and the President of the Conference, Professor Cao Bingyuan delivered a welcoming speech. Professor Witold Pedrycz, a Canadian academician and member of the ICFIE Association, delivered a congratulatory message. Due to the epidemic, Zhang Jingzhong, Honorary Chairman of the Guangdong Provincial Operations Research Association and Academician of the Chinese Academy of Sciences, presented a written speech with written congratulation sent by Professor Reza Tavakkoli Moghaddam and Professor S. Hadi, the Chairman and Vice Chairman of the Iranian Operations Research Society, the pioneer and pioneer of fuzzy mathematics in China, Professor Wang Peizhuang from Beijing Normal University, and President Zhang Qiu, the local chairman.

The key speech for the conference was given by Professor Witold Pedrycz, a Canadian academician, Professor Li Zhongfei from Southern University of Science and Technology, and Professor Hao Zhifeng, the President of Shantou University. Following came their report topics

Federated Learning: A Perspective of Granular Computing (Witold Pedrycz)

Prediction of China Mutual Fund Returns Based on Machine Learning (Li Zhongfei)

On the New Thoughts of Metaverse and Operational Optimization (Hao Zhifeng).

The celebration was presided over by Fan Suohai, Vice Chairman and Secretary General of Guangdong Provincial Institute of Operations Research, and Professor Cao Bingyuan, Chairman gave a work report. Professor Wang Shufeng, Vice Chairman and

President of the Logistics Branch, and Hu Boping, Chairman of the Disabled Maker Research Branch, respectively, spoke on behalf of the two branches. Representative of Chairman Fang Bin, a French overseas Chinese entrepreneur, and Dr. Shang Benin from the UK; Chairman Li Xiaofeng of Shenzhen Non-Rabbit Health Technology Co., Ltd. delivered congratulatory speeches on behalf of the business community. During the ceremony, the main achievements and outstanding contributions from Zhang Jingzhong and Cao Bingyuan, the meritorious members of the Guangdong Provincial Institute of Operations Research were showcased with a plaque for the meritorious members awarded. The meeting awarded the prize certificate for the first Science and Technology Excellent Paper by the Guangdong Provincial Institute of Operations Research, separately to Associate Professor Hu Yaohua from Shenzhen University and Professor Yang Xiaopeng from Hanshan Normal University, respectively. FIEBORSC is chaired by Professor Zhong Yubin, Vice Chairman and Secretary General of the Fuzzy Information and Engineering Branch of the China Operations Research Society. General Liu Zengliang, Chairman of the Society, delivered the annual work report of the Society. The key paper presenters of the conference are renowned experts in the field of fuzzy information and engineering in China, pioneers and pioneers of fuzzy mathematics in China, and Professor Wang Peizhuang from Beijing Normal University. His report title is Factor Space for 40 Years; the title of General/Professor Liu Zengliang's report from Beijing National Defense University is Theory and Application of Factor Neural Networks. The conference speakers also included Professor Xu Zeshui from Sichuan University; Professor Yuan Xuehai from Dalian University of Technology; Professor He Qing, Institute of Computing, Chinese Academy of Sciences; Professor Guo Sicong from Liaoning University of Engineering and Technology, and Professor Yu Fusheng from Beijing Normal University; Professor Zhong Yubin from Guangzhou University; Professor Chen Shuishui from Jimei University; and Professor Li Taifu from Chongqing University of Science and Technology. 62 academic papers were exchanged in writing, including the theory and application of fuzzy mathematics, fuzzy information, and engineering, reflecting some new phased research achievements of the society in the theory and application of fuzzy information and engineering. Experts and scholars attending the meeting unanimously believe that the 12th FIEBORSC Academic Annual Conference will play an important role in promoting research on fuzzy mathematics and system theory and applications in China.

This book includes 32 high-quality research papers from the two sessions. Covering topics in the fields of certainty, stochastic uncertainty, and fuzzy uncertainty, especially the accessible language communication achievements of Chen Yongwen and others, the world's first; Professor Wang Shufeng's digital logistics was first proposed in China; Dr. Zuo Peijun's key to prevention and treatment of COVID-19 is forward-looking and highly praised. Finally, strong support for world-renowned publishing houses Springer Natural and Tsinghua University IEEE International Magazine Fuzzy Information and Engineering; The unremitting efforts of the International Fourth Conference and a Celebration, the FIEBORSC Organizing Committee, and the strong assistance from the domestic emerging enterprises in their original era, we sincerely appreciate the high importance attached by Guangzhou University. In particular, all the organizers of the

two sessions and all the presenting delegates were fearless of the danger of COVID-19 positive, and their selfless dedication and strong support for the conference will be forever in history.

