

Mathematicians Don't Work With Numbers

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For my grandchildren Isaac and Reagan

Preface*

Once I spent a month in a physical rehab facility. Upon learning I'm a mathematician one attendant said, "You must be good with numbers."

While opening a new account, the bank officer curiously asked what kind of work I was retired from. Hearing I'm a mathematician she nodded sagely and said, "A numbers man."

Arghhh!

No! The title of this book states a truth. Mathematicians don't work with numbers!

This book tells what mathematics is really about and its broad relevance to our everyday world. It does not teach mathematics per se. There are too many such books already.

One early reviewer constructively criticized a certain item in this book as oversimplified. Of course, it is! This entire book is an oversimplification of mathematics. That is the only way to present its ideas to a public that has never seen math except in arrays of equations. No one wants that.

If it is true that mathematicians don't work with numbers, and it is indeed true, what do they work with and how

is it relevant to others? I believe this is the first book to ask and answer those questions while having no equations, at least if you stick to the starred (*) sections. These form an acceptable subset to answer the two questions posed.

Each section is a mathematical “vignette” describing one aspect of mathematics. A few vignettes do use numbers, but you will see they are incidental to the principles involved. Each vignette is independent of the others with few exceptions so you can pick and choose. They are not in order of difficulty, and none of the more difficult vignettes are starred.

My personal favorite is Public Key Encryption I.

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I thank my former colleague who is also a good friend, Maria Agoston, for reading a near final manuscript and making many useful comments for greater clarity to the intended audience.

Introduction*

By now many know that NASA mathematician Katherine Johnson, the lead character in the movie *Hidden Figures*, worked with numbers. Except she didn't because that is not what mathematicians do.

Mathematicians use numbers the way novelists use letters. Most time authoring a novel is spent typing letters, but this is not the essence of what it is to be a novelist. The individual letters are typed while the author thinks on a different scale, a grander scale invisible to the observer. So with mathematicians.

Another real-life mathematician was Mark Kac, recruited to work on the Manhattan Project, the secret WWII project that developed the atom bomb. Everyone on the Manhattan Project had to reduce their theoretical results to numbers, for otherwise a machinist would not know what to make nor how big to make it. No theoretical methods here, numbers were paramount.

Kac captured this sense of mathematicians figuratively getting their hands dirty by humorously commenting that he was reduced to working with numbers and worse, some had decimal points! The horror, the shame!—understood only by a fellow mathematician.

The Manhattan Project was a special case and that is not what mathematicians normally do. They avoid numbers like the plague.

In the end, you will have a better appreciation for the work of Katherine Johnson, Mark Kac, and others, an appreciation for what schools didn't teach you and how it's all relevant.

Contents

A Citizen's Dilemma*	1
The Structure of Mathematics*	3
A Danger Scale*	5
Public Key Encryption I*	9
Public Key Encryption II*	13
Fractals	15
Graphs*	19
Military Math*	23
Misusing Statistics: A Rant*	29
Estimation*	33
Ramanujan	35
Hardy	39
It's Obvious*	43
Chaos*	45
Recognition	47
Map Coloring	51

Groups*	55
Topology I*	59
Topology II*	63
Nonexistence Proofs	67
Existence Proofs	71
Can't Be Computed	75
Can't Be Proved	79
Games*	83
X-Ray Vision*	87
The Greatest Three	91
Being Choosy*	95
To Infinity and Beyond*	99
The Largest Hotel Ever*	103
Tied Up in Knots*	105
Probably	107
Rush Hour Traffic*	111
Fermat and His Last Theorem	115
A Million Bucks*	119
Russia Versus America	123
Cardano, Viète, and Notation	127
President James Garfield	129
Error-Correcting Codes*	133
Mercator Maps*	137
Ball and Saddle Geometries	139
The Spherical Earth	143
Consistency	145

Can So Be Proved	149
My Face Is Complex, Yours Is Imaginary	153
Cycloids	157
C***** of Variations*	161
Amazing Waves*	165
How to Push a Pendulum*	169
Surprising Theorems	171
Mathematical Aesthetics*	175
Katherine Johnson’s Math*	179
My Career as a Mathematician*	183
Karen’s New Perspective*	187
Afterword*	189
Doing Metric Right	191
Movies About Real Mathematicians*	193
Further Reading*	195



A Citizen's Dilemma*

Karen is a young woman of about 30. She received acceptably good grades in high school math courses but finds those course contents to be of no use in her profession as a conference organizer. Her friends feel the same way, and they include a commercial real estate agent, a coffee bean buyer, a graphic artist, a pharmacist, a hospice administrator, and a retail store owner. Any math beyond arithmetic just doesn't seem relevant to this circle of friends.

Karen is concerned about issues of task scheduling for every conference she is hired to organize, but surely that has nothing to do with math. She also wonders what strategy to use to best price her services given the pricing and different services offered by her competition but picking a strategy (as opposed to picking a price) seems unrelated to math.

Of course, as aware young citizens, Karen and her friends are concerned with what they see in the news about data security breaches, the climate, international confrontations,

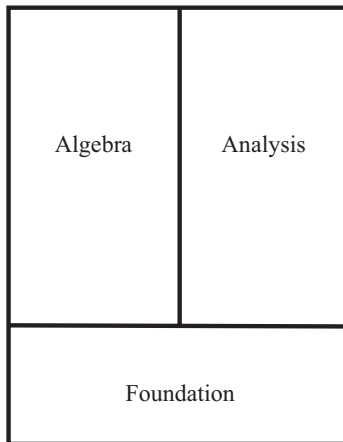
and other issues. Closer to home, some have decided to start a family and wonder about the dangers to children, which seem to have been reevaluated since they were young. Still, a citizen's basic understanding of all these matters has little to do with mathematics.

Or does it?



The Structure of Mathematics*

You may have little or no concept of any structure to the field of mathematics. My description here is a bit simplified but perhaps the mathematical world will forgive me.



The foundation of mathematics contains logic, set theory, proof theory, and several other basic components used to weave the rest of mathematics. The superstructure consists of two main parts, analysis and algebra.

Analysis is the study of the continuous. This includes trigonometry, high school algebra, calculus, and every other part of mathematics that deals with continuous phenomena.

By Algebra, I do not at all mean high school algebra. Algebra as shown in the diagram is almost everything that is not analysis. It studies the discontinuous properties of objects using tools specially designed for discontinuous phenomena. The most basic such tool is group theory about which I have written a vignette, so it's ok to find this perplexing for now.

A few branches of mathematics such as number theory (the theory of the integers) do not fit neatly into this structure. The set of integers is a discontinuous object, yet analysis is widely used in number theory.

While most vignettes are drawn from the analysis half of the superstructure, several are drawn from the foundation, and a very few are drawn from algebra.