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Identifiability and Observability in Epidemiological Models

A Primer

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The World requires at least ten years to understand a new idea, however important or simple it may be.
Ronald Ross (1902 Nobel Prize)

Preface

In Mathematical Epidemiology, many papers have the following structure:

- A model is proposed.
- Some parameters are given, extracted from the literature.
- Remaining unknown parameters are estimated by fitting the model to some observed data.

Fitting is done usually by using an optimization algorithm with the use, for example, of a least square method or a maximum likelihood estimation. To validate the estimation of parameters, one can use noisy synthetic simulated data obtained from the model for given values of the parameters, to check that the algorithm is able to reconstruct from the data the values of these parameters with accuracy.

One objective of this book is to show that this procedure is not always safe and that an examination of the identifiability of parameters is a prerequisite before a numerical determination of parameters. We will review different methods to study identifiability and observability and then consider the problem of numerical identifiability. Our touchstone will be the most famous, but simple, model in Mathematical Epidemiology, the SIR model of Kermack and Mckendrick [73]. This model received renewed attention with the COVID-19 pandemic [106]. Parameter identifiability analysis addresses the problem of which unknown parameters of an ODE model can uniquely be recovered from observed data. We will show that, even for very simple models, identifiability is far from being guaranteed.

The problem of identifiability for epidemiological models is relatively rarely addressed. For instance, a search in the Mathematical Reviews of the American Mathematical Society[1] for 2020 with `epid* AND identifiability` gives only 4 papers, while `epidem* AND parameter` returns 68 publications. Only a small subset of the later publications addresses the problem of identifiability. In particular, the following publications consider the problem of identifiability

[1] https://mathscinet.ams.org/mathscinet.

in epidemiological models: [19, 33, 50, 51, 68, 72, 81, 87, 91, 99, 107, 119–121, 130, 132]. However, the majority of these papers were published elsewhere than in Biomathematics journals. Note that we make a distinction between publications that address directly the parameter estimation problem in epidemiological models (such as in references [13, 20, 21, 27–29, 37, 56, 57, 63, 101, 113, 132] for instance) and works that study explicitly the identifiability property of models. As explained in this book, this is an intrinsic property to be studied prior to determination of parameters values.

The question of observability, i.e. the ability to reconstruct state variables of the model from measurements, is often considered separately from the problem of identifiability. Either model parameters are known, or an identifiability analysis is performed prior to the study of observability. Indeed, the concepts of identifiability and observability are closely related, as we show in this book. However, for certain models, it is possible to reconstruct state variables with observers, while the model is not identifiable. In other situations, we show that considering jointly identifiability and observability with observers can be a way to solve the identifiability problem. This is another illustration of the utility of the concept of observers. This is why we shall dedicate a fair part of this monograph to reviewing the concept of observers and their practical constructions in epidemiology.

This book is aimed at scientists, researchers and graduate students, who use or develop mathematical models for epidemiology, and who are not yet familiar with the concepts of control science (detectability, observability, observers) applied to this field.

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