Bing-Yuan Cao
Yu-Bin Zhong

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Hadi Nasser, Iran
Hao Zhifeng, China

General Assembly I: The 10th International Conference on Fuzzy Information and Engineering (ICFIE'2022).

The 2nd International Conference of Operations Research and Management (ICORG'2022).

The 6th Annual Meeting of Guangdong Provincial Operations Research Society.

The 3rd Annual Meeting of China, Guangdong, Hong Kong and Macau Operations Research Society.

Also the 10th Anniversary Celebration of Guangdong Operational Research Society (GDORS).

December 28, 2022

<https://www.csaeep.com/#/meeting/>

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About the Editors

Bing-Yuan Cao is a professor at Guangzhou University. Doctoral (postdoctoral) supervisor; Lingnan Second Class Chair Professor at Foshan University of Science and Technology; President of the Fuzzy Information and Engineering (FIE) Branch of the China Operations Research Society, the Guangdong Provincial Operations Research Society, and the Guangdong–Hong Kong–Macao Operations Research Society; Chairman of the International FIE Association (soon to be established) and lifelong editor in chief of the FIE magazine at Tsinghua University. In 1987, he was the first to propose “Fuzzy Geometric Programming” and published over 190 papers (many in Top and Q1 journals). He was also the editor in chief of over ten Springer books, received two International FIE Society Achievement Awards, three Provincial Science and Technology Awards, one Springer Most Popular Textbook by Readers, and ranked 5th in the 2021 Optimization Top 100 by Elsevier SciVal.

Prof. Shu-Feng Wang is Director of the Pearl River Delta Regional Logistics Research Center, Guangdong Baiyun University, Professor of Logistics Economics, and Master Supervisor. His main research interests include transportation economics, logistics economics, digital economics, and international trade. In the past five years, he has focused on the evolution and system integration of digital logistics theory, the efficiency of regional logistics resource allocation, the systematic reconstruction of the industrial value chain in the Guangdong–Hong Kong–Macao Greater Bay Area, the generation mechanism of systematic enterprise operation capability and its cultivation research.

He has presided over and completed more than 20 scientific research projects including provincial and ministerial projects and published five academic works and teaching materials such as “Regional Logistics Theory and Empirical Research,” “Transportation Management,” “Logistics System Planning and Design Theory and Method,” and more than 50 academic papers.

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**Annual Meeting of Fuzzy Information
and Engineering**



Variable Universe Fuzzy Control Based on Adaptive Error Integral for Uncertain Nonlinear Systems with Time-Delay

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Abstract. This article deals with variable universe fuzzy control (VUFC) based on adaptive error integral for uncertain nonlinear systems with time delay. A novel fuzzy control strategy is firstly addressed for the system to guarantee asymptotic stability. The strategy concludes in three modules: first, introduced an extended state observer to observe the extended state variables and estimate the unknown total disturbance and the unmodeled dynamics of the controlled plant; second, employ a tracking differentiator to track the dynamic characteristics of the input signal as quickly as possible in the strategy; and thirdly, develop a variable universe fuzzy control for the external-loop system with the observer and the differentiator. Compared with the existing results, the proposed strategy relaxes the restrictions on the conventional domain by the self-regulation of the tracking error domain during the operation of the system. Furthermore, to enhance the control accuracy of the controller, we define an extended adaptive error integral which can not only remain superior to the traditional error integral but also overcome the inferior. In addition, the boundedness of all signals of the presented systems is analyzed and proved by the Lyapunov theory. In the end, several simulation examples and some comparisons are presented to highlight the novelty and feasibility of the proposed method.

Keywords: Variable Universe Fuzzy Controller · Extended State Observer · Tracking Differentiator · Extraction-Expansion Factor · Extended Adaptive Error Integral

1 Introduction

Due to its complexity and diversity, the phenomenon of lag (time delay) always exists in the motion law of the systems. Therefore, the research work on time delay has attracted extensive attention from scholars and experts in the field of control, including system stability analysis, Smith predictor, internal model control, adaptive control, fault-tolerant control, fuzzy control, etc. [1–8]. Among the methods, due to the strong robustness, the influence of disturbance and parameter variation on the control effect is greatly weakened, and fuzzy control is especially suitable for the control of nonlinear, time-varying,

and pure lag systems, which has been widely studied by scholars. But the performance of the early fuzzy controller is not high enough for the time delay system. A fuzzy robust tracking control controller for uncertain nonlinear time-delay systems is proposed, which ensures ideal tracking performance in the sense that all closed-loop signals are consistent and ultimately bounded [9]. A continuous-time fuzzy compensation control method is proposed to eliminate the uncertainty of the system with time delay [10]. A robust fuzzy predictive control scheme with interval delay is proposed, and experimental results show that the scheme has high control accuracy [11].

As we all know, the control rules are the core of the fuzzy controller, whether it's correct or not directly affects the performance of the controller, and the number of its is also an important factor to weigh the performance of the controller. However, the design of a fuzzy controller is highly dependent on expert experience, and the performance of the control system will be greatly reduced when the existing experience is deviated or even wrong [12]. In addition, the accuracy of traditional fuzzy control is limited by fuzzy division. If the number of fuzzy partitions is increased to improve the control accuracy, the steady state error will be increased and the reliability of the controller will be reduced. To overcome the shortcomings of the conventional fuzzy control mentioned above, Li proposed variable universe fuzzy control (VUFC) and pointed out that the discourse universe of fuzzy variables can be extract or expand under the condition of reducing or increasing systematic errors [13]. To some extent, it solves the problem of over-reliance on expert experience, and can precisely strengthen the control of certain target areas. The ability of local target densification control makes the variable universe fuzzy method widely studied by scholars, and successfully applied in robot control [14], neural control engineering [15], etc. However, the input of variable universe fuzzy controller usually uses the difference (system error) between the input signal and the output value and the difference (system error change rate) between the derivative of the input signal and the output value. When the input signal and output value contain higher noise, the error and error change rate will interact with each other. If the extraction-expansion factor of the input variable of VUFC is simply chosen as a function of error or error change rate, the input universe will be adjusted repeatedly and the convergence speed of the system will be reduced. In the current related studies, the control methods of contract-expansion factors are not uniform: [16–21] mathematical function, integral regulation, multi-population genetic algorithm, fuzzy neural network, and backpropagation algorithm are respectively used to describe and optimize the gain of contract-expansion factors. Observer-based variable theory fuzzy controller [22, 23] is proposed for chaotic systems and unknown dead zone nonlinear systems respectively. These control strategies improve the control accuracy of the system to a certain extent, but the influence between error and error change rate has not been completely solved, especially when the input signal contains higher noise, the problem of extracting the input differential signal has not been solved. In addition, the study of VUAF for uncertain nonlinear systems with time delay is very little.

In this paper, a novel variable universe fuzzy controller is proposed for uncertain nonlinear systems with time delay. That is, based on introducing the extended state observer and tracking differentiator [24], an extended adaptive error integral is designed to reduce the influence of approximation error and time delay on tracking error. This is

because both the extended state observer and the tracking differentiator are constructed using the error approximation principle. The innovations of this paper are as follows:

- A nonlinear extended state observer driven by error is introduced to observe the extended state variables and estimate the unknown total disturbance and the unmodeled part of the controlled system. This observer does not depend on the exact model object and is more practical in engineering.
- An error-driven tracking differentiator is introduced to track the dynamic characteristics of the input signal as fast as possible to obtain approximate external differential signals and smooth signal.
- A variable universe fuzzy control method based on extended adaptive error integration is proposed to ensure the stability of the system. The method is suitable for complex time-delay systems, and an extended error integral is defined by using the advantages of error integral, stretching factor, and variable domain, which greatly improves the control accuracy of the proposed controller.
- The superiority of the controller is verified by simulation and comparison tests.

2 Methodology Development

2.1 Model of Uncertain Nonlinear Systems with Time Delay

The equation of uncertain nonlinear system with time delay [25] is described as:

$$\begin{cases} \dot{\mathbf{x}} = \hat{f}(t - \tau, d, \mathbf{x}, \mathbf{u}) \\ y = \bar{g}(\mathbf{x}, \mathbf{u}, t) \end{cases} \quad (1)$$

where τ , y , d , \mathbf{x} and \mathbf{u} are time delay, input, state variable, external disturbance and output of system respectively. $\hat{f}(\mathbf{x}, \mathbf{u}, t - \tau, d)$ is the state with respect to the related variables, $\bar{g}(\mathbf{x}, \mathbf{u}, t)$ is output functions with respect to the related variables.

The control objective of this paper is to find a fuzzy tracking controller such that the state of the system (1) follows a given stable reference model while keeping all closed-loop signals ultimately bounded all the time.

2.2 Estimation of the System

In uncertain nonlinear systems, a key challenge is the estimation state. As a special case of uncertain nonlinear systems, the state estimation of uncertain nonlinear systems with time delay is also very important. In this study, the system state can be expanded from the original n -dimensional state $x_0 = [x_1, x_2, \dots, x_n]$ to an $n + 1$ -dimensional vector $x = [x_1, x_2, \dots, x_n, x_{n+1}]$. The basic principle of the ESO algorithm is shown in a) and has been described in detail in previous studies [24, 26, 27], and [28]. The ESO algorithm can successfully estimate the normal and perturbed system states and compensate the estimated states and total perturbations, which provides an opportunity for the efficient design of VUFC strategy.

1) Coordinate Transformation of the Original System

A border nonlinear time-varying dynamic system is with the following form:

$$y^{(n)}(t) = g\left(y^{(n-1)}(t), y^{(n-2)}(t), \dots, y(t), w(t)\right) + b(x, t)u(t) \quad (2)$$

where $y(t)$ (simply y), $u(t)$ (simply u), $w(t)$ (simply w) and $b(x, t)$ (simply b) are the output, input, external disturbance and high-frequency gain of the dynamic system respectively. Let b_0 be the estimated value of b , $f(t) = g\left(y^{(n-1)}, y^{(n-2)}, \dots, y, w\right) + (b - b_0)u$ be the so-called “total disturbance”. Then, Eq. (3) is transformed into Eq. (3)

$$y^{(n)} = f\left(y^{(n-1)}, y^{(n-2)}, \dots, y, w\right) + b_0u \quad (3)$$

Expand $f(t)$ into a new state of system and assume that $f(t)$ is differentiable, let $\dot{f}(t) = x_{n+1}(t)$. The system in Eq. (3) can be described in a state space Eq. (4)

$$\begin{cases} \dot{x}_i = x_{i+1}(t) & i = 1, \dots, n-1 \\ \dot{x}_n(t) = g\left(y^{(n-1)}, y^{(n-2)}, \dots, y, w\right) + bu \\ y = x_1(t) \end{cases} \quad (4)$$

2) Extended State Observer

A To better estimate the state of Eq. (4), the nonlinear extended state observer (NESO) with y and u as inputs is designed as:

$$\begin{cases} e(t) = z_1(t) - y(t) \text{ or simply } e = z_1 - y \\ z_1(t+1) = z_1(t) + h(z_2(t) - \beta_{01}e) \\ z_2(t+1) = z_2(t) + h(z_3(t) - \beta_{02}fe_1) \\ \vdots \\ z_n(t+1) = z_n(t) + h(z_{n+1}(t) - \beta_{0n}fe_{n-1} + b_0u(t)) \\ z_{n+1}(t+1) = z_{n+1}(t) + h(-\beta_{0n+1}fe_{n-1}) \\ fe_{i-1} = fal(e, \alpha_{i-1}, \delta) \\ fal = \begin{cases} e\delta^{\alpha-1} & |e| \leq \delta \\ |e|^\alpha \text{sign}(e) & |e| > \delta \end{cases} \end{cases} \quad (5)$$

When the observer gain β_{0i} is adjusted properly, $z_i(t)$ can respectively tracks $x_i(t)$ ($i = 1, 2, \dots, n+1$). Choose the following control rate to compensate for the estimated total disturbance $f(t)$:

$$u(t) = \frac{u_0(t) - z_{n+1}(t)}{b_0} \quad (6)$$

where $u_0(t)$ will be analyzed later. Equation (6) is substituted into Eq. (3) to get Eq. (7).

$$y^{(n)}(t) = f(t) - z_{n+1}(t) + u_0(t) \approx u_0(t) \quad (7)$$

Then, Eq. (7) means that the model in Eq. (3) becomes an integral series object after estimating and compensating the total disturbance $f(t)$ through the NESO $u_0(t)$.

Because of the complexity of the actual environment, for the input signal accompanied by random noise, the differential method is used directly to obtain its differential will amplify the noise. In order to obtain the smooth input signal and its differential signal, a tracking differentiator is introduced in this paper.

2.3 Acquisition of Smooth Input Signal and Its Differential Signal

The function $fhan$ is to obtain a smooth control trajectory by tracking-differentiator (TD). The algorithm of TD as is follows:

$$\left\{ \begin{array}{l} fh(t) = fhan(v_1(t) - v_0(t), v_n(t), r(t), \hat{h}_0(t)) \\ v_i(t+1) = v_i(t) + \hat{h}v_{i+1}(t) \quad i = 1, 2, \dots, n-1 \\ v_n(t+1) = v_n(t) + \hat{h}fh(t) \\ \left. \begin{array}{l} d = \hat{h}_0 r \\ d_0 = \hat{h}_0 d \\ v(t) = v_1(t) - v_0(t) + \hat{h}_0 v_n(t) \\ a_0 = \sqrt{d^2 + 8r|v(t)|} \\ fhan : \quad a(t) = \begin{cases} v_n(t) + v(t)/\hat{h} & |v(t)| \leq d_0 \\ v_n(t) + \frac{1}{2(a_0-d)\text{sign}(v(t))} & |v(t)| > d_0 \end{cases} \\ sat = \begin{cases} \frac{a(t)}{d} & |a(t)| \leq d \\ \text{sign}(a) & |a(t)| > d \end{cases} \\ fh = -rsat(a(t), d) \end{array} \right. \end{array} \right. \quad (8)$$

where $v_0(t)$ is the input trajectory, $v_1(t)$ is the tracking trajectory of the input trajectory, $v_i(t)$ ($i = 1, 2, \dots, n$) is the differential trajectory of the tracking trajectory, r is the speed factor, \hat{h} is the sampling period, \hat{h}_0 is the filter factor, $fhan$ is the fastest tracking function.

The smooth tracking signal and differential signal are further used in VUFC strategy, which is beneficial to solve the contradiction between VUFC overshoot and rapidity. The design process is as follows.

2.4 A Novel Variable Universe Adaptive Fuzzy Controller

It is well known that fuzzy control is suitable for control systems that are hard to model. And the design of a fuzzy controller depends mostly on the fuzzy inference rule base on the knowledge of field experts. In addition, the stability of fuzzy controller has been analyzed in Refs. [29, 30]; and has been widely used such as [31]. However, traditional fuzzy controller is not ideal for high-precision control situations. Therefore, variable universe adaptive fuzzy control is proposed in [32], which can effectively control nonlinear systems. The basic principle of a variable universe adaptive fuzzy controller is briefly stated in this section (Refer to [33] for details)

1) Traditional Fuzzy Controller

Fuzzy controller combines the error (e_i) between the differential signal obtained by TD and the state estimation value estimated by NESO. The $e_i(t)$ is

$$e_i = v_i - z_i \quad (i = 1, \dots, n) \quad (9)$$

According to Eq. (9), the input state error of fuzzy controller is $e^T = (e_1, e_2, \dots, e_n)$. Let $E_i = [-I_i, I_i]$ be the universe of input variable e_i , $Y = [-Q, Q]$ be the universe of output variable u , $\mathcal{A}_i = \{A_{ij}\}$ be a fuzzy partition on \mathcal{X}_i , and $\mathcal{B} = \{B_j\}$ be a fuzzy partition on \mathcal{Y} . Each element of \mathcal{A}_i is considered a linguistic variable value of e_i , and each element of \mathcal{B} is considered a linguistic variable value of u ($i = 1, 2, \dots, n; j = 1, 2, \dots, m$). Suppose that we have m fuzzy rules, each of which is in the following form:

$$\text{If } e_1 \text{ is } \mathcal{A}_{j1}, e_2 \text{ is } \mathcal{A}_{j2}, \dots, e_n \text{ is } \mathcal{A}_{jn}, \text{ then } y_j \text{ is } B_j \quad (10)$$

Let e_{ji} be the peak point of \mathcal{A}_{ji} , and u_j be the peak point of B_j , ($i = 1, 2, \dots, n; j = 1, 2, \dots, m$), [13, 33, 34]. According to [33], the fuzzy logic system (i.e., the fuzzy controller) based on Eq. (10) can be expressed as

$$y(e) = F(e) \triangleq \sum_{j=1}^m \prod_{i=1}^n \mathcal{A}_{ij}(e_i) y_j \quad (11)$$

2) Contraction-Expansion Factors of Universes

The contraction-expansion factor of the universe of the fuzzy controller with n inputs and one output is described as follows.

A *contraction-expansion factor* on $\mathcal{X} = [-E, E]$ is a function $\xi: \mathcal{X} \rightarrow [0, 1]$, $e \mapsto \xi(e)$, which satisfies the following conditions:

- i. Evenness: $\xi(e) = \xi(-e)$;
- ii. Near zero: $\xi(0) = \varepsilon$, where ε is a small positive real constant;
- iii. monotonicity: strictly monotonically increasing on $[0, E]$;
- iv. compatibility: $|e| \leq \xi(e)E$.

For any $e \in \mathcal{X}$, $\mathcal{X}(e) \triangleq \xi(e)\mathcal{X} \triangleq [-\xi(e)E, \xi(e)E] \triangleq \{\xi(e)e' | e' \in \mathcal{X}\}$ is called a variable universe of $[-E, E]$. \mathcal{X} is the initial universe of the variable universes whose situation changing is shown in Fig. 1.

Two typical contraction-expansion factors of universe $[-E, E]$ are

$$\xi(e) = (|e|/E)^\tau + \varepsilon, \quad \tau > 0 \quad (12)$$

$$\xi(e) = 1 - \lambda \exp(-ke^2), \quad \lambda \in (0, 1), \quad k > 0 \quad (13)$$

where ε is an arbitrary positive constant.

3) Variable Universe Adaptive Fuzzy Controller

Suppose that $\xi_i(e_i)$ and $\beta(u)$ are the contraction-expansion factors of the initial universes

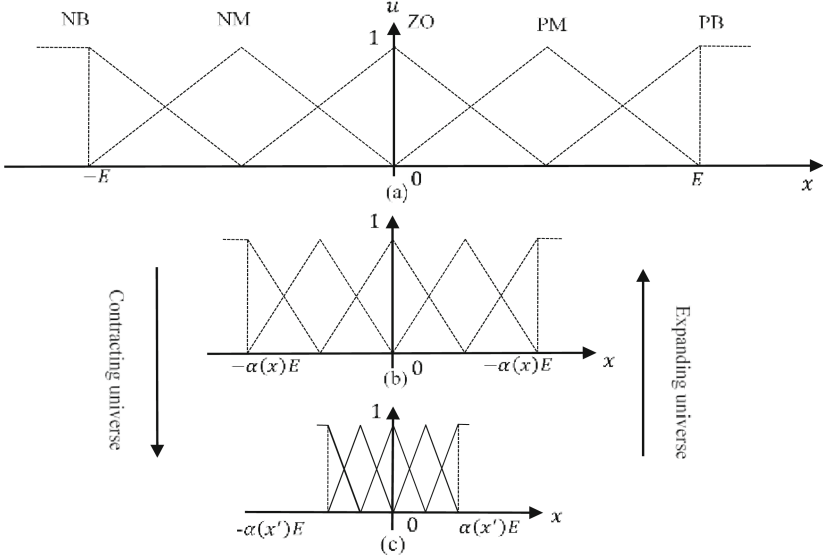


Fig. 1. Principle of a variable universe: (a) initial universe and its fuzzy portion; (b) & (c): contracting/expanding.

of \mathcal{X}_i and Y respectively. By means of the conclusions in Ref. [33], a variable universe adaptive fuzzy controller based on Eq. (11) can be written as follows:

$$y(e(t+1)) = \beta(t) \sum_{j=1}^m \prod_{i=1}^n A_{ij} \left(\frac{e_i(t)}{\xi_i(e_i(t))} \right) y_j \quad (14)$$

where $e(t) \triangleq (e_1(t), e_2(t), \dots, e_n(t))^T$, $\xi_i(e_i(t)) = 1 - \lambda \exp(-ke_i(t)^2)$, $\beta(t) = 1 - \lambda \exp(-ku(e(t))^2)$.

For convenience, we write

$$\omega(e(t)) \triangleq \sum_{j=1}^m \prod_{i=1}^n A_{ij} \left(\frac{e_i(t)}{\xi_i(e_i(t))} \right) y_j \quad (15)$$

Then Eq. (14) is expressed as

$$y(e(t+1)) = \beta(t)\omega(e(t)) \quad (16)$$

Compare with linear active disturbance rejective control (LADRC), the nonlinear active disturbance rejective control (NADRC) with strong noise suppression and robustness has higher control performance for multivariable and strongly coupled nonlinear systems [35]. In recent years, the LADRC and NADRC method has been widely applied and achieved a well effects [36]. But the nonlinear PD in NADRC is not the best choice. As well known, a defect of the PID controller is a stronger dependency on the model and a smaller dynamic adjustment range. To overcome the defect, in this section, we propose a novel variable universe adaptive fuzzy control which is designed by employing a variable universe adaptive fuzzy controller that does not depend on the model and improves

the accuracy of control. Furthermore, to improve the control accuracy of the proposed control strategy, we will define an extended adaptive error integral in following section.

3 The Asymptotically Stable of the Variable Universe Adaptive Fuzzy Analyzed by Lyapunov Theory

3.1 Feasibility Analysis of the Proposed Method

In this section, we first set up the control objectives, and then discuss how to develop a novel variable universe adaptive fuzzy to achieve these control objectives.

According to [37, 38], if the state vector $x = (x_1, x_2, \dots, x_n)^T = (y, \dot{y}, \dots, y^{(n-1)})^T \in R^n$ is available measurement in the systems (1), the systems have model information accurately. The control objective is to force y to follow a given bounded reference trajectory, $v_0(t)$, under the constraint that all trajectories involved must be bounded. However, the principle of the proposed control strategy is to use the error drive to stabilize the system, that is, use the output $z_i(t)$ of the NESO to track $x_i(t)$ ($i = 1, 2, \dots, n+1$), use $v_i(t)$ to track $z_i(t)$ ($i = 1, 2, \dots, n$) and $v_1(t)$ to track the input signal $v_0(t)$. In this way, the system keeps circulating the diagnostic error indirectly forces y to track given bounded reference signal, $v_0(t)$. Therefore, we only need to analyze the stability of the error feedback controller, namely just needs to analyze the stability of the variable universe fuzzy controller. Specially, we have

Control Objectives

Determine the parameter of feedback control $u(x)$ meet the following conditions:

- i) In the sense, the closed-loop system must be globally stable, that is, all variables, $v(t)$, $x(t)$, $z(t)$ and $u_0(x)$, must be uniformly bounded; i.e., $|v_i(t)| \leq M_{v_i} < \infty$, $|x_i(t)| \leq M_{x_i} < \infty$, $|z_i(t)| \leq M_{z_i} < \infty$ ($i = 1, 2, \dots, n$) and $|u_0(x)| \leq M_u < \infty$ for all $t \geq 0$, where M_{v_i} , M_{x_i} , M_{z_i} and M_u are given in advance.
- ii) Under the constraint of i), the estimated tracking error $e(t) \equiv v(t) - z(t)$, should be as small as possible.

Now, we analyze how to construct an adaptive fuzzy controller $u_d(x)$ to realize these control objectives.

First of all, let $e(t) = (v_1(t) - z_1(t), v_2(t) - z_2(t), \dots, v_n(t) - z_n(t))^T = (e_1(t), e_2(t), \dots, e_n(t))^T$ and $k = (k_n, k_{n-1}, \dots, k_1)^T \in R^n$ be such that all roots of the polynomial $h(s) = s^n + k_1 s^{n-1} + \dots + k_n$ are in the open left-half plane. Under the function g and b are known, then the control law

$$\bar{u}(t) = 1/b \left[-g(t) + v_{n+1}(t) + k^T e(t) \right] \quad (17)$$

is applied to Eq. (2) resulting in

$$e_{n+1}(t) + k_1 e_n(t) + \dots + k_n e_1(t) = 0 \quad (18)$$

which shows that $\lim_{t \rightarrow \infty} e(t) = 0$ is a key objective of control. On account of $g(t)$ and b are unknown, the optimal control, $\bar{u}(t)$ cannot be realized. Our purpose is to design a variable universe fuzzy system extremely close to this optimal control.

Supposed that the control u is summation of an adaptive fuzzy control $u_a(x)$ and an adjustable control $u_b(x)$:

$$u = u_0(x) + u_b(x) \quad (19)$$

where $u_a(x)$ is a variable universe fuzzy system in the form of (16), and $u_b(x)$ will be discussed later in this section. If we put the Eq. (19) into the Eq. (3), we have

$$x^{(n)} = f(x) + b[u_a(x) + u_b(x)] \quad (20)$$

Now add the substructure $b\bar{u}$ to Eq. (20) and after some direct operations, the error equation governing the closed-loop system is

$$e_n(t) = -k^T e + [\bar{u}(t) - u_a(x) - u_b(x)] \quad (21)$$

or, equivalently

$$\dot{e} = E_c e + b_c [u^* - u_c(x) - u_s(x)] \quad (22)$$

where

$$E_c = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ -k_n & k_{n-1} & \cdots & \cdots & \cdots & \cdots & -k_1 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b \end{bmatrix} \quad (23)$$

Define $V_e = \frac{1}{2} e^T P e$, where P is a symmetric positive definite matrix satisfying the Lyapunov equation [39].

$$E_c^T P + P E_c = -Q \quad (24)$$

where $Q > 0$. Using Eq. (24) and the error equation Eq. (22), we have

$$\begin{aligned} \dot{V}_e &= -\frac{1}{2} e^T Q e + e^T P b_1 [\bar{u}(t) - u_a(x) - u_b(x)] \\ &\leq -\frac{1}{2} e^T Q e + \left| e^T P b_1 \right| (|\bar{u}(t)| + |u_a|) - e^T P b_1 u_s \end{aligned} \quad (25)$$

Our task now is to design u_b such that $\dot{V}_e \leq 0$. To realize $\dot{V}_e \leq 0$, we need the following assumption:

Assumption: We can define a function $\bar{f}(x)$ and a constant \bar{b} such that $|f(x)| \leq \bar{f}(x)$ and $0 < \bar{b} \leq b$.

We construct the compensator $u_b(x)$ as follows [39]:

$$u_b(x) = I_1^* \text{sgn}\left(e^T P b_1\right) \left[|u_a| + \frac{1}{\bar{b}} \left(\bar{f}(x) + |v_{n+1}| + |k^T e| \right) \right] \quad (26)$$

Where $I_1^* = 1$ if $V_e > \bar{V}$ (\bar{V} is a constant specified according to the needed), and $I_1^* = 0$, if $V_e \leq \bar{V}$. Because $b > 0$, $\text{sgn}(e^T P b_1)$ can be determined; thus, the adjustable control, u_b , of Eq. (26) can be achieved. Put Eq. (26) and Eq. (17) into Eq. (25) and considering the $I_1^* = 1$ case, we have

$$\begin{aligned} \dot{V}_e = & -\frac{1}{2} e^T Q e + \left| e^T P b_1 \right| \left[\frac{1}{\bar{b}} \left(|f| + |v_{n+1}| + |k^T e| \right) + |u_a| \right. \\ & \left. - |u_a| - \frac{1}{\bar{b}} \left(\bar{f}(x) + |v_{n+1}| + |k^T e| \right) \right] \leq -\frac{1}{2} e^T Q e \leq 0 \end{aligned} \quad (27)$$

For the same reason, $I_1^* = -1$, the conclusion is also valid. Therefore, using the adjustable control u_b of Eq. (26), we always have $V_e < \bar{V}$. Because $P > 0$, the boundedness of V_e heralds the boundedness of x .

Equation (26) implies that the u_b is nonzero only when the error function V_e is greater than the positive constant \bar{V} . Nevertheless, largely control is undesirable because it may increase the implementation cost. So, we do not choose this strategy because the u_b is usually very large. But, if the system tends to be unstable, then the compensator u_b begins to force $V_e < \bar{V}$. But the strategy must manual adjustment, which brings a lot of inconvenience to the application of the controller.

Next, we replace the $u_a(x) + u_b(x)$ by the variable universe fuzzy system (18) multiplied by a special adaptive error integral.

3.2 Variable Universe Fuzzy Controller Based on Extended Adaptive Error Integral

Many uncertain disturbance factors exist in an actual system, if a disturbance is added at a certain moment, the static error of the system will increase rapidly and the stability of the system will deteriorate. Meanwhile, the accumulation of error integral over time makes the output change of the controller increase and the system becomes sluggish. From the feasibility analysis of the variable universe adaptive fuzzy in Sect. 3.1, we see that it is necessary to add a compensator when the change of the control quantity is very large. But a compensator is added will increase the difficulty of controller design. In this section, *an extended adaptive error integral is proposed*, that is, taking the range of error integral $[-I, I]$ (Eq. (27)) and the output range of the fuzzy controller $[-P, P]$ (Eq. (28)) as Cartesian product will give universe $[-R, R]$, and name $[-R, R]$ as the range of the extended error integral. Then the range of extended error integral multiplied by the extraction-expansion factor defined on $[-R, R]$ gives the extended state error integral

with variable range which named as *extended adaptive error integral*. The extended adaptive error integral not only limits the change of the control quantity, eliminates state error, and enhances the disturbance rejection of the systems, but also will not increase the difficulty of controller design.

The detailed design process of the error feedback controller of an extended adaptive error integral is divided into four steps as follows:

Step 1 *Extended contraction-expansion factor*

The error integral is calculated as:

$$\bar{\beta}(t) = \bar{\beta}\left(e^T(t)\right) = K_I \int_0^t e^T(\tau) d\tau + \bar{\beta}(0) \quad (28)$$

Let

$$[-I, I] = \{\bar{\beta}(t) | t < +\infty\} \quad (29)$$

be the range of all error integrals;

Let

$$[-P, P] = \{\omega\left(e^T(t)\right) | t < +\infty\} \quad (30)$$

is the output range of the variable universe fuzzy controller.

An *extended contraction-expansion factor* on $X = [-R, R] = [-I, I] \times [-P, P]$ is a function $\rho: X \rightarrow [0, 1]$, $x \mapsto \rho(x)$ which satisfies the following conditions:

- i. Evenness: $\rho(x) = \rho(-x)$;
- ii. monotonicity: strictly monotonically increasing on $[0, R]$;
- iii. compatibility: $|x| \leq \rho(x)E$.

where $[-I, I] = \{\bar{\beta}(t) | t < +\infty\}$, $[-P, P] = \{\omega\left(e^T(t)\right) | t < +\infty\}$.

A practical extended contraction-expansion factor is given below:

$$\rho(t) = 1 - P_I \exp\left(-\sum_{i=1}^n \lambda_i \rho_i^2 - \Upsilon(\omega(t))^2\right) \quad (31)$$

where $P_I \in (0, 1)$ and $\lambda_i, \Upsilon(\geq 0)$ are parameters given in advance ($i = 1, 2, \dots, n$).

Step 2 *Extended adaptive variable universe*

For any $x \in X$, $X(x) \triangleq \rho(x)X \triangleq [-\rho(x)R, \rho(x)R] \triangleq \{\rho(x)x' | x' \in X\}$ is called a variable universe of $[-R, R]$. X is the initial universe of the variable universe.

Step 3 *Extended adaptive error integral*

The extended adaptive error integral $\tilde{\rho}(t)$ is defined as follows:

$$\tilde{\rho}(t) \in [-\rho(t)R, \rho(t)R] \quad (32)$$

Step 4 *Design of error feedback controller of an extended adaptive error integral*

Let $\tilde{\rho}(t) = \beta(t)$, then Eq. (16) can be expressed as

$$u_0 = \tilde{\rho}(t)\omega\left(e^T(t)\right) \quad (33)$$